A brief comparison of optical pathlength difference and various definitions for the interferometric phase

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Abstract. In this paper we discuss that the phase readout in low noise laser interferometers can significantly deviate from the underlying optical pathlength difference (OPD). The cross coupling of beam tilt to the interferometric phase readout is compared to the OPD. For such a system it is shown that the amount of tilt to phase readout coupling depends strongly on the involved beams and their parameters, as well as on the detector properties and the precise definition of the phase. The unique single element photodiode phase is therefore compared to three common phase definitions for quadrant diodes. It is shown that neither phase definition globally shows the least amount of cross coupling of angular jitter.

1. Introduction
A frequent use of laser interferometers is to sense distance variations. Assume a very simple and perfect Mach-Zehnder interferometer where a mirror in one beam path is rotated around the beam reflection point. For simulations and interpretation the interferometer can then be reduced to the setup shown in Fig. 1. Here, the rays labeled z_{MB} and z_p indicate the propagation axis of measurement and reference beam respectively. In this paper, we discuss this very simple setup and its resulting phase readout. We highlight the fact that the resulting phase readout can deviate significantly from what might be expected from the change in the optical path, even in absence of standard noise sources such as laser frequency noise, electronic readout noise or thermal noise. This significant deviation originates from the phase- and intensity profiles of laser beams which cannot be sufficiently represented by plane waves or rays and secondly from incomplete beam detection and beam clipping on finite photodiodes as well as on the actual definition of the interferometric phase in the case of quadrant diodes.
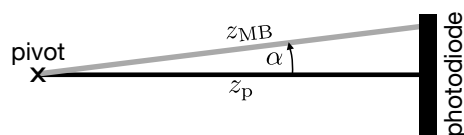

Figure 1: Effective beam paths in a simple laser interferometer, if a mirror in the measurement beam path rotates around the beam reflection point (thus coinciding with the pivot) by $\alpha/2$. For $\alpha = 0$ both beams impinge orthogonally on the center of the photodiode.

2. Interferometric phase vs optical pathlength difference (OPD)

The difference between OPD and interferometric phase can be seen for instance by looking at the tilt to length (TTL) coupling in the given simplified setup.

With no medium involved, the OPD can be computed geometrically: it is the difference between the distances z_{MB} and z_p the measurement and reference beam propagate between pivot and diode (see Fig. 1):

$$\text{OPD} = z_{MB} - z_p = \left(\frac{1}{\cos(\alpha)} - 1 \right) z_p \approx \frac{\alpha^2}{2} z_p \,. \tag{1}$$

Assuming rotation angles in the order of $100\,\mu$rad and a distance z_p of about $20\,$cm, the OPD is in the order of nanometers. For picometer interferometers such as LISA and LISA Pathfinder, it can be seen from this very simple and general example, that the coupling of tilt (i.e. angular jitter) to the OPD can be a significant effect. However, the OPD is not what is measured in these missions - or generally in interferometers, but the interferometric phase. We define here the interferometric phase as the phase that can be extracted from the photocurrent resulting from the two interfering beams on a photodiode (see [1] for a mathematical description of the interferometric phase).

To show that the interferometric phase can deviate significantly from the OPD, we will show two special cases.

Example case 1: We assume that both beams are circular fundamental Gaussian beams. Let the waist of both beams be located halfway between pivot and photodiode and the distance z_p between pivot and photodiode is chosen to be twice the Rayleigh range of the beams. The wavefronts of both beams on the detector surface can then be approximated by a section of a sphere which has its center in the pivot of the rotation. This means, that a mirror tilt maps the wavefront onto itself. For small rotations we therefore expect negligible cross coupling from tilt to the interferometric phase. We have shown previously [1] with a numerical simulation, that for this special case the coupling is well below pm-level.

Example case 2: Assume the same case as before, but let the center of rotation be an arbitrary point. If the detector is a large single element detector which does not clip either beam, and if the parameters of the interfering beams are matched, the coupling of tilt into the interferometric phase is negligible, as we will show below.

We have shown before [2, eq (34)] that the resulting phase is then $\phi \approx \alpha^2 z/(4z_0)$, where z_0 is the Rayleigh range of both beams and z the distance from their waist. In order to compare the interferometric phase to the OPD, it needs to be converted to a length, which we call the longitudinal pathlength signal $\text{LPS} = \phi/k$. For typical wavenumbers k in the order of $2\pi/(1\,\mu\text{m}) \approx 10^7$/m, tilt angles in the order up to a few hundred μrad, and assuming that Rayleigh range z_0 and distance z from the waist are of comparable order, we see that the tilt to length coupling is negligible:

$$\text{LPS} \approx \frac{\alpha^2}{k} \frac{z}{4z_0} \approx \frac{(10^{-4})^2}{10^7}\,\text{m} \ll 1\,\text{pm} \,. \tag{2}$$

In both examples the OPD lies in the order of nanometers while the phase readout is negligible. In example case 1 this is due to the very specific choice of beam parameters. This shows that the nature of the beams (e.g. circular fundamental Gaussian beams) needs to be accounted for when predicting the interferometric phase. In example case 2 the vanishing cross coupling results from the cancellation of two effects [3], which holds for arbitrary but matched fundamental Gaussian beams that are detected without clipping by a single element photodiode.

If either beam is clipped for instance by the slits of a quadrant photodiode (QPD) or because the diode is insufficiently large ($\lesssim$ 3 times spot size on the detector), a significant coupling can occur (see Fig. 2(a)). This shows that clipping effects and therefore the detector shape have significant impact on the phase readout.

Interferometric phase of a quadrant diode   Quadrant photodiodes (QPDs) are often used for phase detection in interferometers, because their use allows to locate the centroid as well as the relative angle of the incident beams[4, 5]. Any quadrant diode naturally provides not one but four photocurrents, which need to be combined in order to generate one phase signal. Consequently, there are several options to define a phase signal for quadrant diodes. One possibility to describe these signals is via the complex amplitude C of the photocurrent, which we define as the integral over the detector surface of the product of the electric field of the measurement beam and complex conjugated electric field of the reference beam and which is given by:

$$C := \int \mathrm{d}S E_m E_r^* = \bar{P} c \exp(i\phi) =: A \exp(i\phi) . \tag{3}$$

Here, $\bar{P}$ is the time averaged detected power and c the measured contrast of the interference. The complex amplitude C can be determined experimentally for instance by a single bin discrete Fourier transform, as done in LISA Pathfinder [6]. The interferometric phase ϕ is then simply the argument of the complex amplitude C. If a large single element detector is assumed, S can be set to the detector plane. If a quadrant detector is assumed, there are four finite integrals to be performed, resulting in four complex amplitudes C_n and consequently four interferometric phases ϕ_n, and four contrasts c_n (n=1,..,4). The sum of the complex amplitudes plus the hypothetical complex amplitudes of the slits would exactly be equal to the complex amplitude of a single element detector of identical diameter. Therefore, the definition of a QPD phase closest to the phase of a single element photodiode is:

$$\phi_{\mathrm{LPF}} = \arg\left(\sum_{n=1}^{4} C_n\right) = \arg\left(\sum_{n=1}^{4} A_n \exp(i\phi_n)\right) . \tag{4}$$

This phase definition was chosen in LISA Pathfinder (LPF). Alternatively the arithmetic mean of the quadrant phases ϕ_n could be computed:

$$\phi_{\mathrm{AP}} = \frac{1}{4}\sum_{n=1}^{4} \phi_n = \frac{1}{4}\arg\left(\prod_{n=1}^{4} C_n\right) . \tag{5}$$

Here, the product notation on the right hand side is a better choice for implementation, since it is more robust to phase jumps than the arithmetic mean on the left hand side.

While ϕ_{AP} might be an intuitive choice, it has clear disadvantages compared with the LPF definition: in the case of poorly centered beams the phases of all quadrants contribute equally to ϕ_{AP}, even though the phase of one or several quadrants might originate from a very low number of photons, while the phases of other quadrants might originate from strong signals. This case is naturally compensated in ϕ_{LPF}, because the summation of the complex amplitudes is effectively a vector addition.

The power imbalance problem in the averaged phase can be accounted for, by weighing the quadrant phases for instance by the amplitudes $A_n := \bar{P}_n c_n$:

$$\phi_{\mathrm{WAP}} = \frac{A_1 \phi_1 + A_2 \phi_2 + A_3 \phi_3 + A_4 \phi_4}{A_1 + A_2 + A_3 + A_4} . \tag{6}$$

(a) Example 2.0

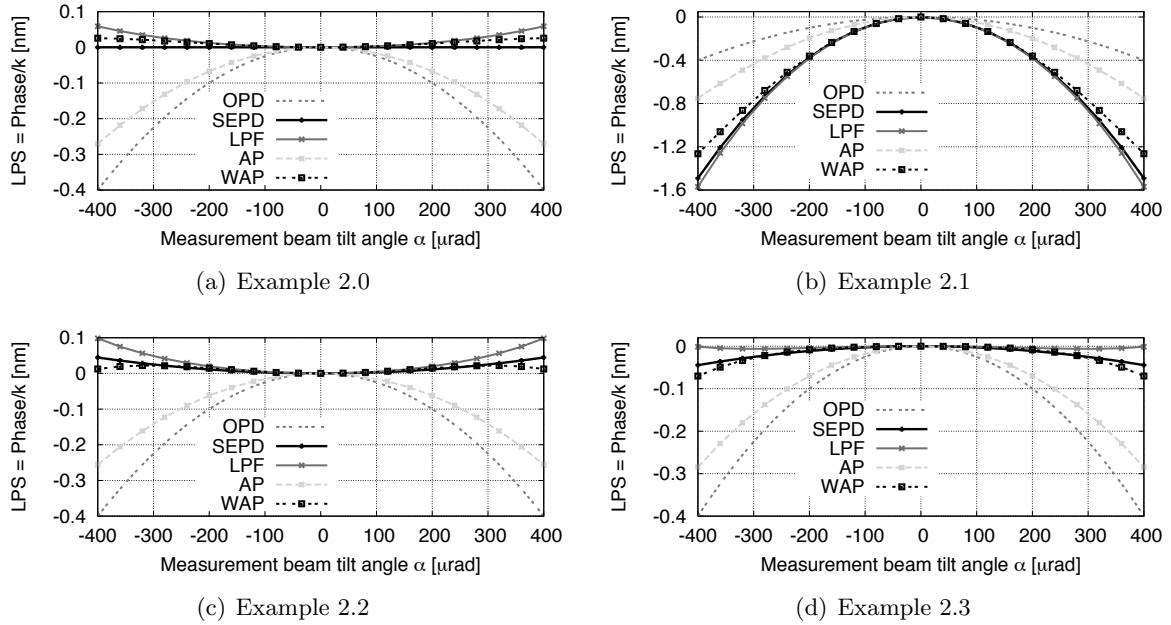(b) Example 2.1

(c) Example 2.2

(d) Example 2.3

Figure 2: Longitudinal pathlength signal (LPS) for the setup of example case 2 with the settings listed in Tab. 1. These graphs were generated with the numerical software tool IfoCAD[7]. Subfig. (a) is the matched beam parameter case described in example case 2. Subfig.(b)-(d) show the same setup for different choices of mismatched beam parameters.

Parameter description	value		Parameter description	value
diameter of circular QPD	10 mm		measurement beam power	1.2 mW
photodiode gap width	70 μm		reference beam power	0.7 mW
λ: wavelength laser beams	1064 nm		z_p: distance from pivot to photodiode	5 mm

	Ex 2.0	Ex 2.1	Ex 2.2	Ex 2.3
distance RB from waist to QPD [m]	-0.4	-0.4	0	0
distance MB from waist to QPD [m]	-0.4	-0.6	0	0
RB Rayleigh range [m]	0.5	3.137	2.5	2.0
MB Rayleigh range [m]	0.5	4.124	2.0	2.5

Table 1: Parameter settings for Figure 2. RB: reference beam, MB: measurement beam

The ϕ_{WAP} is a first order approximation of ϕ_{LPF}, since the first series expansion of the complex amplitude is $C \approx A + iA\phi$:

$$\phi_{\mathrm{LPF}} \approx \arg\left(\sum_{n=1}^{4}(A_n + iA_n\phi_n)\right) \approx \arctan\left(\frac{A_1\phi_1 + A_2\phi_2 + A_3\phi_3 + A_4\phi_4}{A_1 + A_2 + A_3 + A_4}\right) \quad (7)$$

$$\approx \frac{A_1\phi_1 + A_2\phi_2 + A_3\phi_3 + A_4\phi_4}{A_1 + A_2 + A_3 + A_4} = \phi_{\mathrm{WAP}} . \quad (8)$$

There are of course numerous further options to define the phase readout for quadrant detectors. The definitions $\phi_{\mathrm{LPF}}, \phi_{\mathrm{AP}}$ and ϕ_{WAP} are those we regard as most intuitive and physically closest to the phase readout of a single element detector. A direct comparison of these signals is shown

in Fig. 2 for variations of example case 2. These numerical simulations were generated with the software tool IfoCAD[7], the given phase definitions and the settings listed in Tab. 1. More detailed information on the IfoCAD algorithms to compute interferometric phases are given in [1, 8]. Subfig. 2(a)) shows the case of matched beam parameters. As discussed above, the LPS of a single element diode (SEPD) does not sense the measurement beam tilt, provided that the detector is sufficiently large that the beams are fully detected - which is the case in this example. However, all here defined phase signals of a quadrant diode show a significant amount of TTL cross coupling. Since we assume identically shaped SEPDs and QPDs, ϕ_{LPF} equals ϕ_{SEPD} in the hypothetical limit that the gap width goes to zero. Therefore one can say that any difference occurring between ϕ_{SEPD} and ϕ_{LPF} results fully from the clipping at the insensitive gaps between the quadrants of the QPD. Therefore, Subfig. 2(a) shows the vanishing TTL coupling on SEPDs as described above as well as the TTL cross coupling on QPDs originating from incomplete beam detection.

For mismatched beam parameters, the amount of TTL cross coupling of each phase signal depends on the exact setup and the defined beam parameters, as shown in Fig 2(b)-2(d). It can be seen that either of the three QPD LPS signals can show the least amount of TTL cross coupling. In other words, neither signal is globally an optimal definition for a phase readout. The beam parameters in the examples were chosen to highlight this fact, but are otherwise arbitrary. A detailed comparison of the phase signals is work in progress and will be discussed in a separate paper.

3. Conclusions

For high performance interferometers, such as LISA Pathfinder, and the Laser Ranging Instrument (LRI) of GRACE Follow-On, it is necessary to distinguish carefully the optical pathlength difference from the interferometric phase, since these parameters could deviate significantly, depending on beam parameters and setup. For future high performance interferometers, such as in space based gravitational wave detectors like LISA and DECIGO, and in future interferometric geodesy missions beyond GRACE Follow-On, the fact that the cross coupling of angular jitter to phase noise depends on the phase definition could be used to reduce the resulting phase noise. For this however, dedicated analyses need to be performed which also include effects such as the robustness of the phase signal to phase jumps, coupling of other optical noise types or the robustness to quadrant failure.

Acknowledgments

4. References

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