Exciton-mediated photothermal cooling in GaAs membranes

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# Exciton-mediated photothermal cooling in GaAs membranes

**André Xuereb**[1,5]**, Koji Usami**[2]**, Andreas Naesby**[2]**,
Eugene S Polzik**[2] **and Klemens Hammerer**[3,4]

[1] Centre for Theoretical Atomic, Molecular and Optical Physics, School of Mathematics and Physics, Queen's University Belfast, Belfast BT7 1NN, UK
[2] QUANTOP—Danish National Research Foundation Center for Quantum Optics, Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen, Denmark
[3] Institut für Gravitationsphysik, Leibniz Universität Hannover, Callinstraße 38, D-30167 Hannover, Germany
[4] Institut für Theoretische Physik, Leibniz Universität Hannover, Appelstraße 2, D-30167 Hannover, Germany
E-mail: andre.xuereb@qub.ac.uk

**Abstract.** Cooling of the mechanical motion of a GaAs nano-membrane using the photothermal effect mediated by excitons was recently demonstrated by some of the authors (Usami *et al* 2012 *Nature Phys.* **8** 168) and provides a clear example of the use of thermal forces to cool down mechanical motion. Here, we report on a single-free-parameter theoretical model to explain the results of this experiment which matches the experimental data remarkably well.

[5] Author to whom any correspondence should be addressed.

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## 1. Introduction and motivation

Micro- and nanomechanical systems in the quantum regime offer exciting perspectives for fundamental tests of quantum physics as well as for quantum technological applications. It is a unique feature of these systems that they can be strongly coupled to a plethora of other quantum systems: optomechanics explores interactions with light based on radiation pressure [1, 2], dipole gradient [3, 4] or photothermal forces [5–7]; electromechanics investigates the coupling of electronic and mechanical degrees of freedom [8, 9]; and magnetic forces can couple a mechanical oscillator to magnetic moments [10], even of single electrons [11]. It is this versatile nature of mechanical systems which makes them attractive as basic building blocks for hybrid quantum systems [12]. In the current paper we elaborate on the recent findings reported in [13] demonstrating the interplay of photonic and electronic degrees of freedom in a micromechanical semiconductor membrane.

The photothermal effect [5–7, 13–20] elegantly overcomes one fundamental limit encountered by any optomechanical cooling mechanism based on the radiation-pressure interaction [1, 2]. That is to say, a single photon of frequency $\omega_L$ can only provide an energy change $\Delta E \sim (v/c)\hbar\omega_L$ upon reflection off a mirror with velocity $v$, and this is due to the Doppler shift [7], but absorption of the same photon, as per the photothermal effect, implies $\Delta E \sim \hbar\omega_L$. This $\Delta E$ defines the *upper limit* of the change in motional energy that can be imparted by a single photon, but it is clear that, in principle, the photothermal effect can give rise to cooling forces that eclipse the radiation-pressure force produced by the same number of photons [13, 19].

In the semiconductor GaAs, the absorption of an above-bandgap optical photon and subsequent decay of the associated bound states [21, 22] is a complicated process; the energy liberated by the photon first creates an electron–hole bound pair (an exciton), which decays by scattering phonons throughout the structure of the material. This scattering process, which manifests itself primarily as the transport of heat and takes place over the *thermalization time* $\tau_{th}$, changes the properties of the material. The drumhead modes of a membrane are critically dependent on these properties, and this process therefore couples the absorption of the light to the motion of these modes. Our main aim in this paper is to describe a phenomenological Hamiltonian model for this process and to predict the cooling or heating effect imparted by
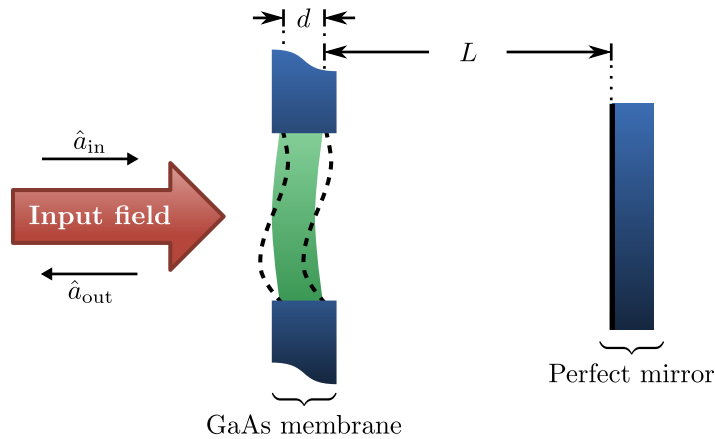
**Figure 1.** Schematic of the experiment. A Fabry–Pérot cavity is bounded by a perfect mirror on one end and a GaAs membrane on the other. Drumhead vibrations of the membrane, indicated by the dashed lines, couple to the cavity field and to the excitons inside the membrane. In the experiment, $L = 2.9\,\text{cm}$, $d = 160\,\text{nm}$, and the membrane is not perfectly flat; this latter feature is important to the mechanism we consider, as explained in section 4. The quantized input ($\hat{a}_{\text{in}}$) and output ($\hat{a}_{\text{out}}$) fields, identified below as superpositions of the free field that couples to the cavity field, are also depicted.

this photothermal interaction. We take particular care to model the details of the experimental system of [13]; we note, for example, that the coupling of the excitonic field to the continuum of input field modes is crucial to describing the experimental data.

This paper is structured as follows. In the next section we shall build our system Hamiltonian and derive the equations of motion of the field operators. Following this, we shall insert a memory kernel in these equations to account for the delay in the thermalization process. After we fit our model to the experimental data, we briefly discuss the possibility of using pure deformation-potential effects to achieve cooling in similar systems, and then conclude.

## 2. System Hamiltonian and equations of motion

Let us start by considering the physical system represented in figure 1: a Fabry–Pérot cavity is bounded by a perfect mirror on one end and a semi-transmissive GaAs membrane on the other. We shall describe the exciton fields inside the membrane by means of a bosonic approximation, which is valid when the exciton population is not too large, and assign to these fields the operators $\hat{c}_\lambda$ (frequency $\omega_\lambda$ and decoherence rate $\gamma_\lambda$), where $\lambda$ is some, possibly continuous, index labelling the modes. Any sum over $\lambda$ is to be interpreted as either a sum, if $\lambda$ is discrete, or an integral, otherwise.

These fields interact with the cavity field, $\hat{a}$ (frequency $\omega_c$ and half-width at half maximum (HWHM) linewidth $\kappa_c$), as well as with an infinity of field modes $\hat{a}_\omega$ that represent the free field that forms the input to the cavity field. The excitonic fields also interact with our chosen mechanical mode, which we describe using the operator $\hat{b}$, and which has a mechanical frequency $\omega_m$ and HWHM linewidth $\kappa_m$.
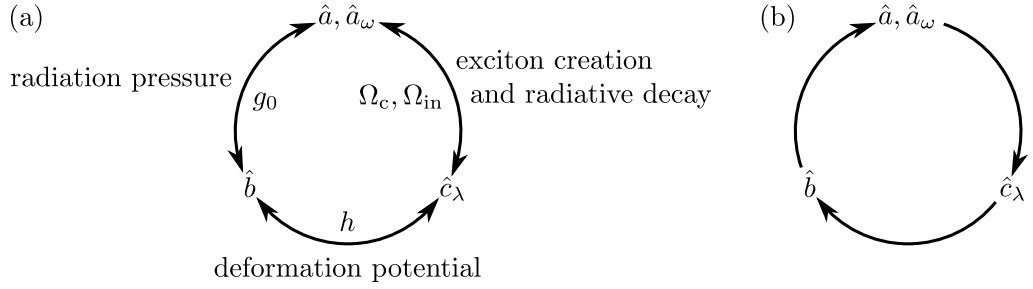
**Figure 2.** (a) The full set of interactions that our model describes; each interaction is labelled by the relevant coupling constant in the effective description of section 3. (b) Adiabatic elimination allows us to describe the system in terms of one cycle of interactions, with the mechanics modifying the optical fields, which act on the excitonic fields, which in turn couple to the mechanics. A memory kernel, used to describe a delayed interaction, will later be introduced into the segment connecting $\hat{c}_\lambda$ and $\hat{b}$.

The Hamiltonian $\hat{H}$ is made up of four different contributions. The free Hamiltonian reads (we shall take $\hbar = 1$ throughout this paper for conciseness of notation)

$$\hat{H}_{\text{free}} = \omega_c \hat{a}^\dagger \hat{a} + \int \mathrm{d}\omega\, \omega \hat{a}_\omega^\dagger \hat{a}_\omega + \sum_\lambda \omega_\lambda \hat{c}_\lambda^\dagger \hat{c}_\lambda + \omega_{\text{m}} \hat{b}^\dagger \hat{b}, \tag{1}$$

and describes the fields in the absence of any interaction or dissipation. The next set of terms describes dissipation, and reads

$$\hat{H}_{\text{diss}} = \mathrm{i}\sqrt{\frac{\kappa_c}{\pi}} \int \mathrm{d}\omega\, (\hat{a}_\omega^\dagger \hat{a} - \hat{a}_\omega \hat{a}^\dagger) + \hat{H}_{\text{mech,diss}} + \hat{H}_{\text{exc,diss}}; \tag{2}$$

we have chosen to write down explicitly only the Hamiltonian describing the cavity field decay. $\hat{H}_{\text{mech,diss}}$ ($\hat{H}_{\text{exc,diss}}$) similarly describes the dissipation of the mechanical (excitonic) operator(s). There are two sets of interaction terms, illustrated pictorially in figure 2(a); the first couples the excitons to the electric field:

$$\hat{H}_{\text{abs}} = \sum_\lambda \left( \int \mathrm{d}\omega\, \frac{\Omega_{\text{in},\lambda}}{\sqrt{\pi \kappa_c}} \hat{a}_\omega + \Omega_{c,\lambda} \hat{a} \right) \hat{c}_\lambda^\dagger + \text{h.c.}, \tag{3}$$

where $\Omega_{c,\lambda}$, which is assumed to be real without loss of generality, is the coupling rate of exciton mode $\lambda$ with the cavity field, and $\Omega_{\text{in},\lambda}$ with the free-field modes. The coupling of the exciton modes to the free field cannot be neglected in this case: the membrane is thick enough for interference effects between the input field and the cavity field to be significant, and $|\Omega_{\text{in},\lambda}/\Omega_{c,\lambda}|$ to be of order unity. The two terms in $\hat{H}_{\text{abs}}$ therefore interfere, leading to an asymmetric cooling spectrum (see section 4, below, and figure 3); this phenomenon is closely linked to the Fano line-shapes observed in optomechanical systems where the mechanical oscillator is coupled both to a cavity field and to the free field [23, 24], although the dominant coupling of the optical fields to the mechanics is indirect in the present case.

The second set of interaction terms describes the coupling of the mechanical motion to the cavity field and to the excitons [25, 26]:

$$\hat{H}_{\text{mech}} = g_0 \hat{a}^\dagger \hat{a} (\hat{b} + \hat{b}^\dagger) + \sum_{\lambda,\lambda'} h_{\lambda,\lambda'} \hat{c}_\lambda^\dagger \hat{c}_{\lambda'} (\hat{b} + \hat{b}^\dagger). \tag{4}$$
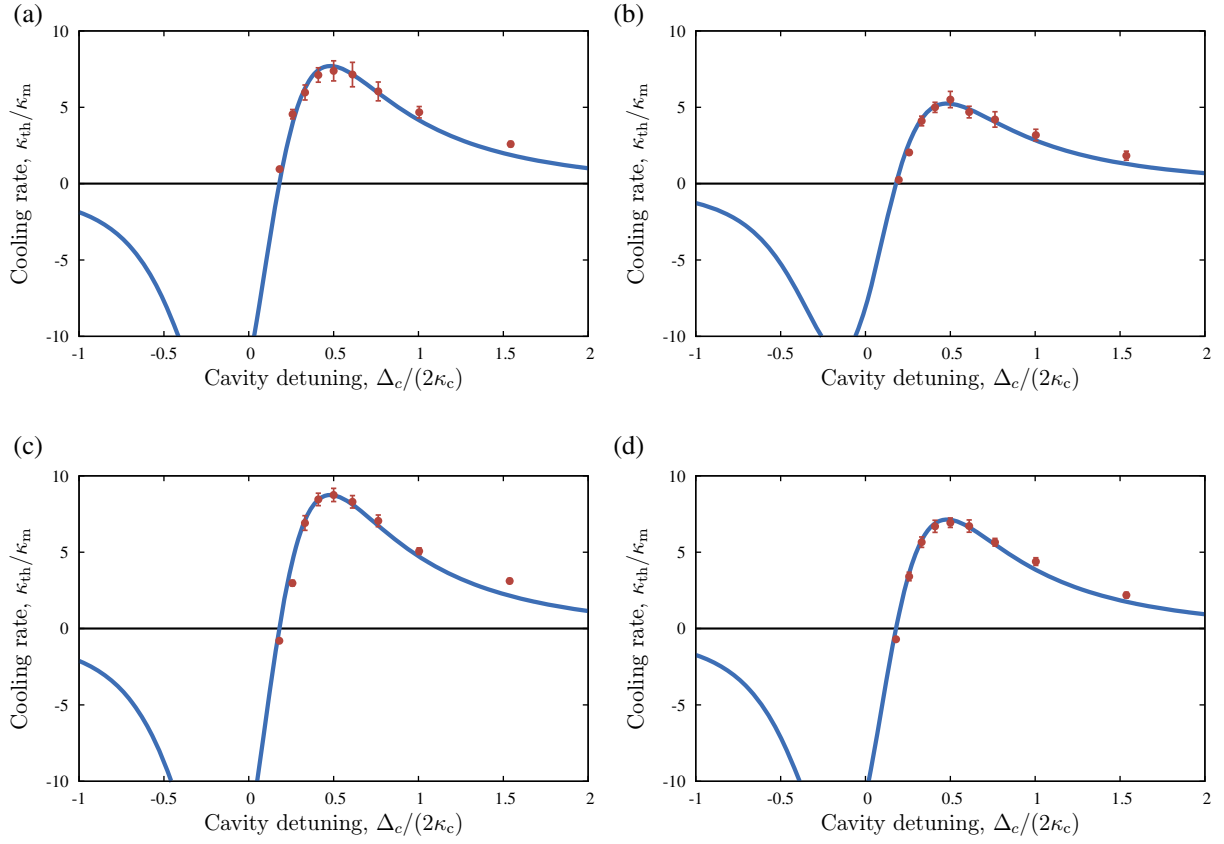
**Figure 3.** Fits to four sets of experimental data; dataset 1 is the set reported in [13]. The four sets of data differed in the location of the driving beam on the membrane and in the driving wavelength $\lambda_L$ used. All other system parameters are reported in the text and in table 1. (a) Dataset 1; $\lambda_L = 870$ nm; (b) dataset 2; $\lambda_L = 852$ nm; (c) dataset 3; $\lambda_L = 852$ nm; (d) dataset 4; $\lambda_L = 852$ nm.

$g_0$ is the usual (radiation-pressure) coupling constant and $h_{\lambda,\lambda'}$ describes the deformation-potential coupling. The photothermal force is not included in equation (4) and will be introduced at a later stage. We omit any direct coupling of the motion to the free field, since such effects would be very small [24] compared to the terms in the preceding equation. Finally, we can write

$$\hat{H} = \hat{H}_{\text{free}} + \hat{H}_{\text{diss}} + \hat{H}_{\text{abs}} + \hat{H}_{\text{mech}} . \tag{5}$$

This Hamiltonian can be used to generate the Heisenberg–Langevin equations of motion for the field operators. We choose to work in a frame rotating at the frequency of the driving field, $\omega_L$, and define the detunings $\Delta_c = \omega_L - \omega_c$ and $\Delta_\lambda = \omega_L - \omega_\lambda$, obtaining the equations of motion

$$\dot{\hat{a}} = -(\kappa_c - i\Delta_c)\hat{a} - \sqrt{2\kappa_c}\hat{a}_{\text{in}} - ig_0\hat{a}(\hat{b} + \hat{b}^\dagger) + i\sum_\lambda (\Omega_{\text{in},\lambda}^* - \Omega_{c,\lambda}^*)\hat{c}_\lambda, \tag{6}$$

$$\dot{\hat{b}} = -(\kappa_m + i\omega_m)\hat{b} - \sqrt{2\kappa_m}\hat{b}_{\text{in}} - ig_0\hat{a}^\dagger\hat{a} - i\sum_{\lambda,\lambda'} h_{\lambda,\lambda'}\hat{c}_\lambda^\dagger\hat{c}_{\lambda'} \tag{7}$$

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and

$$\dot{\hat{c}}_\lambda = -(\gamma - i\Delta_\lambda)\hat{c}_\lambda - \sqrt{2\gamma}\hat{c}_{\lambda,\text{in}} - \frac{\Omega_{\text{in},\lambda}}{\kappa_\text{c}} \sum_{\lambda'} \Omega_{\text{in},\lambda'}\hat{c}_{\lambda'} - i\sum_{\lambda'} h_{\lambda,\lambda'}\hat{c}_{\lambda'}(\hat{b} + \hat{b}^\dagger)$$

$$-i(\Omega_\text{in} + \Omega_\text{c})\hat{a} - i\sqrt{\frac{2}{\kappa_\text{c}}}\Omega_\text{in}\hat{a}_\text{in}. \tag{8}$$

The operators $\hat{c}_{\lambda,\text{in}}$ describe the zero-mean Langevin forces associated with the excitonic modes, whereas the optical field input operator $\hat{a}_\text{in}$ is defined as per the usual input–output theory (cf [27, section 5.3]), but we note that the interaction of the excitons with the free field modifies the input–output relation for the system, yielding

$$\hat{a}_\text{out} = \hat{a}_\text{in} + \sqrt{2\kappa_\text{c}}\hat{a} - i\sqrt{\frac{2}{\kappa_\text{c}}}\Omega_\text{in}^\star \sum_\lambda \hat{c}_\lambda. \tag{9}$$

Thus, for example,

$$\hat{a}_\text{in}(t) = \frac{1}{\sqrt{2\pi}} \int_{t_0}^t d\omega\, e^{-i\omega(t-t_0)} \hat{a}_\omega, \tag{10}$$

where $t_0$ is a time in the distant past. The definition of $\hat{a}_\text{out}$ is similar but makes use of a time $t_1$ in the distant future.

At this point we shall make two further assumptions regarding the exciton fields: (i) the quantities $\Omega_{\text{c},\lambda} \to \Omega_\text{c}$, $\Omega_{\text{in},\lambda} \to \Omega_\text{in}$, $h_{\lambda,\lambda'} \to h_0$ and $\gamma_\lambda \to \gamma$ are assumed to be independent of the index $\lambda$; and (ii) we also assume that the exciton density of states is constant in the relevant region. Both of these are valid for the frequency ranges we consider in this paper; appreciable modulations only happen over frequency ranges that exceed any of our frequencies by orders of magnitude (cf figure 3 in [13]). In order to avoid introducing new symbols we shall displace each operator $\hat{o}$ by its mean value $\bar{o}$: $\hat{o} \to \bar{o} + \hat{o}$, where the operator on the right-hand side has zero mean. Therefore, *all operators in the following will have zero mean*. The linearized equations of motion can now be written as

$$\dot{\hat{a}} = -(\kappa_\text{c} - i\Delta_\text{c})\hat{a} - \sqrt{2\kappa_\text{c}}\hat{a}_\text{in} - ig_0\bar{a}(\hat{b} + \hat{b}^\dagger) + i(\Omega_\text{in}^* - \Omega_\text{c}^*)\sum_\lambda \hat{c}_\lambda, \tag{11}$$

$$\dot{\hat{b}} = -(\kappa_\text{m} + i\omega_\text{m})\hat{b} - \sqrt{2\kappa_\text{m}}\hat{b}_\text{in} - ig_0(\bar{a}^\star\hat{a} + \bar{a}\hat{a}^\dagger) - i\sum_\lambda (h^\star\hat{c}_\lambda + h\hat{c}_\lambda^\dagger) \tag{12}$$

and

$$\dot{\hat{c}}_\lambda = -(\gamma - i\Delta_\lambda)\hat{c}_\lambda - \sqrt{2\gamma}\hat{c}_{\lambda,\text{in}} - ih(\hat{b} + \hat{b}^\dagger) - i(\Omega_\text{in} + \Omega_\text{c})\hat{a} - i\sqrt{\frac{2}{\kappa_\text{c}}}\Omega_\text{in}\hat{a}_\text{in}, \tag{13}$$

where a term of higher order in $\Omega_\text{in}$ was dropped in equation (13), and where we defined $h = h_0 \sum_\lambda \bar{c}_\lambda$. The last term in equation (11) represents the coupling of the excitonic modes to the cavity field. It has no contribution to the adiabatic solution detailed in section 3 due to it being of a higher order in $\Omega_\text{in}$ and $\Omega_\text{c}$. We have also absorbed $\bar{b}$ into an effective redefinition of $\Delta_\text{c}$ and $\Delta_\lambda$, and therefore $\bar{b} = 0$.

Solving for the $c$-number component of equations (6)–(8) in steady state, the mean values of the fields are shown to satisfy ($\bar{c}_{\text{in}} = 0$)

$$-(\kappa_{\text{c}} - \text{i}\Delta_{\text{c}})\bar{a} - \text{i}(\Omega_{\text{c}} - \Omega_{\text{in}})^{\star}\sum_{\lambda}\bar{c}_{\lambda} - \sqrt{2\kappa_{\text{c}}}\bar{a}_{\text{in}} = 0, \quad \text{and} \tag{14}$$

$$-(\gamma - \text{i}\Delta_{\lambda})\bar{c}_{\lambda} - \text{i}(\Omega_{\text{in}} + \Omega_{\text{c}})\bar{a} - \text{i}\sqrt{\frac{2}{\kappa_{\text{c}}}}\Omega_{\text{in}}\bar{a}_{\text{in}} = 0. \tag{15}$$

It is now apparent that the equations for the optical and mechanical fields involve only sums of the type $\sum_{\lambda}\hat{c}_{\lambda}$ or $\sum_{\lambda}\bar{c}_{\lambda}$. These sums can be performed easily due to our assumption of $\lambda$-independent constants, yielding

$$\bar{a} = \left(-\sqrt{\frac{2}{\kappa_{\text{c}}}}\bar{a}_{\text{in}}\right)\frac{\kappa_{\text{c}}\gamma + \nu\Omega_{\text{in}}(\Omega_{\text{c}} - \Omega_{\text{in}})^{\star}}{(\kappa_{\text{c}} - \text{i}\Delta_{\text{c}})\gamma + \nu(\Omega_{\text{c}} + \Omega_{\text{in}})(\Omega_{\text{c}} - \Omega_{\text{in}})^{\star}}, \tag{16}$$

and

$$\sum_{\lambda}\bar{c}_{\lambda} = \sqrt{\nu}\left(\sqrt{\frac{2}{\kappa_{\text{c}}}}\bar{a}_{\text{in}}\right)\frac{\text{i}\kappa_{\text{c}}\sqrt{\nu}\Omega_{\text{c}} - \Delta_{\text{c}}\sqrt{\nu}\Omega_{\text{in}}}{(\kappa_{\text{c}} - \text{i}\Delta_{\text{c}})\gamma + \nu(\Omega_{\text{c}} + \Omega_{\text{in}})(\Omega_{\text{c}} - \Omega_{\text{in}})^{\star}}, \tag{17}$$

where the factor $\nu \equiv \gamma\sum_{\lambda}(\gamma - \text{i}\Delta_{\lambda})^{-1} \in \mathbb{R}^{+}$ takes into account the many exciton modes we are interacting with. Just as was done in the equation of motion for $\hat{c}_{\lambda}$, terms of higher order in $\Omega_{\text{in}}$ were again dropped in evaluating $\bar{c}_{\lambda}$.

## 3. Adiabatic elimination and memory kernel

The system we aim to describe has a hierarchy of dynamics determined by $\gamma \gg \kappa_{\text{c}} \gg \kappa_{\text{m}}, \omega_{\text{m}}$. Sequential adiabatic elimination of the fields is therefore possible, first solving the equation of motion for the excitonic fields, on whose time-scale $\hat{a}$ is approximately constant, and then for the cavity field. We concentrate on the regime where the exciton-mediated effect dominates over the radiation-pressure force. At this level of approximation we can describe the web of interactions in our system via the following process:

(i) the position of the mechanical oscillator changes the photon number inside the cavity ($\hat{b}$ affects $\hat{a}$),

(ii) the exciton populations follow the changing cavity field ($\hat{a}$ affects $\hat{c}_{\lambda}$), and finally

(iii) the changing exciton populations modulate the mechanical properties, coupling to $\hat{b}$ ($\hat{c}_{\lambda}$ affects $\hat{b}$).

Figure 2 illustrates the different interactions that take place in the system we are describing, together with the reduced system that results after adiabatically eliminating the optical and excitonic fields.

With the above in mind, and ignoring the contributions from the input field, we can write the adiabatic solution of the cavity field operator familiar from the radiation-pressure cooling literature [28, 29]:

$$\hat{a} \approx -\text{i}g_0\bar{a}\left[\frac{1}{\kappa_{\text{c}} - \text{i}(\Delta_{\text{c}} + \omega_{\text{m}})}\hat{b} + \frac{1}{\kappa_{\text{c}} - \text{i}(\Delta_{\text{c}} - \omega_{\text{m}})}\hat{b}^{\dagger}\right], \tag{18}$$

which is substituted into the equation of motion for $\hat{c}_\lambda$ to give

$$\hat{c}_\lambda \approx -\frac{g_0 \bar{a} (\Omega_c + \Omega_{in})}{\gamma - \kappa_c - i(\Delta_\lambda - \Delta_c)} \left[ \frac{1}{\kappa_c - i(\Delta_c + \omega_m)} \hat{b} + \frac{1}{\kappa_c - i(\Delta_c - \omega_m)} \hat{b}^\dagger \right]. \tag{19}$$

We see from this equation that the dominant function of the excitons is to act as a channel for the absorbed optical energy to interact with the mechanics. Noting that $\gamma$ is much larger than all the other frequencies, we obtain

$$\sum_\lambda \hat{c}_\lambda \approx -\frac{\nu}{\sqrt{\nu}} \frac{g_0 \bar{a} \sqrt{\nu} (\Omega_c + \Omega_{in})}{\gamma} \left[ \frac{1}{\kappa_c - i(\Delta_c + \omega_m)} \hat{b} + \frac{1}{\kappa_c - i(\Delta_c - \omega_m)} \hat{b}^\dagger \right]. \tag{20}$$

Substitution of this solution into the equation of motion for $\hat{b}$ gives us the adiabatic dynamics when the excitons couple to the mechanics through the deformation potential. Because we want to describe the *time-delayed* effect of the excitons on the mechanics, however, we must introduce a memory kernel $\mathcal{M}(t)$ into the equation of motion for $\hat{b}$. This follows the ideas outlined in related treatments of the photothermal effect [5–7, 19], and in our notation corresponds to setting:

$$h_0^\star \sum_\lambda \hat{c}_\lambda \to \eta^\star \int_{-\infty}^\infty \mathcal{M}(t - \tau) \sum_\lambda \hat{c}_\lambda(\tau) \, d\tau, \tag{21}$$

and similarly for the term involving $h_0$. This process changes the physical meaning of these terms and allows us to describe the photothermal effect phenomenologically. We emphasize that $\eta$ does not describe the deformation-potential coupling, but is a phenomenological coupling constant that describes the strength of the delayed interaction linking the exciton fields with the mechanical motion, i.e. the photothermal effect. This is in contrast to equations (4) and (12), which treat the deformation-potential coupling without taking into account thermal expansion.

Physically, equation (21) tells us that the modulation to the mechanical properties takes into account the entire history of the exciton fields. We choose to use an exponentially-decaying memory kernel [7]:

$$\mathcal{M}(t - \tau) = \frac{1}{\tau_{th}} e^{-(t-\tau)/\tau_{th}} \Theta(t - \tau), \tag{22}$$

where $\Theta(t)$ is the Heaviside step function and accounts for the causal nature of the memory kernel; this $\mathcal{M}(t)$ leads to the same expressions as the '$h(t)$' chosen by Metzger *et al*, cf [19], upon integration by parts of the relevant terms. The use of a memory kernel in the equation of motion for $\hat{b}$ can be motivated by making use of an extended model that includes a bath of phonon modes which act as the intermediary between the excitons and $\hat{b}$. Elimination of the these modes in $d\hat{b}/dt$ naturally gives rise to a time-integral of a sum of decaying exponentials, which we identify with an exponentially-decaying memory kernel; this process is outlined in the appendix.

A brief note about the effect of noise terms is due. It lies outside the scope of this paper to consider the effects of noise on the limits of this cooling mechanism; being interested in cooling rates in this paper, we accordingly discard such terms. As discussed in [7], the nature of the photothermal effect does not preclude reaching the ground state, even in the bad-cavity limit, despite the absorption of light in the mechanical oscillator. The thermal noise induced by

the absorbed light can be effectively modelled as a Langevin force term as in, e.g. equation (5) of [5] or equation (15) of [7]. This Langevin force can be viewed as having its physical origins in the thermal fluctuations of the phononic bath described in appendix, which couples the exciton modes to $\hat{b}$.

A unique feature of our setup lies in the interference between the cavity and input fields, discussed after equation (3) above, which could, in analogy with dissipative optomechanics [24], lead to a situation where the effect of the noise originating from the optical fields cancels out, and therefore to a more efficient cooling mechanism and a lower base temperature.

## 4. Optomechanical cooling rate

Proceeding from the previous section along the same lines as standard optomechanical theory, we can now derive a simple expression for the optomechanical cooling rate due to this photothermal effect. Indeed, we can show that the mechanical decay rate changes from $\kappa_{\mathrm{m}}$ to $\kappa_{\mathrm{m}} + \kappa_{\mathrm{th}}$, where for $\tau_{\mathrm{th}} \gg 1/\omega_{\mathrm{m}}$ and $\Omega_{\mathrm{c}} \in \mathbb{R}$

$$
\kappa_{\mathrm{th}} = \frac{P_{\mathrm{in}}}{\hbar\omega_{\mathrm{L}}} \frac{2g_0}{(\kappa_{\mathrm{c}}^2 + \Delta_{\mathrm{c}}^2)\omega_{\mathrm{m}}\tau_{\mathrm{th}}} \frac{\eta_{\mathrm{th}}}{\gamma} \frac{\Omega_{\mathrm{c}}^2}{\gamma} \mathrm{Re} \left\{ \frac{(1 + \Omega_{\mathrm{in}}/\Omega_{\mathrm{c}})(\Delta_{\mathrm{c}}\Omega_{\mathrm{in}}^{\star}/\Omega_{\mathrm{c}} + \mathrm{i}\kappa_{\mathrm{c}})}{\kappa_{\mathrm{c}} - \mathrm{i}(\omega_{\mathrm{m}} + \Delta_{\mathrm{c}})} \right.
$$
$$
\left. + \frac{(1 + \Omega_{\mathrm{in}}^{\star}/\Omega_{\mathrm{c}})(\Delta_{\mathrm{c}}\Omega_{\mathrm{in}}/\Omega_{\mathrm{c}} - \mathrm{i}\kappa_{\mathrm{c}})}{\kappa_{\mathrm{c}} - \mathrm{i}(\omega_{\mathrm{m}} - \Delta_{\mathrm{c}})} \right\}, \tag{23}
$$

with $P_{\mathrm{in}} = \hbar\omega_{\mathrm{L}} |\bar{a}_{\mathrm{in}}|^2$ being the input power coupled into the cavity (in watts), the sum over $\lambda$ was absorbed into the phenomenological coupling constant $\eta_{\mathrm{th}} = \nu\eta$, and we have absorbed $\sqrt{\nu}$ into each of $\Omega_{\mathrm{c}}$ and $\Omega_{\mathrm{in}}$. Equation (23) excludes higher-order terms in $\Omega_{\mathrm{c}}$ and $\Omega_{\mathrm{in}}$, as does our adiabatic solution. It is worth noting that the sign and magnitude of $\eta_{\mathrm{th}}$ depend on the shape of the membrane. The membrane in the experiment has a slight curvature, such that any thermal expansion has a well-defined effect on its effective position $\hat{x} \equiv (\hat{b} + \hat{b}^{\dagger})/\sqrt{2}$; if the membrane were to be flipped over, the sign of $\eta_{\mathrm{th}}$ would change. For a perfectly flat membrane, the membrane 'would not know' which way to buckle under thermal expansion; $\eta_{\mathrm{th}}$ would then be zero and other terms would be expected to dominate.

The expression for $\kappa_{\mathrm{th}}$ depends critically on $\Omega_{\mathrm{in}}$, whose relationship to $\Omega_{\mathrm{c}}$ is fixed by the geometry of the cavity and membrane. In the good-cavity limit, which is valid whenever the finesse of the cavity is $\gg 1$, and taking into account the large refractive index of GaAs, we obtain

$$
\frac{\Omega_{\mathrm{in}}}{\Omega_{\mathrm{c}}} = -\frac{\mathrm{i}}{\sqrt{2}} \mathrm{e}^{\mathrm{i}(k_{\mathrm{L}}d/2 - 2L\Delta_{\mathrm{c}}/c)} \sin(k_{\mathrm{L}}d/2), \tag{24}
$$

where $d$ is the thickness of the membrane, $L$ the length of the cavity, and $k_{\mathrm{L}} = \omega_{\mathrm{L}}/c$. $\Omega_{\mathrm{c}}$ itself can be fixed by observing the fraction $f_{\mathrm{abs}}$ of power absorbed by the membrane, since it can be shown that on cavity resonance ($\Delta_{\mathrm{c}} = 0$) and for $\Omega_{\mathrm{c}}^2 \ll \kappa_{\mathrm{c}}\gamma$,

$$
f_{\mathrm{abs}} = \frac{4\Omega_{\mathrm{c}}^2}{\gamma\kappa_{\mathrm{c}}}. \tag{25}
$$

It is worth noting the physical significance of equation (25): in our model $\Omega_{\mathrm{c}}$ and $\Omega_{\mathrm{in}}$ effectively give rise to the imaginary part of the refractive index of the membrane, conventionally

**Table 1.** Experimental parameters and coupling constant extracted from the data. The datasets are numbered as per figure 3.

| | $\lambda_L$ (nm) | $P_{in}$ ($\mu$W) | $\kappa_m$ (s$^{-1}$) | $f_{abs}$ (%) | $\Omega_c^2/\gamma$ ($2\pi$ MHz) | $\eta_{th}/\gamma \times 10^2$ |
|---|---|---|---|---|---|---|
| 1 | 870 | 20 | 1.8 | 50 | 32.3 | 7.5 |
| 2 | 852 | 25 | 2.2 | 55 | 35.4 | 4.6 |
| 3 | 852 | 25 | 2.2 | 55 | 35.4 | 7.6 |
| 4 | 852 | 25 | 2.2 | 55 | 35.4 | 6.2 |

labelled $\kappa$. $f_{abs}$ therefore exhibits a wavelength-dependence, as does $\kappa$, cf table 1. Independent experimental measurements give us values for: $\kappa_c$, $\Omega_c^2/\gamma$, $\omega_m$, and $\tau_{th}$. $g_0$ is fixed by the geometry and by the reflectivity of the membrane, whereas $P_{in}$, $\omega_L$, and $\Delta_c$ are determined by the experiment. The only independent fit parameter in equation (23) is therefore the photothermal coupling strength $\eta_{th}/\gamma$.

## 5. Fit to experimental data

We shall now use $\kappa_{th}$ to model four sets of experimental data. The experimental runs differ in the transverse location of the membrane vis-à-vis the cavity field, and the values of $\eta_{th}$ obtained for the four sets are consistent with the coupling of the excitons to the (2, 1) drumhead mode of the membrane.

The independently-determined parameters used in the model and common to every dataset were as follows: $L = 2.9$ cm, $d = 160$ nm, $g_0 = 2\pi \times (-5.1)$ Hz, $\kappa_c = 2\pi \times 258$ MHz, $\omega_m = 2\pi \times 23.4$ kHz and $\tau_{th} = 6.6$ ms. Other parameters differed between datasets and are listed in table 1. This table also lists the photothermal coupling rates resulting in the fits shown in figure 3. These coupling rates, when plotted as a function of the position of the driving beam on the membrane, are consistent with a membrane displacement profile matching the (2, 1) drumhead mode, cf figure 4, and therefore conform to our expectations. This dependence of the photothermal force on the position of the driving field was noted previously, and is indeed crucial to the results in [17]. In conclusion, the experimental data presented above are consistent with a single fit parameter, corresponding to a maximal coupling strength $\eta_{th}^{max}/\gamma \approx 0.1$, if we also take into account the location of the driving beam on the membrane. Further data would be necessary, however, to verify this hypothesis rigorously.

## 6. Cooling through electronic stress

Let us now consider a different system where the mechanism that provides the cooling force is no longer the thermal stress set up by decaying excitons but pure electronic stress. For a system such as the one we described above to enter this regime, the photothermal effect must be switched off. In the case of GaAs, this can be achieved by operating at a temperature of about 12 K [13]. Under these conditions, the excitons interact with the mechanics directly, and only through deformation potential, as expressed in the model of section 2. In an equivalent picture, we may say that the effective memory time is zero, and the memory kernel reduces to a delta-function: $\mathcal{M}(t) \rightarrow \delta(t)$.
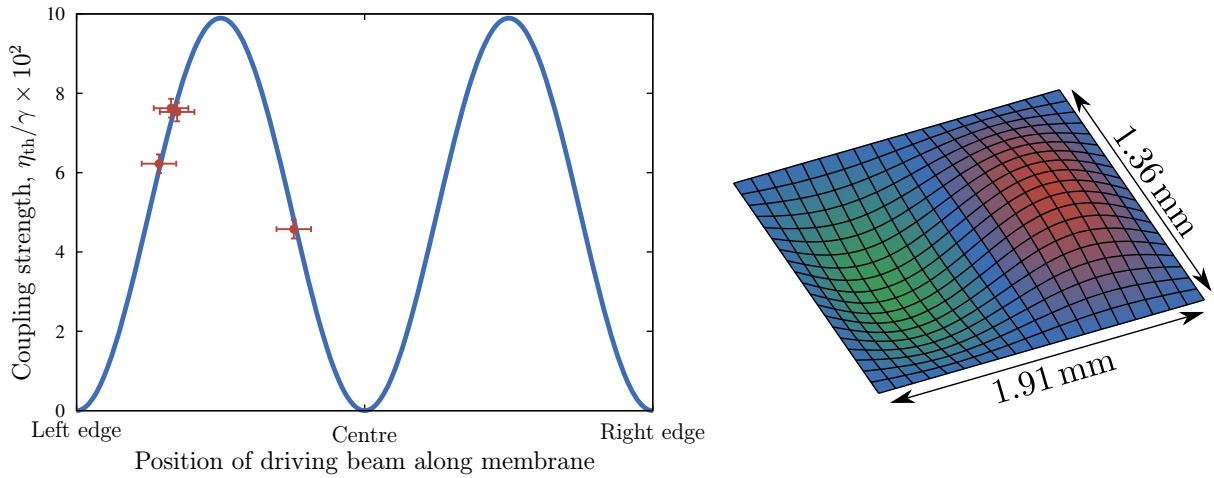
**Figure 4.** (a) The coupling parameters that fit the four datasets are consistent with the excitation of the (2, 1) mode of the membrane (shown in (b)); the solid curve represents the sinusoidal profile of the squared amplitude of this mode. (b) Vibrational profile of the (2, 1) mode of the membrane.

By placing the membrane *inside* a cavity [30] ($\Omega_{in} = 0$), the finesse of the cavity can be made significantly larger if level of absorption in the membrane is lowered, leading to a correspondingly smaller $\Omega_c$. This can be done in GaAs by running the experiment at longer wavelengths, e.g. at 884 nm, where the absorption is significantly lower than at 870 nm.

## 7. Conclusion

We have explored optomechanical cooling through the photothermal effect in a semiconductor membrane. Our model uses a coupling similar to the deformation-potential coupling but makes use of a memory kernel to model the long thermalization time typical of such structures. The introduction of the memory kernel was based on entirely phenomenological grounds, following [5–7, 19], but we justify the use of an exponentially-decaying kernel by introducing additional phononic degrees of freedom that are then eliminated. The resulting model only has one free parameter, with all others being determined independently or by the geometry of the situation, and provides a remarkably good fit to the experimental data.

By using different forms of the memory kernel we can also compare the different physical mechanisms in promoting optomechanical cooling. Thus, for example, an instantaneous memory kernel $\mathcal{M}(t) = \delta(t)$ reduces our description to one taking into account pure deformation-potential effects, similarly to what was originally envisioned in [31]. Such a mechanism could be important under conditions where the photothermal effect is cancelled out, e.g., at temperatures where the membrane undergoes no photothermal deformation [13].

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## Appendix. Indirect exciton–motion coupling, and emergence of memory kernel

The model presented in section 2 serves to explain the physical processes occurring in a more transparent manner. As far as the interaction of the $\hat{c}_\lambda$ with $\hat{b}$ is concerned, however, this model does not capture the fact that the process takes place indirectly. In this appendix, we will examine a more detailed Hamiltonian that leads to the same effective equation of motion for $\hat{b}$. We shall introduce a phononic bath of modes $\hat{d}_\mu$ (oscillation frequency $\omega_\mu$ and amplitude decay rate $\kappa_\mu$) that serve as intermediaries between $\hat{c}_\lambda$ and $\hat{b}$. Physically, the $\hat{d}_\mu$ account for the nonzero temperature of the lattice making up the membrane. The interaction terms between the three systems can be written as [25, 26]

$$\sum_{\lambda,\lambda',\mu} k_{\lambda,\lambda',\mu} \hat{c}_\lambda^\dagger \hat{c}_{\lambda'} (\hat{d}_\mu + \hat{d}_\mu^\dagger) + \sum_{\mu,\mu'} l_{\mu,\mu'} (\hat{d}_\mu^\dagger \hat{d}_{\mu'} \hat{b} + \hat{d}_\mu \hat{d}_{\mu'}^\dagger \hat{b}^\dagger), \tag{A.1}$$

with the $k_{\lambda,\lambda',\mu}$ and $l_{\mu,\mu'}$ being coupling frequencies whose values we shall not specify or calculate. Hermiticity requires that $k_{\lambda',\lambda,\mu} = k_{\lambda,\lambda',\mu}^*$ and $l_{\mu',\mu} = l_{\mu,\mu'}^*$. Any sums over $\lambda$ or $\mu$ may be either discrete or continuous, as the case requires. Equation (A.1) mediates the interaction between the excitons and $\hat{b}$, and therefore replaces the second term in $\hat{H}_{\text{mech}}$. The second sum in equation (A.1) represents a coupling of the bulk motion ($\hat{b}$) to the phonon modes of the lattice ($\hat{d}_\mu$); this coupling, which represents the lattice anharmonicity, takes the form of a $\hat{b}$-dependent shift in the frequency of the lattice phonon modes.

We shall now proceed to eliminate the phononic bath modes. The equation of motion for the $\hat{d}_\mu$ reads

$$\dot{\hat{d}}_\mu = -(\kappa_\mu + i\omega_\mu)\hat{d}_\mu - i\sum_{\lambda,\lambda'} k_{\lambda,\lambda',\mu} \hat{c}_\lambda^\dagger \hat{c}_{\lambda'} - i\sum_{\mu'} (l_{\mu,\mu'} \hat{d}_{\mu'} \hat{b} + l_{\mu',\mu} \hat{d}_{\mu'} \hat{b}^\dagger), \tag{A.2}$$

where we have not written down the input noise terms, expressed in terms of the anti-Hermitian operators $\hat{d}_{\mu,\text{in}}$ in the quantum Brownian-motion damping model [32], since these terms have no effect on the cooling *rate* but help to determine the lowest mechanical occupation number that can be achieved through this cooling mechanism.

The last set of terms in equation (A.2) leads, both directly and through the $\hat{c}_\lambda$, to a renormalization of $\omega_{\text{m}}$ and $\kappa_{\text{m}}$ due to the absorbed optical power and finite temperature of the lattice, and we may therefore safely ignore it, linearize the equation of motion, and finally write

$$\dot{\hat{d}}_\mu = -(\kappa_\mu + i\omega_\mu)\hat{d}_\mu - i\sum_{\lambda,\lambda'} (k_{\lambda,\lambda',\mu} \bar{c}_\lambda^\star \hat{c}_{\lambda'} + k_{\lambda',\lambda,\mu} \bar{c}_\lambda \hat{c}_{\lambda'}^\dagger). \tag{A.3}$$

Formally, then, the solution for $\hat{d}_\mu$ is given by

$$\hat{d}_\mu(t) = -i \int_{-\infty}^t e^{-(\kappa_\mu + i\omega_\mu)(t-\tau)} \sum_{\lambda,\lambda'} [k_{\lambda',\lambda,\mu} \bar{c}_{\lambda'}^\star \hat{c}_\lambda(\tau) + k_{\lambda,\lambda',\mu} \bar{c}_{\lambda'} \hat{c}_\lambda^\dagger(\tau)] d\tau, \tag{A.4}$$

noting once more that we are ignoring input noise fields. The Hamiltonian above therefore gives the following contribution to the linearized equation of motion for $\hat{b}$:

$$
\begin{aligned}
\dot{\hat{b}} = &-\mathrm{i}\sum_{\mu,\mu'}(l_{\mu,\mu'}\bar{d}_{\mu'}^{\star}\hat{d}_{\mu}+l_{\mu',\mu}\bar{d}_{\mu'}\hat{d}_{\mu}^{\dagger})\\
= &-\mathrm{i}\int_{-\infty}^{t}\sum \mathrm{e}^{-\kappa_{\mu}(t-\tau)}\left\{-\mathrm{i}\mathrm{e}^{-\mathrm{i}\omega_{\mu}(t-\tau)}l_{\mu,\mu'}\bar{d}_{\mu'}^{\star}\left[k_{\lambda',\lambda,\mu}\bar{c}_{\lambda'}^{\star}\hat{c}_{\lambda}(\tau)+k_{\lambda,\lambda',\mu}\bar{c}_{\lambda'}\hat{c}_{\lambda}^{\dagger}(\tau)\right]\right\}\mathrm{d}\tau\\
&-\mathrm{i}\int_{-\infty}^{t}\sum \mathrm{e}^{-\kappa_{\mu}(t-\tau)}\left\{\mathrm{i}\mathrm{e}^{\mathrm{i}\omega_{\mu}(t-\tau)}l_{\mu',\mu}\bar{d}_{\mu'}\left[k_{\lambda,\lambda',\mu}^{\star}\bar{c}_{\lambda'}^{\star}\hat{c}_{\lambda}(\tau)+k_{\lambda',\lambda,\mu}^{\star}\bar{c}_{\lambda'}\hat{c}_{\lambda}^{\dagger}(\tau)\right]\right\}\mathrm{d}\tau\\
= &-\mathrm{i}\int_{-\infty}^{t}\sum \mathrm{e}^{-\kappa_{\mu}(t-\tau)}\left[-\mathrm{i}\mathrm{e}^{-\mathrm{i}\omega_{\mu}(t-\tau)}l_{\mu,\mu'}\bar{d}_{\mu'}^{\star}+\mathrm{c.c.}\right]k_{\lambda',\lambda,\mu}\bar{c}_{\lambda'}^{\star}\hat{c}_{\lambda}(\tau)\mathrm{d}\tau\\
&-\mathrm{i}\int_{-\infty}^{t}\sum \mathrm{e}^{-\kappa_{\mu}(t-\tau)}\left[-\mathrm{i}\mathrm{e}^{-\mathrm{i}\omega_{\mu}(t-\tau)}l_{\mu,\mu'}\bar{d}_{\mu'}^{\star}+\mathrm{c.c.}\right]k_{\lambda,\lambda',\mu}\bar{c}_{\lambda'}\hat{c}_{\lambda}^{\dagger}(\tau)\mathrm{d}\tau,
\end{aligned} \tag{A.5}
$$

with the sums running over $\lambda$, $\lambda'$, $\mu$, and $\mu'$; 'c.c.' denotes the complex conjugate of the preceding term. We now make the formal replacement

$$
\sum_{\lambda',\mu,\mu'}\mathrm{e}^{-\kappa_{\mu}(t-\tau)}\left[-\mathrm{i}\mathrm{e}^{-\mathrm{i}\omega_{\mu}(t-\tau)}l_{\mu,\mu'}\bar{d}_{\mu'}^{\star}+\mathrm{c.c.}\right]k_{\lambda',\lambda,\mu}\bar{c}_{\lambda'}^{\star}\to\eta^{\star}\left(\sum_{\lambda}\bar{c}_{\lambda}^{\star}\right)\mathcal{M}(t-\tau), \tag{A.6}
$$

where $\mathcal{M}(t)$ is a causal memory kernel that we choose to have a decaying exponential form, and where $k_{\lambda',\lambda,\mu}$ is assumed to be independent of $\lambda$ and $\lambda'$. Finally, then, the contribution to the equation of motion for $\hat{b}$ is

$$
\begin{aligned}
\dot{\hat{b}} = &-\mathrm{i}\eta^{\star}\left(\sum_{\lambda}\bar{c}_{\lambda}^{\star}\right)\int_{-\infty}^{\infty}\mathcal{M}(t-\tau)\sum_{\lambda}\hat{c}_{\lambda}(\tau)\mathrm{d}\tau\\
&-\mathrm{i}\eta\left(\sum_{\lambda}\bar{c}_{\lambda}\right)\int_{-\infty}^{\infty}\mathcal{M}(t-\tau)\sum_{\lambda}\hat{c}_{\lambda}^{\dagger}(\tau)\mathrm{d}\tau.
\end{aligned} \tag{A.7}
$$

The memory kernel in the equation of motion for $\hat{b}$ therefore arises naturally from this more complete, albeit still phenomenological, model.

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