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Nonequilibrium localization in quantum Hall systems at very low frequencies

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We have measured the conductivity σ_{xx} of GaAs/GaAlAs Corbino devices under nonequilibrium conditions in the quantum Hall regime at low frequencies from dc up to 10 kHz. At very low frequencies up to 10–20 Hz, σ_{xx} shows a steep decrease from its dc value towards a reduced saturation value. Measurements of the temperature dependence of σ_{xx} from 70 mK to 4.2 K revealed a dominance of a conductivity with a temperature dependence $\sigma_{xx} \sim \exp\{-(T_0/T)^{1/2}\}$ typical for variable-range hopping (VRH) at temperatures below 1 K and an additional contribution of thermal activation above 2 K. The characteristic temperature T_0 as determined from the VRH-like temperature plot of the conductivity showed also an increase with the frequency and a pronounced decrease with increasing voltage below the breakdown of the quantum Hall effect (QHE). We attribute these results to an effective suppression of the delocalization of electrons in alternating external fields due to a reduced heating and scattering between localization centers. The time scale for this process can be as long as 100 ms. Our data show that the background conductivity σ_{xx} at subcritical voltages affects the critical voltage for the breakdown of the QHE.

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I. INTRODUCTION

The conductivity mechanisms in Landau-quantized systems were widely investigated both experimentally and theoretically for the last three decades. In particular, the relation between the localization length ξ and the effective scattering length L_Φ near the mobility edges of the Landau level (i.e., in the plateau-transition region) was addressed in the scaling theory.[1–5] Various experiments were dedicated to the universality of the scaling exponent (see, e.g., Refs. 6–8). Measurements of the transition region between quantum Hall (QH) plateaus up to frequencies in the GHz range revealed the universality of scaling exponents up to these frequencies, concomitant with a reduction of the plateau width at higher frequencies.[9,10] In the plateau centers, measurements of ξ are generally difficult, due to the low values of σ_{xx} in the QH regime. However, at higher frequencies ξ has been measured recently, showing values of the localization length in the plateau centers approaching the magnetic length.[11] At higher temperatures—or driving voltages—respectively, it is possible to obtain measurable values of σ_{xx} at integer-filling factors (e.g., Ref. 12). It was shown that a monotonous increase of σ_{xx} with the voltage (of ρ_{xx} with the sample current) occurs before the quantum Hall effect (QHE) finally breaks down. It is a long-standing question how far the breakdown of the QHE itself is related to this increase of σ_{xx} in the subcritical regime. In earlier papers, a direct relation between the critical value of the voltage or current and the subcritical values of σ_{xx} or ρ_{xx} was postulated[12–14] in the framework of a phenomenological hot-electron model, whereas this relation was excluded in a model presented in a more recent paper.[15]

Usually, the conductivity in subcritical QH systems is attributed to variable-range hopping (VRH) for low temperatures below 1 K (see, e.g., Refs. 16–18). Increasing the temperature, thermal activation (TA) over the Landau gap begins to contribute and finally dominates the temperature dependence of σ_{xx}.[19–21] At intermediate temperatures $1\ \mathrm{K} < T < 4.2\ \mathrm{K}$, it was shown that both TA and VRH can coexist.[22]

In this study, we have investigated the conductivity σ_{xx} of QH devices in Corbino geometry as a function of the temperature between 70 mK and 4.2 K, of the source-drain voltage below the critical value, and of the frequency from dc up to 10 kHz. At very low frequencies below 20 Hz, the measured values of the nonequilibrium σ_{xx} drop steeply from their dc value to a saturation value, which is reached between 10 and 20 Hz and persists until some kHz. Thus, the localization under nonequilibrium conditions is stronger, and hence the stability of the QHE is higher compared to the dc case. This is a rather unexpected result in view of the recent findings concerning the ac QHE at kHz frequencies: precision measurement revealed an increasing loss of precision of the QH quantization, in particular at the plateau flanks.[23]

A low-frequency decay of the conductivity σ_{xx} under nonequilibrium conditions (further called nonequilibrium conductivity) was predicted in earlier papers to explain the steep increase of the maximum breakdown voltage and of the dynamical breakdown hysteresis, as observed in oscillation generators based on Corbino QH devices.[24,25] In our previous paper, ac measurements of the I-V characteristics of the QH breakdown in Corbino devices directly revealed higher upper limits of the breakdown hysteresis at frequencies of some Hz as compared to the dc curves.[25] Thus, the measurements in this study of the frequency dependence of the nonequilibrium conductivity σ_{xx} give direct experimental evidence for the predicted decrease of σ_{xx} with increasing frequency, and that σ_{xx} at subcritical voltages dominantly determines the critical voltage for the breakdown of the QHE. With this, our measurements cannot be explained by a model, postulating a breakdown of the QHE independent of the prebreakdown evolution of the conductivity.[15] Instead, our measurements confirm the prediction of the hot-electron model (as developed in Ref. 25) for the relation between the breakdown of the QHE and the nonequilibrium conductivity σ_{xx} in the prebreakdown (subcritical) regime. Further, we found a dominance of conductivity with a temperature dependence $\sigma_{xx} \sim \exp\{-(T_0/T)^{1/2}\}$ that is typical for variable-range hopping (VRH) at temperatures below 1 K and an additional contribution of thermal activation above 2 K. With this, our results show that also very low energy excitations (of the order of k_BT) can determine the breakdown, not only the interband excitations (of the order of $\hbar\omega_c$).

II. EXPERIMENTAL DETAILS

On two wafers of GaAs/GaAlAs heterojunctions (wafer *A*: electron density $n_s = 2.7 \times 10^{11}$ cm^{-2}, Hall mobility $\mu_H = 1.0 \times 10^5$ cm^2/V s; wafer *B*: $n_s = 4.8 \times 10^{11}$ cm^{-2}, $\mu_H = 1.8 \times 10^5$ cm^2/V s), Corbino samples with an inner contact radius r_i of 100 μm and three different radii r_a of the outer contact of 150, 200, and 300 μm (i.e., three widths of the two-dimensional (2D) channel of 50, 100, and 200 μm) were patterned (accordingly we refer to the samples as *A*100, *B*50, etc.). All samples were characterized by dc Shubnikov-de Haas measurements and by I-V characteristics. [For the values $V_{\min}$ and $V_{\max}$, limiting the hysteresis of the QH breakdown in the dc regime, see, e.g., the inset in Fig. 1(a).]

From the enhanced amplitude of the relaxation oscillations previously observed in the bistable regime of the QH breakdown in Corbino devices,[24] we deduced an enhanced dynamical hysteresis of the breakdown. Therefore, we confirmed this by taking the frequency dependence of the I-V curves at low ac frequencies and found a steep increase of the hysteresis in the frequency range from dc up to about 20 Hz (for details, see Ref. 25). This behavior was consistent for all samples used in this study (see a typical example in Fig. 1).

III. RELATION BETWEEN THE NONEQUILIBRIUM CONDUCTIVITY σ_{xx} AND THE BREAKDOWN OF THE QHE: THE HOT-ELECTRON MODEL WITH BACKGROUND CONDUCTIVITY

An explanation of the existence of the hysteresis of the current-voltage characteristics of QH devices at the breakdown of the quantum Hall effect can be derived from the hot-electron model (HEM).[12,13] In this model, the hysteresis arises from the metastability of the power balance between gain (electrical energy fed to the system per time and area), $p_{\text{gain}} = \sigma_{xx}(T_{\text{el}})E_r^2$ (σ_{xx} is the longitudinal conductivity of the QH system, E_r is the radial electric field in the Corbino device), and the energy-loss rate $p_{\text{loss}} = [\varepsilon(T_{\text{el}}) - \varepsilon(T_L)]/\tau_{\text{rel}}$, which describes the relaxation of the energy per area $\varepsilon(T_{\text{el}})$ at the elevated electron temperature T_{el} back to the energy per area $\varepsilon(T_L)$ at the lattice temperature T_L. The relaxation time τ_{rel} depends on the electron temperature for scattering at phonons[13] and, as shown later, by scattering at impurities.[14,26]

The energy per area $\varepsilon(T)$ of the electron system as a function of temperature can be analytically calculated for the Fermi energy E_F situated in the middle of the Landau gap, if there is a constant background density of (localized) states D_{BG}, and if the relation $k_BT \ll \hbar\omega_c$ holds (k_BT: thermal energy, $\hbar\omega_c$: cyclotron energy). This is valid up to temperatures of 12–15 K for our samples, whereas the breakdown value $V_{\max}$ is typically determined by electron temperatures of about 5–7 K. Thus, low-energy excitations of the order of k_BT within the localized states of the density D_{BG} determine the value of $V_{\max}$ within this approach. In contrast, a recent model attributes the breakdown to excitations at the higher energy of the order of $\hbar\omega_c$ only, regarding D_{BG} as unimportant in this respect.[15] An essential aim of this study is to confirm the validity of the approach considering low-energy excitations within localized states of the density D_{BG} as given above for the samples investigated in our experiments.

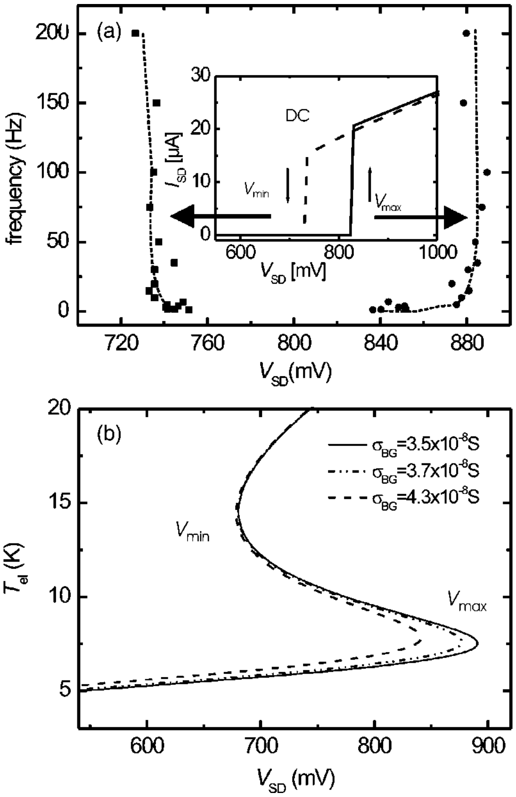

FIG. 1. Dynamic hysteresis of the QH breakdown as a function of the frequency (sample *A*100, $\nu = 2$). (a) Separate plot of the hysteresis limits $V_{\max}$ and $V_{\min}$ as a function of the frequency. Inset: corresponding I-V characteristics (dc), showing the hysteresis in the zero-frequency limit. (b) Modeling of the change of the hysteresis, as observed for sample *A*100, by a variation of the background conductivity σ_{BG} (hot-electron model, see text).

From the power-balance equation, the relation between the radial electric field and the electron temperature can be evaluated, if the temperature dependence of the conductivity σ_{xx} is known,

$$\sigma_{xx}(T_{\text{el}}) = \sigma_0 \exp\{-\Delta/k_BT_{\text{el}}\} + \sigma_{\text{BG}} \tag{1}$$

The first term in the sum of Eq. (1) describes the thermal activation (the activation gap is $\Delta = \hbar\omega_c/2$), and the second term σ_{BG} is a background conductivity, which can be attrib-

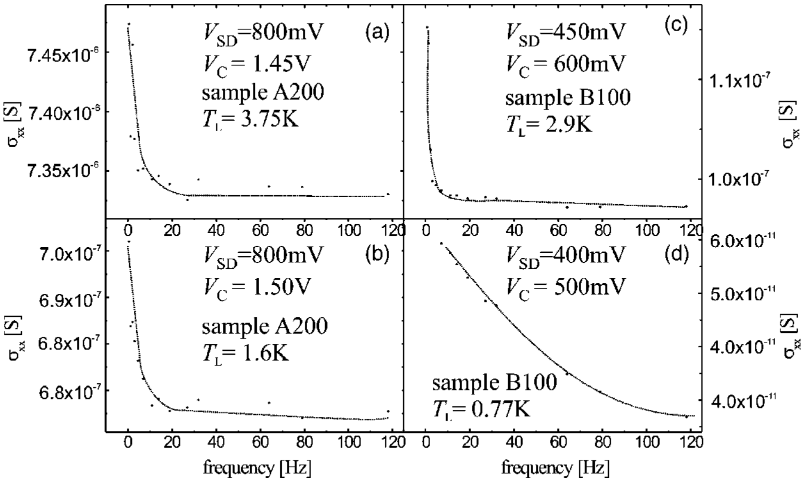

FIG. 2. Longitudinal conductivity σ_{xx} as a function of the frequency for the samples *A*200 (a) and (b) and *B*100 (c) and (d), measured at different temperatures (see legend) at filling factor $\nu=2$. The values of V_{SD} (amplitude of the ac voltage) are set well below the critical value V_{C} of the dc-QH breakdown (see legend).

uted, for example, to hopping (see also Ref. 14). Whereas the first term dominates at higher temperatures above about 12 K and determines $V_{\min}$, the background conductivity σ_{BG} dominates $V_{\max}$ at electron temperatures of about 5–7 K [see Fig. 1(b)]. Thus, a neglecting of σ_{BG} (or D_{BG})[15] has no essential influence on the determination of $V_{\min}$, but leads to values for $V_{\max}$ being too small for our samples (see below).

With an increase in σ_{BG}, $V_{\max}$ exhibits a pronounced decrease [see Fig. 1(b)] up to the complete disappearance of the hysteresis ($V_{\max} \approx V_{\min}$) at $\sigma_{\mathrm{BG}} = 1.3 \times 10^{-7}$ S $= 1.7 \times 10^{-3} \sigma_{xy}$. If we attribute σ_{BG} to a hopping conductivity, we can formulate σ_{BG} as follows for variable-range hopping (VRH):[16–18,27]

$$\sigma_{\mathrm{BG}} = \sigma_0(T) \exp\left\{ - \left(\frac{T_0}{T} \right)^{1/2} \right\} \quad \text{with} \tag{2}$$

$$\sigma_0(T) = S_0/T \quad \text{and} \tag{2a}$$

$$T_0 = \frac{C_{\mathrm{hop}} e^2}{k_{\mathrm{B}} 4\pi\varepsilon\varepsilon_0 \xi} \tag{2b}$$

(S_0: constant conductivity prefactor; T_0: characteristic temperature; C_{hop}: hopping constant). The temperature exponent $\frac{1}{2}$ for variable-range hopping is, in contrast to the exponent $[1/(1+d)]d$ dimensionality of the system) of Mott hopping,[28] independent of the dimensionality and related to a soft Coulomb gap near the Fermi energy.[29]

The essential parameter for σ_{BG} is the localization length ξ, if hopping conductivity is assumed to be the essential mechanism. An increase in the localization length (decrease of T_0) yields, in accordance with Eq. (2), an increase in the hopping conductivity. Consequently, from the observed increase of the hysteresis with the frequency as reported in Ref. 25 a reduction of σ_{BG} was predicted, accompanied by an increase in T_0. In general, an increase in T_0 can be related to a decrease in ξ, meaning a more effective localization. However, it is questionable how far Eq. (2b), which was deduced in the linear-response limit, can be applied quantitatively at rather high voltages near the breakdown of the QHE. Therefore, in this study we have measured the conductivity σ_{xx} as a function of the bias voltage, frequency, and temperature. Our data show clearly a trend of a suppressed delocalization of electrons already at low frequencies.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

As discussed previously,[25] the HEM with background conductivity predicts that the steep increase in the critical voltage $V_{\max}$ of the QH breakdown observed at low frequencies is due to a reduction of the longitudinal conductivity σ_{xx} by about $1-10 \times 10^{-8}$ S (depending on the sample properties).

Fig. 2 shows the dependence of σ_{xx} on the frequency up to 120 Hz, as measured on the samples *A*200 [Figs. 2(a) and 2(b)] and *B*100 [Figs. 2(c) and 2(d)] at different temperatures (3.75 K and 1.6 K for *A*200; 2.9 K and 0.77 K for *B*100). The low-frequency drop of σ_{xx} (from dc to 10–20 Hz) was found between 2×10^{-8} S [see Fig. 2(c)] and 1.4×10^{-7} S [Fig. 2(a)] at temperatures above 1.5 K. (Please note that σ_{xx} is 4–5 orders smaller at 0.77 K [Fig. 2(d)], measured above $f \geqslant 7$ Hz with a highly sensitive amplifier, which made it, however, impossible to resolve the far steeper change of σ_{xx} at frequencies $f < 7$ Hz in this case).

Consequently, the measured decrease in the nonequilibrium values of σ_{xx}, which occurs in the same frequency range as the previously measured increase in the dynamical hysteresis of the QHE breakdown, directly confirms the prediction of Ref. 25. This is straight experimental evidence that σ_{xx} at subcritical voltages ($V_{\mathrm{SD}} < V_{\min}$) directly determines the breakdown of the QHE.

Further, the fact that the HEM provides a decrease of σ_{xx} which is required to explain the increase of the critical voltage $V_{\max}$ in agreement with our measurements, proves that $V_{\max}$ is determined by the contribution σ_{BG} [Eq. (1)], which is dominant at lower temperatures ($T_{\mathrm{el}} \approx 6$ K). For these temperatures, the thermal energy of the electrons $k_{\mathrm{B}}T \approx 0.5$ meV is about a factor of 10 smaller than $\hbar\omega_{\mathrm{c}}/2$. Therefore, for typical sample parameters ($\hbar\omega_{\mathrm{c}} \approx 10$ meV, Landau-level broadening $\Gamma \approx 0.035\hbar\omega_{\mathrm{c}}$ and density of localized states $D_{\mathrm{BG}} \approx 0.2\ \mathrm{m}^*/\pi\hbar^2$) the contribution of the thermal activation to the enhanced energy per area $\varepsilon(T_{\mathrm{el}})$ is approximately by a factor of 10^2 smaller than the contribution of the

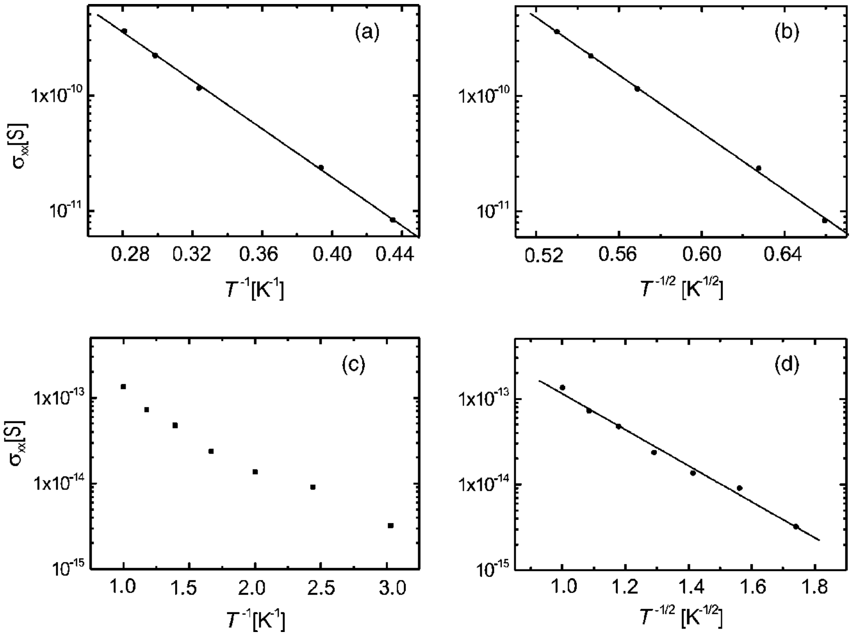

FIG. 3. Longitudinal conductivity σ_{xx} (dc) at filling factor $\nu=2$ as a function of the temperature. (a) Arrhenius plot $\sigma_{xx}=f(1/T)$, sample $A200$, $V_{\mathrm{SD}}=360$ mV, $T>2$ K. (b) VRH plot $\sigma_{xx}=f(1/T^{1/2})$ of the same data as in (a). (c) Arrhenius plot $\sigma_{xx}=f(1/T)$, sample $A100$, $V_{\mathrm{SD}}=804$ mV, $T<1$ K. (d) VRH plot $\sigma_{xx}=f(1/T^{1/2})$ of the same data as in (c).

low-energy excitations ($2\,k_{\mathrm{B}}T$ around the Fermi energy) within the localized states, characterized by D_{BG}. This means, that the value $V_{\max}$ is determined by low-energy transitions (between localized states in the vicinity of the Fermi energy). This is a rather unexpected result, as the breakdown of the QHE is usually attributed to higher energy differences (from the localized states near the Fermi energy to the next higher extended state—for an overview, see Ref. 30). In particular, in a recent paper[15] the authors concluded that the breakdown does not essentially depend on the density of localized states D_{BG} or on hopping contributions σ_{BG} to σ_{xx}. The measured change of σ_{xx} and $V_{\max}$ with the frequency confirms that this conclusion is not applicable for our sample parameters at electron temperatures below 10 K. Our previous calculations[25] yielded a different behavior of the lower limit of the breakdown hysteresis, $V_{\min}$. As is visible in Fig. 1, $V_{\min}$ was found experimentally to be only weakly dependent on the frequency, in accordance with our calculations. As $V_{\min}$ is determined by electron temperatures of $T_{\mathrm{el}}\approx 15$ K, σ_{BG} is not as important in comparison to the contribution of thermal activation over the Landau gap. Therefore, the conclusion in Ref. 15, that the QH breakdown is dominated by transitions to the extended states (and not by σ_{BG}), is only applicable to our samples with respect to the lower limit of the breakdown hysteresis $V_{\min}$.

Although the order of magnitude of the measured change of σ_{xx} agrees quite well with the prediction of the HEM, the measured saturation values of σ_{xx} at $T>1.6$ K are 1 or 2 orders higher than calculated [larger deviations at higher temperatures, see Figs. 2(a)–2(c)].

In order to explain these larger saturation values of σ_{xx}, one could in principle invoke the idea of an inhomogeneous carrier distribution and the resulting evolution of current-carrying filaments[31,32] at the breakdown of the QHE.[33] These filaments develop preferentially in sample zones with a locally enhanced σ_{xx} (filament) $=\sigma_h$, whereas other parts of the sample can remain in the QH regime at lower values of σ_{xx} (QHE) $=\sigma_\ell$. Thus, the breakdown, also as modeled within the HEM, is dominated by the local properties of the filaments (i.e., by σ_h). If the filaments occupy an angular sector α of the Corbino device [the remaining sector $(2\pi-\alpha)$ is at σ_ℓ], a global measurement of $\sigma_{xx}=\langle\sigma\rangle$ yields

$$\langle\sigma\rangle=\frac{I_{\mathrm{SD}}}{V_{\mathrm{SD}}}\frac{\ln\left(\frac{r_{\mathrm{a}}}{r_{\mathrm{i}}}\right)}{2\pi}=\sigma_\ell+\frac{\alpha}{2\pi}(\sigma_h-\sigma_\ell)<\sigma_h \qquad (3)$$

Thus, inhomogeneities acting as precursors of the filament would lead to subcritical σ_{xx} values measured at values lower than predicted by the HEM. Instead, the experiments yield the opposite trend.

If we assume an additional 2D conduction mechanism to explain the higher saturation values of σ_{xx}, this would directly influence the measured hysteresis and is therefore less probable. It is more likely that the higher saturation values of the conductivity [although absolutely still small—a measured value of $\sigma_{xx}(\mathrm{sat})=6.7\times10^{-7}$ S corresponds to $8.6\times10^{-3}\sigma_{xy}$] are due to parallel conduction in the GaAlAs doping layer of the heterostructure wafers. Such a parallel channel would only influence the measured value $\langle\sigma\rangle$, but neither σ_{xx} in the 2DES nor the breakdown properties of the QHE in this system. This assumption is supported by the strong decrease of $\sigma_{xx}(\mathrm{sat})$ when decreasing the temperature below 1 K (see below).

To clarify the conduction mechanisms which contribute to σ_{xx}, we measured the temperature dependence in two different temperature ranges (between 2 K and 4 K in a He^4 cryostat, and between 330 mK and 1 K in a $\mathrm{He}^3/\mathrm{He}^4$ dilution refrigerator) at various fixed voltages V_{SD} below the critical breakdown value and fixed frequencies from 7 to 27 Hz.

In Fig. 3, we see two different plots of $\sigma_{xx}(T)$ [$\ln\sigma_{xx}=f(T^{-1})$ assuming thermal activation, and $\ln\sigma_{xx}=f(T^{-1/2})$ assuming VRH] for the two temperature ranges (wafer A). At temperatures above 2 K, both plots yield reasonable linearity [Figs. 3(a) and 3(b)]. Consequently, TA and VRH are not

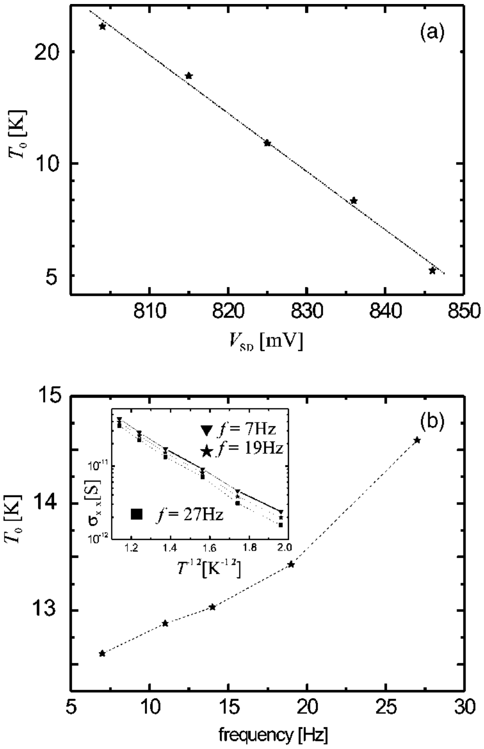

FIG. 4. Characteristic temperature T_0 obtained from VRH plot (see inset), (a) as a function of the subcritical dc voltage V_{SD} (70 mK$<T<$770 mK, sample A200) and (b) as a function of the frequency (70 mK$<T<$770 mK, sample B100, subcritical ac amplitude V_{SD}=400 mV).

separable at these temperatures. However, at temperatures below 1 K, the VRH plot yields a better linearity than the TA plot [Figs. 3(c) and 3(d)]. Thus, at these temperatures, VRH may be assumed to be dominant to determine σ_{xx}. The fact that the absolute values of σ_{xx} are smaller than in Fig. 2 is due to the lower electric fields applied (the values given in Fig. 2 denote the amplitudes of the sinusoidal voltage) for the temperature-dependent measurements.

From the VRH plots, the values of the characteristic temperature T_0 can be deduced. Fig. 4 shows the dependence of T_0 on the applied voltage V_{SD} and on the frequency f. As expected, T_0 decreases markedly with increasing subcritical voltage [dc, see Fig. 4(a)]. This indicates a growing delocalization of the electrons with the voltage (already at subcritical voltages $V_{SD}<V_{min}$). The question may arise of whether a hopping conduction can be assumed to persist at rather high subcritical voltages of $V_{SD}\sim 0.5–0.8V_{min}$, and whether a localization length ξ can be deduced from the values of T_0 obtained at higher voltages. Rather few theoretical[18,34] and experimental[22,35,27] studies have addressed the problem of hopping conductivity in QH systems in the presence of higher electric fields. In these studies, the influence of the finite electric field $\boldsymbol{E}$ is described by an effective temperature T_{eff}, which is enhanced by the energy gain of the electrons in the electric field according to

$$k_B T_{eff} = \tfrac{1}{2} e \boldsymbol{E} \xi \tag{4}$$

Taking typical values of our experiment, $|\boldsymbol{E}|=4$ kV/m and an estimated value of $|\boldsymbol{\xi}|\approx 10$ μm {the decrease of T_0 as a function of V_{SD} [Fig. 4(a)] and the increase of T_0 as a function of the frequency [Fig. 4(b)] would correspond to an increase of ξ from 4 μm to 22 μm and to a decrease of ξ from 7.7 μm to 6.6 μm, respectively, if Eq. (2b) were applicable}, one obtains $T_{eff}\sim 200$ K. Such a temperature would not only terminate the hopping regime, but would also result in a complete breakdown of the QHE. However, as the QHE clearly persists at $|\boldsymbol{E}|>4$ kV/m in our samples, the localization length ξ as estimated from the VRH plots of σ_{xx} vs temperature as in Fig. 3(d) cannot be correlated to the energy gain of the electrons. This is because the scattering properties of QH systems are highly anisotropic due to the Hall angle (between current density $\boldsymbol{j}$ and electric field $\boldsymbol{E}$) close to 90°. Under ideal QH conditions in a Corbino device, the radial voltage V_{SD} drives a circular current I_y, and $I_{SD}=0$ holds. Assuming an average distance between two scattering events (most likely at Coulomb potentials of impurities[26]) along the drift direction of the electrons of the order of the localization length ξ, a single inelastic scattering event would displace the electrons, traveling on equipotential lines perpendicular to the radial field, radially by about the cyclotron diameter, $\Delta r\sim 2R_c$. In case of low Landau levels, Δr is of the order of the magnetic length. Correspondingly, the theory of Fogler *et al.*[34] predicts a localization length approaching $2R_c$ in the center of a QH plateau. As it is difficult to measure the localization length at low electric fields at integer-filling factors, this prediction is hard to prove. However, the asymptotic behavior of $\xi(\nu)$ measured in Refs. 11 and 35 shows a trend of a reduction of ξ towards R_c if ν approaches integer values. Thus, if we replace $\boldsymbol{\xi}$ in Eq. (4) by its component in the direction of the electric field, i.e., the magnetic length, we obtain an enhancement of the effective temperature of $\Delta T\sim 0.2–0.3$ K. This would explain the persistence of both the QHE and the VRH regime at bath temperatures below 1 K.

The meaning of a quantity ξ as obtained from a VRH plot according to Eq. (2b) at high electric fields remains a question if the scattering is highly anisotropic. As the order of this quantity corresponds to the typical drift distances between scattering events obtained in earlier studies,[33] ξ(VRH) may stand for the unperturbed circular movement of electrons. Because this distance, at least for an assumed isotropic distribution of scatterers, has also to be overcome radially by the electrons to contribute to I_{SD}, a corresponding value of ξ may also determine the VRH plots of $\sigma_{xx}(T)$. However, this assumption would involve multiple scattering at impurities, displacing the electrons in radial direction by about $2R_c$ for each scattering event, without a complete loss of the phase of the wave function. To our knowledge, there is yet no complete theory of the anisotropic scattering properties in noneqilibrium QH systems available.

As mentioned above, the frequency dependence of the characteristic temperature T_0, obtained from $\sigma_{xx}(T)$ at different frequencies 7 Hz$<f<$27 Hz, would correspond to a small decrease of the localization length ξ. The moderate decrease corresponds to the slight reduction of σ_{xx} as shown in Fig. 2(d) for the same sample B100 at $T<1$ K in the same frequency range. (As mentioned above, measurements of σ_{xx}

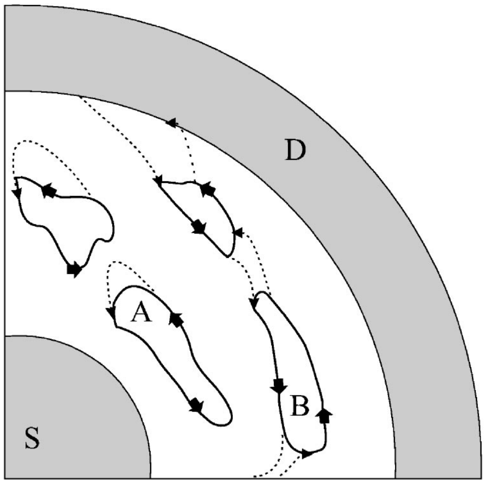

FIG. 5. Schematic picture of the suppression of the delocalization at low frequencies. Electrons start from the compressible islands and tunnel towards neighboring compressible islands. The electric field changes sign before the compressible islands start to overlap (ac, trajectory *A*, no contribution to source-drain current). If the electric field is permanently applied, the electrons accomplish their way between the electrodes via the compressible islands (dc, trajectory *B*, finite source-drain current).

below 7 Hz were technically not possible for $T<1$ K.) From the results we conclude that the delocalization in the QH system becomes less effective in comparison to the dc case if alternating electric fields with high but subcritical amplitudes are applied.

In Fig. 5, we show a scheme of a possible scenario to explain the effect: the closed loops represent localized states (compressible islands) inside the QH system. If a constant electric field is applied or the frequency is sufficiently low, electrons tunnel at a rather high rate between the compressible islands, which increases by this process.[36,37] Thus, with a certain probability they can be transferred from the source (S) to the drain (D) contact via these compressible islands (trajectories B in Fig. 5). If an alternating electric field is applied, the electrons start from any compressible islands and tunnel at a certain rate towards the neighboring compressible islands electrode via the localized states during the presence of one half wave $t_{1/2}$ of the electric field. If this half wave $t_{1/2}$ is short enough, the polarity of the ac field changes before the compressible states start to overlap (trajectories of type A in Fig. 5). The trajectories of type A contribute nothing to the source-drain current and thus to σ_{xx}.[36,37] Furthermore, a redistribution of the compressible islands themselves can be assumed to be responsible for the delocalization of electrons at high electric fields.

As the steep drop of σ_{xx} occurs typically at about 10 Hz (please note that the drop in Fig. 2(d) is by orders of magnitude smaller than in Figs. 2(a)–2(c) and represents the very tiny changes within the saturation of σ_{xx}), the time $t_{1/2}=1/(2f)$ is of the order of 50 ms. This appears rather long in view of the delocalization times of a few nanoseconds as measured in pulsed breakdown experiments.[33] However, the measurements presented here are performed at subcritical electric fields of $E_{\mathrm{SD}}/E_{\mathrm{c}}\approx 0.5$ (E_{c} is the critical electric field for the breakdown of the QHE), whereas the data in Ref. 33 were taken at $1.2<E_{\mathrm{SD}}/E_{\mathrm{c}}<3$. The delocalization times are determined basically by the transition times between compressible islands or the redistribution of those, which are a strong function of the applied field and of the temperature. (Time scales for the redistribution of charges between localized states up to about 20 min at mK temperatures have been observed.[38]) The dissipative current density j_x related to trajectories of the type B is quite small. A simple estimate of $j_x=n_s e\xi(T_{\mathrm{el}},E_{\mathrm{SD}},f)/t_{1/2}$ yields 1.2×10^{-7} A/m for the saturation current density for sample $B100$ at $T_{\mathrm{el}}<1$ K, $E_{\mathrm{SD}}/E_{\mathrm{c}}\approx 0.5$, $f>10$ Hz, far smaller than the corresponding circular current density $j_y=\sigma_{xy}E_{\mathrm{SD}}\approx 0.3$ A/m. The saturation value of $\sigma_{xx}=2.9\times10^{-11}$ S from the estimate above agrees suprisingly well with the saturation value measured for this sample at $T=0.77$ K, $f=27$ Hz, and $V_{\mathrm{SD}}=0.4$ V. In view of the enhanced saturation values as discussed above, this could indicate a strong reduction of the parallel conduction in the doping layer when reducing the temperature from 1.6 K to 0.77 K.

V. SUMMARY

In previous measurements, we have observed a steep increase of the dynamical hysteresis of the breakdown of the QHE with increasing frequency up to 10–20 Hz. This increase stems from an increase of the upper hysteresis limit $V_{\max}$ at an almost unchanged lower hysteresis limit $V_{\min}$ (see Ref. 25). We could predict such a behavior within a hot-electron model (HEM) by the assumption of a background conductivity σ_{BG} of the QH system which drops already at low frequencies. In this study, we proved this by measurements of the conductivity σ_{xx} (at voltages far from equilibrium, but below the critical voltage for the breakdown of the QHE) of GaAs/GaAlAs Corbino devices in the quantum Hall (QH) regime at low frequencies from dc up to 10 kHz. At very low frequencies up to 10–20 Hz, σ_{xx} shows a steep decrease from its dc value towards a reduced saturation value. This decrease of σ_{xx} is of the same order as predicted by the HEM. Measurements of the temperature dependence of the nonequilibrium σ_{xx} from 70 mK to 4.2 K revealed a dominance of a background conductivity with a temperature-dependence-like variable-range hopping (VRH) $\sigma_{xx}\sim\exp\{-(T_0/T)^{1/2}\}$ for temperatures below 1 K and an additional contribution of thermal activation above 2 K. This is a clear evidence that the conductivity at subcritical voltages, in particular the background contribution σ_{BG}, directly affects the maximum breakdown limit $V_{\max}$ of the QHE. This background contribution, and thus $V_{\max}$, are therefore dominated by low-energy excitations between compressible islands. In contrast, the lower limit $V_{\min}$ of the hysteresis is determined by activation processes from localized to extended states. The characteristic temperature T_0 as determined from VRH-like temperature plots of the conductivity also showed an increase with the frequency. We attribute these results to a less-effective delocalization of electrons in alternating external fields with increasing frequency, due to a reduced heating and scattering between compressible islands.

The time scale for the additional ac localization is rather long (50–100 ms) and determines the saturation limit of the nonequilibrium conductivity σ_{xx} of the QH system.

From the results we can conclude for our samples that the breakdown stability of the ac QHE at low frequencies is distinctively higher than in the DC case.

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