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Constraining spacetime nonmetricity with Lorentz-violation methods

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Abstract. In this report, we will give the first constraints on in-matter nonmetricity. We will show how the effective-field-theory (EFT) toolbox developed for the study of Lorentz violation (LV) can be employed for investigations of the "effective LV" background caused by nonmetricity, a geometric object extending the notion of a Riemannian manifold. The idea is to probe for the effects of spacetime nonmetricity sourced by liquid ^{4}He with polarized slow neutrons. We present the first constraints on isotropic and parity-odd nonmetricity components. Further constraints on anisotropic nonmetricity components within this EFT framework may be feasible with proper experimental techniques in the near future.

1. Introduction

The celebrated observation in 2016 of gravitational waves by the LIGO and Virgo Scientific Collaboration [1] has once more underscored the widely held belief that the geometry of spacetime is a dynamical physical entity. Keeping such a geometric foundation, efforts to extend general relativity (GR) proceed often by exploring more general geometries beyond that of a Riemanian spacetime. In general metric-affine gravity [2], the linear connection is $\Gamma^{\alpha}{}_{\beta\gamma} = \{{}_{\beta}{}^{\alpha}{}_{\gamma}\} + K^{\alpha}{}_{\beta\gamma} + Q^{\alpha}{}_{\beta\gamma}$, where $\{{}_{\beta}{}^{\alpha}{}_{\gamma}\}$ denotes the Levi–Civita connection determined purely by the metric tensor $g_{\mu\nu}$, $K^{\alpha}{}_{\beta\gamma} \equiv \frac{1}{2}\left(T^{\alpha}{}_{\beta\gamma} - T_{\beta}{}^{\alpha}{}_{\gamma} - T_{\gamma}{}^{\alpha}{}_{\beta}\right)$ is the contorsion tensor determined by the torsion tensor $T^{\alpha}{}_{\beta\gamma} \equiv \Gamma^{\alpha}{}_{\beta\gamma} - \Gamma^{\alpha}{}_{\gamma\beta}$, and $Q^{\alpha}_{\beta\gamma} \equiv \frac{1}{2}\left(N_{\beta}{}^{\alpha}{}_{\gamma} + N_{\gamma}{}^{\alpha}{}_{\beta} - N^{\alpha}{}_{\beta\gamma}\right)$ is given by the nonmetricity tensor $N_{\alpha\beta\gamma} \equiv -D_{\alpha}g_{\beta\gamma}$. In a historical context, two avenues for extending GR are particularly noteworthy. By relaxing the usual metric-compatibility condition $D_{\alpha}g_{\beta\gamma} = 0$, Weyl interpreted $(N_1)_{\alpha} = g^{\beta\gamma}N_{\alpha\beta\gamma}$ as the electromagnetic 4-potential in a (failed) attempt to unify electromagnetism with gravity [3]. By relaxing the torsion-free condition $\Gamma^{\alpha}{}_{\beta\gamma} = \Gamma^{\alpha}{}_{\gamma\beta}$ instead, Cartan proposed to couple torsion with the intrinsic-spin current of matter fields, an approach now known as Einstein–Cartan theory [4].

Assuming that $T^{\alpha}{}_{\beta\gamma}$ or $N_{\alpha\beta\gamma}$ vary slowly at the scale of the solar system, we may regard them as constant tensor fields in the neighborhood of the Earth. For a local terrestrial experiment, these tensor fields then effectively break Lorentz symmetry even if the original theory is Lorentz invariant [5]. This idea allows us to utilize the theoretical framework developed for the study of CPT- and Lorentz-symmetry violation, the Standard-Model Extension (SME) [6], to explore the

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associated background-tensor–matter couplings. For example, some characteristic LV signals, like sidereal and annual variations of physical observables [7], which stem from the motion of Earth-based laboratories through such solar torsion or nonmetricity backgrounds, may be used to constrain these tensor fields. This idea has already been studied extensively for torsion backgrounds [8] including in-matter torsion effects [9]. Numerous other phenomenological studies on torsion fields are also available in the literature [10–19]. For the case of nonmetricity–matter couplings, an analogous treatment has been given recently in Ref. [20]. As an important geometric object, matter–nonmetricity couplings have also been the focus of various theoretical studies [21]. However, in this paper we adopt the viewpoint of Ref. [20] and regard the presence of nonmetricity as an experimental issue instead.

In particular, we study in-matter nonmetricity couplings by considering a polarized slow-moving neutron beam propagating in liquid ^{4}He. In this set-up, the source of nonmetricity is assumed to be the liquid ^{4}He, and the probe is the neutron beam. The polarized neutrons are produced at the National Institute of Standards and Technology (NIST) Center for Neutron Research and have an approximate Maxwellian distribution with a peak energy of about 3 meV. The liquid ^{4}He was kept at a temperature of around 4 K in a magnetically shielded cryogenic target [22]. The key difference between our study and that in Ref. [20] is that we examine the hypothetical nonmetricity sourced locally by liquid ^{4}He in a terrestrial laboratory. Since the ^{4}He is comoving with the laboratory, this excludes any sidereal or annual effect predicted in studies of nonmetricity sourced by celestial body, *e.g.*, the Sun.

This paper is organized as follows. Section 2 briefly reviews the effective-field-theory framework borrowed from LV studies to describe fermion–nonmetricity couplings. In Sec. 3, we discuss the constraints obtained from the measurement of neutron-spin precession at NIST's Center for Neutron Research. Finally, in Sec. 4, we briefly summarize and discuss the results of this paper. The convention for the metric signature and the Levi–Civita symbol are $\eta^{\mu\nu} = \mathrm{diag}(+,-,-,-)$ and $\epsilon^{0123} = +1$, respectively.

2. Theoretical Preparation

Since we adopt an experimental point of view, as mentioned above, the question as to whether $N_{\alpha\beta\gamma}$ should be minimally coupled (to the dilation and shear currents) [21] or not is not our primary concern. Our starting point is the analysis in Ref. [20] that takes all matter–nonmetricity operators allowed by observer Lorentz symmetry [6] into account. To our knowledge, $N_{\alpha\beta\gamma}$ must be tiny, which means we can safely drop higher-order nonmetricity couplings. The relevant energy in the neutron experiment is comparatively low, which means we may also ignore the composite nature of the neutron and regard it as a point-like particle. The free neutron Lagrangian is $\mathcal{L}_0 = \frac{i}{2}\overline{\psi}\gamma^\mu \overleftrightarrow{\partial}_\mu \psi - m\overline{\psi}\psi$. We also set $g^{\mu\nu} \to \eta^{\mu\nu}$ since we may ignore conventional gravitational effect for our current purposes. To write down systematically fermion–nonmetricity couplings in the SME framework, it will be convenient to decompose $N_{\alpha\beta\gamma}$ into its four Lorentz irreducible pieces [20], *i.e.*,

$$\begin{aligned}
(N_1)_\mu \equiv& -\eta^{\alpha\beta} N_{\mu\alpha\beta}, \qquad (N_2)_\mu \equiv -\eta^{\alpha\beta} N_{\alpha\mu\beta}, \\
S_{\mu\alpha\beta} \equiv& \tfrac{1}{3}\left[N_{\mu\alpha\beta} + N_{\alpha\beta\mu} + N_{\beta\mu\alpha}\right] + \tfrac{1}{18}\left[(N_1)_\mu \eta_{\alpha\beta} + (N_1)_\alpha \eta_{\beta\mu} + (N_1)_\beta \eta_{\mu\alpha}\right] \\
&+ \tfrac{1}{9}\left[(N_2)_\mu \eta_{\alpha\beta} + (N_2)_\alpha \eta_{\beta\mu} + (N_2)_\beta \eta_{\mu\alpha}\right], \\
M_{\mu\alpha\beta} \equiv& \tfrac{1}{3}\left[2N_{\mu\alpha\beta} - N_{\alpha\beta\mu} - N_{\beta\mu\alpha}\right] + \tfrac{1}{9}\left[2(N_1)_\mu \eta_{\alpha\beta} - (N_1)_\alpha \eta_{\beta\mu} - (N_1)_\beta \eta_{\alpha\mu}\right] \\
&- \tfrac{1}{9}\left[2(N_2)_\mu \eta_{\alpha\beta} - (N_2)_\alpha \eta_{\beta\mu} - (N_2)_\beta \eta_{\alpha\mu}\right].
\end{aligned} \tag{1}$$

The nonmetricity tensor can be rebuilt from the above Lorentz irreducible tensors as follows

$$\begin{aligned}
N_{\mu\alpha\beta} =& \tfrac{1}{18}\left[-5(N_1)_\mu\eta_{\alpha\beta} + (N_1)_\alpha\eta_{\beta\mu} + (N_1)_\beta\eta_{\mu\alpha} + 2(N_2)_\mu\eta_{\alpha\beta} - 4(N_2)_\alpha\eta_{\beta\mu} - 4(N_2)_\beta\eta_{\mu\alpha}\right] \\
&+ S_{\mu\alpha\beta} + M_{\mu\alpha\beta}\,.
\end{aligned} \tag{2}$$

With these Lorentz-irreducible pieces, we can list all the relevant Lagrangian contributions constructed first in Ref. [20]:

$$\begin{aligned}
\mathcal{L}_N^{(4)} &= \left[\zeta_1^{(4)}\,(N_1)_\mu + \zeta_3^{(4)}\,(N_2)_\mu\right]\overline{\psi}\gamma^\mu\psi + \left[\zeta_2^{(4)}\,(N_1)_\mu + \zeta_4^{(4)}\,(N_2)_\mu\right]\overline{\psi}\gamma_5\gamma^\mu\psi,\\
\mathcal{L}_N^{(5)} &= -\tfrac{1}{2}\left\{i\left[\zeta_1^{(5)}\,(N_1)^\mu + \zeta_3^{(5)}\,(N_2)^\mu\right]\overline{\psi}\overset{\leftrightarrow}{\partial}_\mu\psi + \left[\zeta_2^{(5)}\,(N_1)^\mu + \zeta_4^{(5)}\,(N_2)^\mu\right]\overline{\psi}\gamma_5\overset{\leftrightarrow}{\partial}_\mu\psi\right\}\\
&\quad -\tfrac{1}{4}i\left[\zeta_5^{(5)}\,M_{\mu\nu}{}^{\rho} - \zeta_6^{(5)}\,{}^*M_{\mu\nu}{}^{\rho}\right]\overline{\psi}\sigma^{\mu\nu}\overset{\leftrightarrow}{\partial}_\rho\psi + \tfrac{1}{2}i\left[\zeta_7^{(5)}\,(N_1)_\mu + \zeta_8^{(5)}\,(N_2)_\mu\right]\overline{\psi}\sigma^{\mu\nu}\overset{\leftrightarrow}{\partial}_\nu\psi\\
&\quad -\tfrac{1}{4}i\,\epsilon^{\lambda\mu\nu\rho}\left[\zeta_9^{(5)}\,(N_1)_\lambda + \zeta_{10}^{(5)}\,(N_2)_\lambda\right]\overline{\psi}\sigma_{\mu\nu}\overset{\leftrightarrow}{\partial}_\rho\psi,\\
\mathcal{L}_N^{(6)} &\supset -\tfrac{1}{4}\zeta_1^{(6)}\,S_\lambda{}^{\mu\nu}\,\overline{\psi}\gamma^\lambda\partial_\mu\partial_\nu\psi + \text{h.c.} - \tfrac{1}{4}\zeta_2^{(6)}\,S_\lambda{}^{\mu\nu}\,\overline{\psi}\gamma_5\gamma^\lambda\partial_\mu\partial_\nu\psi + \text{h.c.}\,,
\end{aligned} \tag{3}$$

where we have defined ${}^*M_{\mu\nu}{}^{\rho} \equiv \frac{1}{2}\epsilon_{\kappa\lambda\mu\nu}\,M^{\kappa\lambda\rho}$ and all $\zeta_l^{(n)}$ are real. To avoid missing any possible nonmetricity–matter couplings, we have to take all Lorentz irreducible pieces of $N_{\mu\alpha\beta}$ into account including $M_{\alpha\beta\gamma}$ and $S_{\alpha\beta\gamma}$ in $\mathcal{L}_N^{(5)}$, $\mathcal{L}_N^{(6)}$. From the study of Lorentz violation [6, 23, 24], we know that some LV coefficients can be eliminated by field redefinitions, and a similar situation arises for some of the above nonmetricity operators. By scrutinizing Lagrangian (3), the operators associated with the couplings $\zeta_1^{(4)}$, $\zeta_3^{(4)}$, $\zeta_1^{(5)}$, $\zeta_2^{(5)}$, $\zeta_3^{(5)}$, $\zeta_4^{(5)}$, $\zeta_7^{(5)}$, and $\zeta_8^{(5)}$ can be removed at least at first order with carefully chosen field redefinitions.

By matching to our concrete experimental set-up, we can refine the remaining Lagrangian further. First note that the ground state of ^{4}He is spin 0. This state can therefore not contribute to anisotropies. Although individual ^{4}He atoms can have specific positions and momenta, no particular arrangement of a large numbers of helium atoms with preferred directions is expected. So, on average, nonmetricity generated by a macroscopic number of ^{4}He atoms is expected to be rotationally invariant, which means we only need to consider isotropic couplings. Components without spatial indices are obviously isotropic, so we have $(N_1)^0$, $(N_2)^0$, and S^{000}. For the mixed-symmetry piece, note that it satisfies the "Bianchi-like" identity

$$M_{\alpha\beta\gamma} + M_{\beta\gamma\alpha} + M_{\gamma\alpha\beta} = 0\,, \tag{4}$$

which indicates that no mixed-symmetry parts contributes. For a more detailed explanation, see Ref. [25]. Next, note that a more suitable framework to deal with slow-neutron effects is nonrelativistic. To extract the nonrelativistic physics we found it convenient to utilize directly a result in Ref. [24] instead of resorting to the usual Foldy–Wouthuysen transformation [26]. The final nonrelativistic Hamiltonian becomes

$$h_s = \frac{\vec{p}^2}{2m} + \left[\left(\zeta_2^{(4)} - m\,\zeta_9^{(5)}\right)(N_1)_0 + \left(\zeta_4^{(4)} - m\,\zeta_{10}^{(5)}\right)(N_2)_0 + \frac{m^2}{2}\zeta_2^{(6)}S_{000}\right]\frac{\vec{p}\cdot\vec{\sigma}}{m}\,, \tag{5}$$

where we have ignored the rest-mass contribution. Note that the term with the square brackets is spin dependent and can be traced back to a parity-odd, time-reversal even operator. A more complete nonrelativistic Hamiltonian including anisotropic contributions can be found in Ref. [25].

3. Experiment and Constraint

As mentioned above, the nonmetricity–neutron coupling operator in Eq. (5) is parity violating (PV). When a transversely polarized slow-neutron beam propagates through such a nonmetricity background, its spin vector rotates. To see this, we give a brief heuristic derivation below. For a beam of neutrons propagating in the z direction with its spin transversely polarized in a direction $\vec{n} = (\cos\alpha, \sin\alpha, 0)$, the initial state can be written as $|\psi_\perp\rangle = \frac{1}{\sqrt{2}}(e^{-i\alpha/2}|+\rangle + e^{i\alpha/2}|-\rangle)$, where

$|\pm\rangle$ represent helicity eigenstates. After passing a distance $L = |\vec{p}|t/m$ through liquid ^{4}He, the presumed source of nonmetricity, the Hamiltonian (5) indicates that, up to a total phase, $|\psi_\perp\rangle$ evolves to

$$|\psi_\perp(t)\rangle = \frac{1}{\sqrt{2}}(e^{-i(\alpha+\phi_{PV})/2}|+\rangle + e^{i(\alpha+\phi_{PV})/2}|-\rangle)\,, \tag{6}$$

where

$$\phi_{PV} = 2\Big[\Big(\zeta_2^{(4)} - m\,\zeta_9^{(5)}\Big)(N_1)_0 + \Big(\zeta_4^{(4)} - m\,\zeta_{10}^{(5)}\Big)(N_2)_0 + \frac{m^2}{2}\zeta_2^{(6)}S_{000}\Big]L\,. \tag{7}$$

Comparing to the initial state $|\psi_\perp\rangle$, ϕ_{PV} is just the spin rotation angle. In neutron physics, this phenomena is called neutron optical activity and is quantified by the rotary power $\frac{d\phi_{PV}}{dL}$. With an ingenious experimental design to cancel residual magnetic effects and other common-mode signals, this quantity was measured at NIST's Center for Neutron Research at the 1-σ level:

$$\frac{d\phi_{PV}}{dL} = +1.7 \pm 9.1(\text{stat.}) \pm 1.4(\text{sys}) \times 10^{-7}\,\text{rad/m}\,. \tag{8}$$

For details of this measurement, refer to Refs. [27–30]. Conversion to natural units together with Eq. (7) yields the following nonmetricity measurement:

$$\Big(\zeta_2^{(4)} - m\,\zeta_9^{(5)}\Big)(N_1)_0 + 2\Big(\zeta_4^{(4)} - m\,\zeta_{10}^{(5)}\Big)(N_2)_0 + m^2\,\zeta_2^{(6)}S_{000} = (3.4 \pm 18.2) \times 10^{-23}\,\text{GeV}\,. \tag{9}$$

We interpret this result as the 2-σ constraint

$$\Big|2\Big(\zeta_2^{(4)} - m\,\zeta_9^{(5)}\Big)(N_1)_0 + 2\Big(\zeta_4^{(4)} - m\,\zeta_{10}^{(5)}\Big)(N_2)_0 + m^2\,\zeta_2^{(6)}S_{000}\Big| < 3.6 \times 10^{-22}\,\text{GeV}\,. \tag{10}$$

Without extremely fine-tuned cancellations between the various nonmetricity couplings in Ref. (10), we may give the following individual bounds:

$$\begin{aligned}
&|\zeta_2^{(4)}(N_1)_0| < 10^{-22}\,\text{GeV}, \qquad |\zeta_4^{(4)}(N_2)_0| < 10^{-22}\,\text{GeV}\,,\\
&|\zeta_9^{(5)}(N_1)_0| < 10^{-22}, \qquad |\zeta_{10}^{(5)}(N_2)_0| < 10^{-22}, \qquad |\zeta_2^{(6)}S_{000}| < 10^{-22}\,\text{GeV}^{-1}.
\end{aligned} \tag{11}$$

The above limits, to our knowledge, provide the first measurement of $\zeta_2^{(6)}S_{000}$ as well as the first measurement of any nonmetricity component inside matter.

4. Summary

In this work, using the SME and a measurement of neutron rotary power, we have obtained the constraint (10) on isotropic nonmetricity–neutron couplings. Without any fine-tuned cancellation, this constraint can be further interpreted as bounds on various nonmetricity components (11). Note that the well-known PV physics in the usual Standard Model (SM) also leads to neutron spin rotation, but no quantitative result from first-principle calculations is available for this conventional effect due to the intractability of the strong interactions. However, from the most decent estimate in the literature [31], $(d\phi_{PV}/dL)_{\text{SM}} = -6.5 \pm 2.2 \times 10^{-7}\,\text{rad/m}$, which is still less than the statistics of the measurement (8), we conclude that the constraint (10) we extracted on nonmetricity is reliable. The SM background prevents more stringent bounds on nonmetricity by only improving the statistics but keeping the same methodology. Within the general theoretical framework of the SME, complementary constraints on nonmetricity components may be feasible in the near future including limits on anisotropic components.

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