COMPEL - The international journal for computation and mathematics in electrical and electronic engineering

Geometric dynamics of nonlinear circuits and jump effects

Tina Thiessen, and Wolfgang Mathis,

Geometric dynamics of nonlinear circuits and jump effects

Tina Thiessen and Wolfgang Mathis
Faculty of Electrical Engineering and Computer Science, Institute of Theoretical Electrical Engineering, Leibniz Universität Hannover, Hannover, Germany

Abstract
Purpose – This paper seeks to give an outline about the geometric concept of electronic circuits, where the jump behavior of nonlinear circuits is emphasized.

Design/methodology/approach – A sketch of circuit theory in a differential geometric setting is given.

Findings – It is shown that the structure of circuit theory can be given in a much better way than by means of a description of circuits using concrete coordinates. Furthermore, the formulation of a concrete jump condition is given.

Originality/value – In this paper, an outline is given about the state of the art of nonlinear circuits from a differential geometric point of view. Moreover, differential geometric methods were applied to two example circuits (flip flop and multivibrator) and numerical results were achieved.

Keywords Circuits, Differential geometry

Paper type Research paper

1. Introduction
It is known that the structural aspects of classical mechanics can be represented in an elegant manner using modern differential geometry[1];, e.g. the monograph of Arnold (1988). Although the theory of nonlinear dynamical circuit is another useful concept for modelling dynamical systems, a complete geometrical theory is still missing. But some aspects of circuit theory were discussed in a differential geometrical setting (Brayton and Moser, 1964; Smale, 1972; Desoer and Wu, 1972; Matsumoto *et al.*, 1981; Ichiraku, 1979; Chua, 1980) and some other authors. In this paper, an outline about the geometric concept of electronic circuits is given where we emphasize the jump behavior of nonlinear circuits.

Classical approaches for analysing electrical circuits are based on fundamental physical laws; that is the constitutive relations of circuit elements and Kirchhoff's laws for describing the interconnections of circuit elements – the so-called circuit topology. The constitutive relations can be decomposed into resistive and reactive relations where the resistive relations are formulated by currents and voltages whereas the reactive relations are formulated by currents and voltages as well as their derivatives. As a result we obtain a system of differential equations that describes the dynamics of a considered circuit. Moreover, we are able to study the exceptional behavior of such a circuit and characterize it by certain conditions.

However, if we are interested in the description of an entire class of circuits without knowing the details about its circuit topology, a geometric setting is more suitable. In the

The author would like to thank the German Research Foundation (DFG) for the financial support of this project.

COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering
Vol. 30 No. 4, 2011
pp. 1307-1318

0332-1649
DOI 10.1108/03321641111133217

following, the derivation of the descriptive equations as well as the motivation of the differential geometric setting are illustrated by means of a simple tunnel diode circuit due to Brayton and Moser (1964) with a S-shaped nonlinearity $g(u)$ (Figure 1).

For the analysis of the tunnel diode circuit we use following equations:

$$u = -Ri_L + U_0 - u_L, \quad i = g(u_C), \tag{1}$$

$$L\frac{di_L}{dt} = u_L, \quad C\frac{du_C}{dt} = i_C. \tag{2}$$

By means of Kirchhoff's equations:

$$u = u_C, \quad i_L = i + i_C, \tag{3}$$

we eliminate u_L and i_C and obtain the dynamical descriptive equations of this circuit:

$$u_C = -Ri_L + U_0 - L\frac{di_L}{dt}, \quad i_L = g(u_C) + C\frac{du_C}{dt}. \tag{4}$$

2. Differential geometric concept of circuits

In order to understand the fundamental ideas behind the geometrical theory of nonlinear dynamical circuits, we reinterpret the description of the tunnel diode circuit from a geometric point of view. For this purpose, we consider the static behavior by means of an associated resistive circuit. Then, we add dynamical circuit elements (capacitors and inductors) to the resistive circuits. These two steps can be interpreted in a geometrical sense.

For deriving the associated resistive circuit we apply the following procedure:

(1) All capacitors have to be replaced by open-circuits.

(2) All inductors have to be replaced by short-circuits.

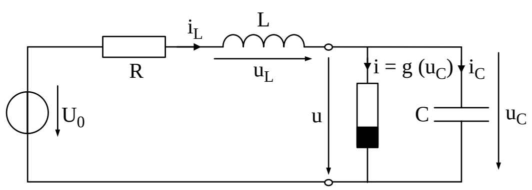


Figure 1.
Tunnel diode circuit

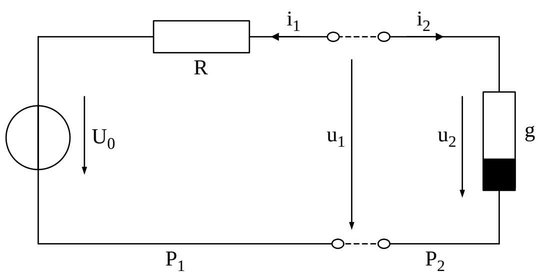


Figure 2.
Resistive circuit composed of P_1 and P_2

As a result, we get a nonlinear resistive circuit, Figure 2, where the operational points have to be determined.

In this simple case, the operational points can be obtained graphically. For this purpose we decompose the circuit in the sub-circuits P_1 and P_2 and formulate their constitutive relations, separately. We obtain:

$$-i_1 = \frac{1}{R}(U_0 - u_1), \quad i_2 = g(u_2), \tag{5}$$

where we use the assigned directions of currents and voltages. Now, by means of Kirchhoff's equations:

$$i_1 + i_2 = 0, \quad u_1 = u_2, \tag{6}$$

the descriptive equations of P_1 and P_2 can be connected in an algebraic manner.

From a geometric point of view the constitutive relations of P_1 and P_2 are interpreted as subsets in a higher dimensional embedding space using their own coordinates i_1, i_2, u_1 and u_2 where the curves of the constitutive relations are intersections with the corresponding coordinate planes. Using Kirchhoff's equations and introducing new variables $i := i_2 = -i_1$ and $u := u_1 = u_2$, these relations can be projected into a common representation space – the $i - u$-space in our example. From a geometric point of view it is suitable to interpret these two subsets $\mathcal{O}_1$ and $\mathcal{O}_2$ in a two-dimensional embedding space $\mathbb{R}^2$ (Figure 3) and obtain the well-known load line representation.

In a more general geometric setting $\mathcal{O}_1$ and $\mathcal{O}_2$ can be interpreted as sets in the $2n$-dimensional embedding space $\mathbb{R}^{2n}$, where n is the number of branches. The intersection $\mathcal{O}_1 \cap \mathcal{O}_2$ can be denoted as the state space $\mathcal{S}$ of the resistive circuit.

Although this load line representation and the corresponding definition of the state space $\mathcal{S}$ is helpful in simple circuits, it is based on a mixture of resistive constitutive relations and Kirchhoff's laws. Therefore, a more systematic definition of $\mathcal{S}$ is needed. For this purpose, we define the Kirchhoffian space $\mathcal{K}$ as the set of all currents and voltages which satisfies Kirchhoff's laws. Moreover, the Ohmian space $\mathcal{O}$ is defined as the set of all currents and voltages which satisfies all resistive constitutive relations. Then the state space $\mathcal{S}$ of a circuit is defined as the intersection $\mathcal{S} := \mathcal{K} \cap \mathcal{O}$. In contrast to the former definition the concept with the Kirchhoffian and Ohmian space has the advantage that $\mathcal{K}$ has a vector space structure in linear and nonlinear circuits since Kirchhoff's laws are homogeneous equations. Note, circuits with linear resistors and independent sources have an Ohmian space with an affined structure such that also $\mathcal{S}$

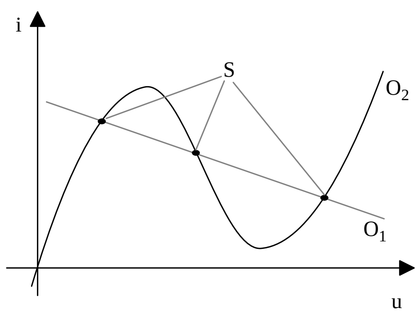


Figure 3.
Load line representation of a resistive circuit

is affine. This is the reason why it is better to denote this class of circuits "affine circuits" instead of "linear networks" as usual.

If the state space $\mathcal{S}$ of a resistive circuit consists of more than one state, it cannot be a realistic model because a physical system cannot exist in more than one state. Obviously, these situations can be solved if the time t is introduced as an additional variable such that we generalize our physical axiom for the state space: a physical system cannot be in more than one state at the same time. It follows that the behavior of a system is represented by t-parametrized curves in the state space $\mathcal{S}$. The set of all admitted curves in the state space of a circuit is denoted as circuit dynamics.

In the following, we restrict our considerations to differential systems and circuits where the dynamics is defined by the set of all solutions of the descriptive differential equations on a sufficient smooth state space $\mathcal{S}$. Therefore, the following questions arise:

- Under which conditions $\mathcal{S}$ is a smooth manifold?
- How a dynamics on $\mathcal{S}$ can be created?

If we assume that $\mathcal{S}$ is a smooth manifold, the dynamics of a circuit can be generated by differential equations. In order to formulate the descriptive differential equations of a circuit we have to introduce dynamical elements. Based on fundamental physical laws, the relationships between currents and voltages of inductors and capacitors are given by means of differential relations. Therefore, from a more general point of view, these differential equations are formulated in $i_L - u_L$ and $i_C - u_C$ coordinates planes of the embedding space $\mathbb{R}^{2n}$. In a differentiable geometric setting, these differential equations have to be formulated on the state space $\mathcal{S}$ such that a "lifting" process of the differential equations on $\mathcal{S}$ is needed.

We already emphasize that for defining the dynamics of a circuit it is very essential that the state space $\mathcal{S}$ is a smooth manifold. A set possesses the structure of a differentiable (smooth) m-dimensional manifold if it is locally equivalent to a $\mathbb{R}^m$. A concrete representation of a manifold can be given by means of a chart (map) that maps a part of $\mathcal{S}$ into $\mathbb{R}^m$. A detailed discussion about differentiable manifolds can be found in the monograph of Guillemin and Pollack (1974). The Kirchhoffian space $\mathcal{K}$ possesses the structure of a vector space and, therefore, of a differentiable manifold, but in general the Ohmian space $\mathcal{O}$ is not a differentiable manifold. However, even if $\mathcal{O}$ wears the structure of a differentiable manifold, it is not obvious that the state space $\mathcal{S}$ wears this structure. If we consider a circuit by its descriptive equations, it means that the intersection of the solution sets of the Kirchhoffian equations and the Ohmian equations is a smooth manifold if these equations are "local" independent. From a geometric point of view this means that the intersection of $\mathcal{K}$ and $\mathcal{O}$ is "transversal" or in a more technical setting: if $\mathcal{K}$ and $\mathcal{O}$ are two submanifolds of $\mathbb{R}^{2n}$ we call $\mathcal{K}$ and $\mathcal{O}$ transversal, if the following condition is satisfied:

$$x \in \mathcal{K} \cap \mathcal{O} : T_x\mathcal{K} \oplus T_x\mathcal{O} = T_x\mathbb{R}^n \tag{7}$$

Now, we are able to characterize the standard situation in nonlinear dynamical circuits. The state space $\mathcal{S}$ is a smooth manifold (Figure 4) if we have:

- the Ohmian space $\mathcal{O}$ is a smooth manifold; and
- the state space $\mathcal{S} = \mathcal{K} \cap \mathcal{O}$ is not empty as well as $\mathcal{K}$ and $\mathcal{O}$ are transversal.

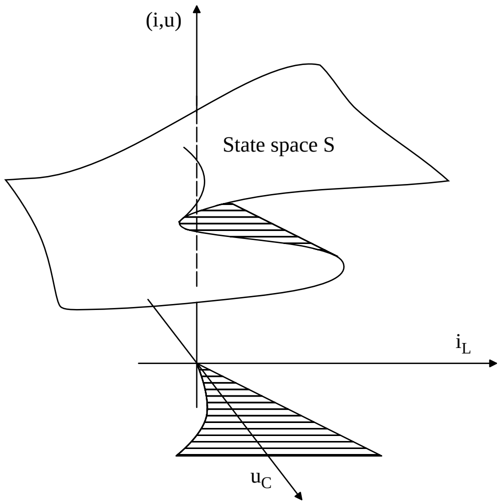


Figure 4.
Folded state space

These properties can be satisfied if we apply a suitable remodelling technique with resistive elements. Therefore, this situation is typical or so-called generic.

If the state space $\mathcal{S}$ is a smooth manifold the dynamics of a circuit can be defined. A dynamics on $\mathcal{S} = \mathbb{R}^m$ is defined by a set of differential equations $\dot{\xi} = X(\xi)$ where X is vector field on $\mathbb{R}^m$. In order to construct a vector field X on a smooth manifold $\mathcal{S}$, a more general approach is needed. We restrict ourself to circuits with λ capacitors and γ inductors. At first, we define a one-form Ω and a two-tensor G on the space of currents of inductors and voltages of capacitors. Then a projection map $\pi : \mathcal{S} \rightarrow \mathbb{R}_i^\lambda \oplus \mathbb{R}_u^\gamma$ is chosen that maps a certain part of $\mathcal{S}$ to the coordinate planes of the inductors and capacitors, respectively. Now we use the map π^* to "lift" or "pull-back" Ω and G on the state space $\mathcal{S}$. This operation is local because there are situations where $\mathcal{S}$ is folded just like in Figure 4. In this case, there is more than one part of $\mathcal{S}$ that can be mapped to the same part of the coordinate planes. With respect to the local dynamics of a circuit, the following theorem is fundamental.

If the Ohmian space $\mathcal{O}$ is a smooth manifold, the Kirchhoffian space $\mathcal{K}$ and the Ohmian space $\mathcal{O}$ are transversal and a pullback map π^* exists such that a one-form $\omega := \pi^*\Omega$ and a nondegenerated two-tensor $g := \pi^*G$ can be defined, then there exists locally a unique vector field $X : \mathcal{S} \rightarrow T(\mathcal{S})$ which satisfies:

$$g(X, Y) = \omega(Y), \tag{8}$$

for all smooth vector fields Y. With this locally defined vector field X we are able to define the (local) dynamics of a circuit by means of $\dot{\xi} = X \circ \xi$.

3. Nongeneric circuit behavior

3.1 Singular points and jumps

There are several cases where a locally defined vector field X does not exists. If $\mathcal{S}$ is a smooth manifold then it is essential that g is nondegenerated. The bilinear map

$g := \pi^*G$ can be interpreted as an inner product such that the assumed non-degeneracy of g follows from the condition $g(X, Y) = 0$ for all $Y \Leftrightarrow X = 0$. Therefore, a degeneracy of g results from defects of π^* or G. G is degenerated if $L(i)$ or $C(u)$ is zero for some i and u, respectively, where these nongeneric cases can be remodelled by parasitic reactances. A defect π^* is related to a dependency of the dynamic variables. With respect to the Kirchhoffian space $\mathcal{K}$ a defect of π^* corresponds to loops of capacitors and independent voltage sources or so-called cutsets of inductors and independent current sources. With respect to the Ohmian space $\mathcal{O}$ a defect of π^* is related to a zero of du_R/di_R or di_R/du_R such that above-mentioned loops and meshes arise. Also in these cases, a remodelling process is available in order to obtain a generic situation of the circuit dynamics. For further details the reader is left to Mathis (1987).

These considerations can be discussed in a more concrete manner if circuit topology is included. For this purpose we have to restrict ourself to RLC circuits. Then interconnections of a circuit can be described by oriented graphs and its boundary and co-boundary operators or assuming a coordinate system (a chart) by its incidence matrices. If we assume that a proper tree of a graph exists (i.e. a circuit including all capacitor branches and no inductor branches), then no so-called "forced degeneracies" arise. These forced degeneracies are defects of the dynamics related, e.g. to meshes of capacitors and cut-sets of inductors, which should be excluded from our discussion.

It is shown by Ichiraku (1978) that a point (i, u) of the state space $\mathcal{S}$ is a singular point if and only if the characteristic manifold $\mathcal{O}_R$ and the affine subspace $\mathcal{K}_R$ are not transverse at $(i_R, u_R) := \pi_R(\mathbb{R}^{2n})$ where π_R is the natural projection from the embedding space $\mathbb{R}^{2n}$ to the currents and voltages of the resistors. $\mathcal{K}_R$ is the Kirchhoffian space and $\mathcal{O}_R$ is the Ohmian space of the resistive circuit obtained from the given one by open-circuitting all inductor branches and short-circuiting all capacitor branches.

3.2 Chart representation of circuits and jump phenomena

In this section, we will discuss how degeneracy and jump effects of a concrete electronic circuit can be analysed. For this purpose, a suitable chart has to be chosen in order to represent $\mathcal{S}$ and the dynamics of a circuit by means of a DAE system:

$$\mathbf{B}(\mathbf{x})\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, \mathbf{y}), \tag{9}$$

$$0 = \mathbf{f}(\mathbf{x}, \mathbf{y}), \tag{10}$$

where $\mathbf{x}$ is the vector corresponding to the capacitor voltages and inductor currents and $\mathbf{y}$ is a vector of additional voltages and currents (Thiessen, *et al.*, 2010; Mathis *et al.*, 2009a, b). $\mathbf{B}(\mathbf{x})$ is a matrix related to the dynamical elements and $\mathbf{g}$ represents a nonlinear vector field with respect to $\mathbf{x}$ and $\mathbf{y}$. The solution set of the algebraic equation (10) represents the state space $\mathcal{S}$ of the circuit under consideration in the chosen chart, whereas the differential equation (9) represent its dynamical behavior.

As mentioned in the last section, a generic dynamics of a circuit do not exist at points where the projection map π^* has singularities. This means with respect to the DAE representation that points exists where the local solvability to $\mathbf{y}$ is not guaranteed. These points are specified by the following condition:

$$det\left(\frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{y})}{\partial \mathbf{y}}\right) = 0. \tag{11}$$

Therefore, we assume that equation (11) is a necessary jump condition (Nielsen and Willson, 1980; Tchizawa, 1984; Ichiraku, 1979) which can be formulated in a geometric setting if it is projected to the state space $\mathcal{S}$. We will give a sufficient condition for a jump point only in a heuristic sense: jump points are those points specified by equation (11) and that are an inner point of a set including Lyapunov stable and unstable points. The set of all points which fulfils these two conditions is called "jump set".

The corresponding "hit set" is the intersection of the "bundle" of all jump spaces at points of the jump set and the state space $\mathcal{S}$. Under the natural physical constraints, the energy of capacitors and the charge of inductors is preserved such that the voltage across a capacitance or the current through an inductance have inertia through the jump process and do not change. Another restriction is a fixed state, e.g. by an input voltage. Thus, the jump space is predefined and the trajectories "hit" by a continuation a stable point on the manifold $\mathcal{S}$. Obviously, for this construction an embedding space is needed.

4. Examples

4.1 Example: flip flop

In this section, the flip flop circuit shown in Figure 5 is analysed from a geometric point of view. We use the Ebers-Moll model in forward mode (Ebers and Moll, 1954) to reasonably model the nonlinear bipolar transistors.

The design parameters are $R_{c1} = R_{c2} = 2\,\mathrm{k\Omega}$, $R_{b1} = R_{b2} = 100\,\mathrm{k\Omega}$, $R_V = 50\,\mathrm{k\Omega}$, $I_S = 6.73\,\mathrm{fA}$, $V_T = 26\,\mathrm{mV}$, $\alpha_F = 0.99$ and $U_0 = 9\,\mathrm{V}$. The emitter current is declined by $i_{Fx} = I_S \cdot (e^{(U_{Dx}/V_T)} - 1)$. We neglected the reverse mode, the collector-emitter and the parasitic base-emitter capacitances. By Kirchhoff's law we can derive the algebraic constraints of the circuit:

$$0 = \Bigg[U_0 + (R_{c2} + R_{b1}) \cdot \frac{U_{in} - u_{D1}}{R_V} - \left(R_{c2} \cdot a_F \cdot I_S \cdot \left(e^{(u_{D2}/V_T)} - 1\right)\right.$$
$$\left. + (R_{c2} + R_{b1}) \cdot (1 - a_F) \cdot I_S \cdot \left(e^{(u_{D1}/V_T)} - 1\right) + u_{D1}\right)\Bigg] \cdot \frac{1}{R_{c2} + R_{b1}}, \qquad (12)$$

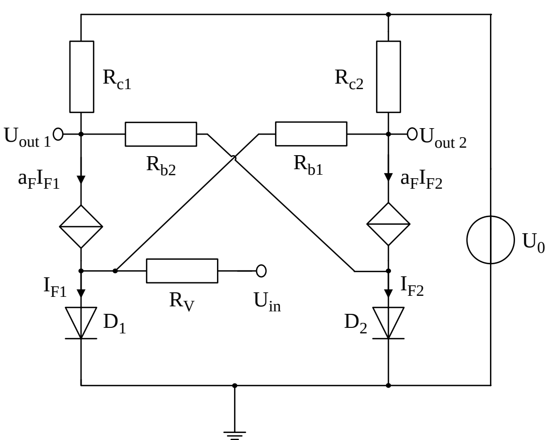


Figure 5.
Flip-flop circuit

$$0 = \left[U_0 - \left(R_{c1}\cdot a_F\cdot I_S\cdot\left(e^{(u_{D1}/V_T)} - 1\right)\right.\right.$$
$$\left.\left. +(R_{c1}+R_{b2})\cdot(1-a_F)\cdot I_S\cdot\left(e^{(u_{D2}/V_T)}-1\right) + u_{D2}\right)\right]\cdot\frac{1}{R_{c1}+R_{b2}}. \tag{13}$$

In equations (12) and (13), u_{D1} and u_{D2} represent the voltages across the diodes D_1 and D_2, respectively, and U_{in} is the input voltage. For a geometrical interpretation one can numerically determine the state space of the system given by the intersection of the solution sets of equations (12) and (13). For this purpose, one has to assume a certain range for one of the coordinates, e.g. u_{D1} and determine the other two afterwards with numerical methods (e.g. Newton-Raphson method).

From a geometric point of view, one has to look for a specific coordinate system where a fold can be identified. Because of the fixed input voltage one knows, that the input voltage cannot jump. So, it is obvious that one can identify a fold in the transfer characteristic of the system, i.e. the coordinate system $U_{out} - U_{in}$, which represents the projected state space of the circuit. In Figure 6, one can see the S-shaped transfer characteristic of the output signal U_{out2}. The flip flop circuit switches its outputs, when getting an appropriate input signal. One can also identify the unstable operating points by the negative slope in U_{out2} in Figure 6.

To identify the switching points one can determine the necessary jump condition given in Section 3 by equation (11):

$$\left[(R_{c2}+R_{b1})\cdot(1-a_F)\cdot\frac{I_S}{V_T}\cdot e^{(u_{D1}/V_T)} + 1 + \frac{R_{c2}+R_{b1}}{R_V}\right]$$
$$\cdot\left[(R_{c1}+R_{b2})\cdot(1-a_F)\cdot\frac{I_S}{V_T}\cdot e^{(u_{D2}/V_T)} + 1\right] = \frac{R_{c1}\cdot R_{c2}\cdot I_S^2\cdot a_F^2}{V_T^2}\cdot e^{(u_{D1}+u_{D2})/V_T}. \tag{14}$$

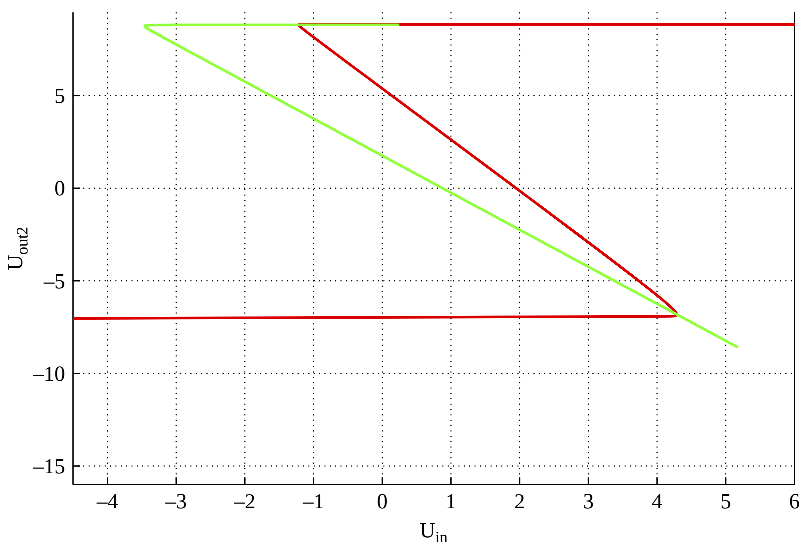


Figure 6.
Output 2 (S-shaped) with necessary jump condition (V-shaped)

The intersection of the transfer characteristic and the solution set of the necessary jump condition consist of unstable points of the state space (Figure 6). The sufficient condition is also fulfilled and the output voltage leaves the state space to jump to the hit set.

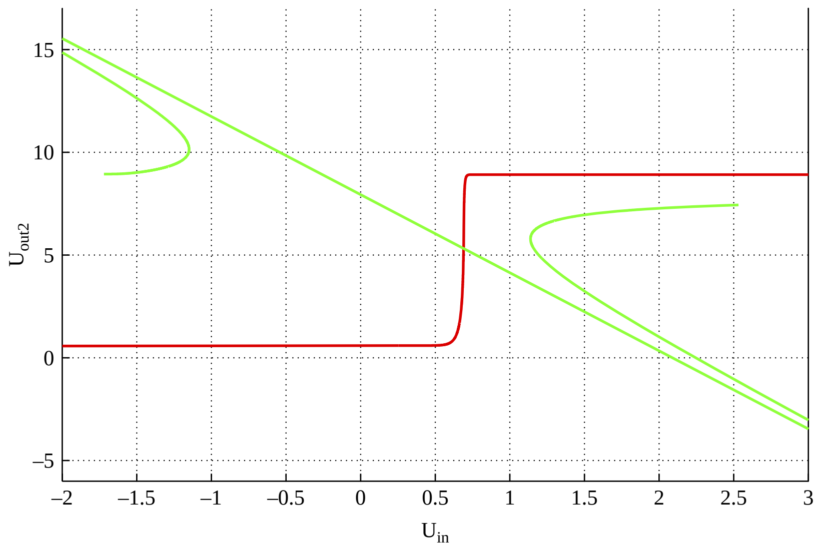
There are also parameter settings where the flip flop does not have a switching behavior. These parameter settings lead to a transfer curve shown in Figure 7, which is not any more S-shaped and has no Lyapunov unstable parts. The solution set of the necessary jump condition has no intersection with the transfer characteristic. The path intersecting with the transfer characteristic belongs to a pole in the evaluation of the jump condition. So the output voltage of the flip flop is just the amplified input signal and has no jump behavior.

4.2 Example: multivibrator

To analyse the multivibrator circuit shown in Figure 8, we also use the Ebers-Moll model in forward mode. The design parameters are $R_1 = R_4 = 10\,\Omega$, $R_2 = R_3 = 47\,\mathrm{k}\Omega, C_1 = C_2 = 10\,\mu\mathrm{F}$, $I_S = 6.73\,\mathrm{fA}$, $V_T = 26\,\mathrm{mV}$, $\alpha_F = 0.99$ and $U_0 = 9\,\mathrm{V}$. By Kirchhoff's law, we can derive the algebraic constraints of the circuit to:

$$0 = \frac{U_0 - u_{D1}}{R_3} + \frac{U_0 - u_{D1} - u_{C1}}{R_4} - a_F \cdot I_S \cdot \left(e^{(u_{D2}/V_T)} - 1\right) - (1 - a_F) \cdot I_S \cdot \left(e^{(u_{D1}/V_T)} - 1\right) \quad (15)$$

$$0 = \frac{U_0 - u_{D2}}{R_2} + \frac{U_0 - u_{D2} - u_{C2}}{R_1} - a_F \cdot I_S \cdot \left(e^{(u_{D1}/V_T)} - 1\right) - (1 - a_F) \cdot I_S \cdot \left(e^{(u_{D2}/V_T)} - 1\right) \quad (16)$$



Note: Jump conditions not fulfilled

Figure 7.
Output 2 with necessary jump condition

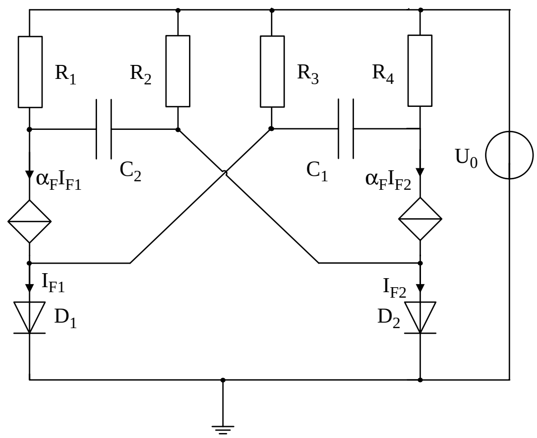


Figure 8.
Multivibrator circuit

and the dynamical equations to:

$$C_1 \cdot \dot{u}_{C1} = \left(U_0 - u_{C1} - u_{D1} - R_4 \cdot a_F \cdot I_S \cdot \left(e^{(u_{D2}/V_T)} - 1 \right) \right) \cdot \frac{1}{R_4} \tag{17}$$

$$C_2 \cdot \dot{u}_{C2} = \left(U_0 - u_{C2} - u_{D2} - R_1 \cdot a_F \cdot I_S \cdot \left(e^{(u_{D1}/V_T)} - 1 \right) \right) \cdot \frac{1}{R_1}. \tag{18}$$

Here, u_{D1} and u_{D2} represent the voltages across the diodes D_1 and D_2, respectively, and u_{C1} and u_{C2} represent the voltages across the capacitances.

To identify the switching points one can determine the necessary jump condition:

$$\begin{aligned} 0 = & -\frac{R_1 \cdot R_4 \cdot I_S^2}{V_T^2} \cdot e^{(u_{D1}+u_{D2})/V_T} \\ & + \left(\frac{R_1 \cdot I_S \cdot (1-a_F)}{V_T} \cdot e^{(u_{D1}/V_T)} + \frac{R_1}{R_2} + 1 \right) \cdot \left(\frac{R_4 \cdot I_S \cdot (1-a_F)}{V_T} \cdot e^{(u_{D2}/V_T)} + \frac{R_4}{R_3} + 1 \right). \end{aligned} \tag{19}$$

Here, it is also important to choose the right coordinate system to identify the fold in the state space. As mentioned in Section 3, the voltages across a capacitance cannot jump because of the physical constraints. Consequently, it is reasonable to choose a coordinate system in which just one coordinate can jump. In Figure 9, we choose the projection of the state space in the coordinate system $u_{D1} - u_{C1} - u_{C2}$. One can also see the associated jump set represented by the circles. To verify our results we have regularized the multivibrator circuit by parasitic capacitances parallel to the diodes and calculated the resulting limit cycle. As one can see, parts of the limit cycle lies on the state space manifold and leaves the state space in some specific point of the jump set.

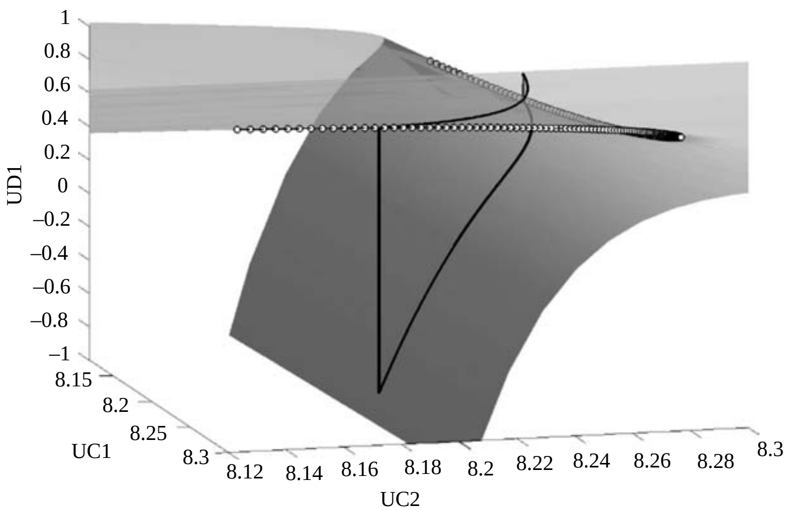


Figure 9.
Projection of the state space in the coordinate system $u_{D1} - u_{C1} - u_{C2}$; limit cycle (black line) and calculated jump set (circles)

5. Concluding remarks

In this paper, an outline is given about the state of the art of nonlinear circuits from a differential geometric point of view, where a rather simple nonlinear RLC circuit and its elementary geometric description is used for introducing an abstract geometric setting. After a discussion of the generic dynamics behavior of circuits based on this geometrical framework, necessary conditions of so-called jump effects are formulated. Finally, jump behavior is illustrated by means of two typical circuits.

Note

1. In this paper, we omit the technical details of the theory of differentiable manifolds. The interested reader is left to Guillemin and Pollack (1974).

References

Arnold, V. (1988), *Mathematical Methods of Classical Mechanics*, Springer, New York, NY.

Brayton, R. and Moser, J. (1964), "A theory of nonlinear networks-I and II", *Quarterly of Applied Mathematics*, Vol. 22 Nos 1-33, pp. 81-104.

Chua, L. (1980), "Dynamic nonlinear networks: state-of-the-art", *IEEE Transactions on Circuits and Systems*, Vol. CAS-27 No. 11, pp. 1059-87.

Desoer, C. and Wu, F. (1972), "Trajectories of nonlinear RLC networks: a geometric approach", *IEEE Transactions on Circuit Theory*, Vol. 19 No. 6, pp. 562-71.

Ebers, J. and Moll, J. (1954), "Large-signal behavior of junction transistors", *Proceedings of the IREE*, Vol. 42, pp. 1761-72.

Guillemin, V. and Pollack, A. (1974), *Differential Topology*, Prentice-Hall, Englewoods Cliffs, NJ.

Ichiraku, S. (1978), "On singular points of electrical circuits", *Yokohama Mathematical Journal*, Vol. 26, pp. 151-6.

Ichiraku, S. (1979), "Connecting electrical circuits: transversality and well-posedness", *Yokohama Mathematical Journal*, Vol. 27, pp. 111-26.

Mathis, W. (1987), *Theorie nichtlinearer Netzwerke*, Springer, New York, NY.

Mathis, W., Blanke, P., Gutschke, M. and Wolter, F. (2009a), "Analysis of jump behavior in nonlinear electronic circuits using computational geometric methods", *Nonlinear Dynamics and Synchronization, 2009. INDS'09 2nd International Workshop on*, pp. 89-94.

Mathis, W., Blanke, P., Gutschke, M. and Wolter, F.-E. (2009b), "Nonlinear electric circuit analysis from a differential geometric point of view", *ITC-Fachbericht-ISTET 2009*, p. 30167.

Matsumoto, T., Chua, L.O., Kawakami, H. and Ichiraku, S. (1981), "Geometric properties of dynamic nonlinear networks: transversality, local-solvability and eventual passivity", *IEEE Transactions on Circuits and Systems*, Vol. CAS-28 No. 5, pp. 406-28.

Nielsen, R. and Willson, A.N.J. (1980), "A fundamental result concerning the topology of transistor circuits with multiple equilibria", *Proceedings of the IEEE*, Vol. 68 No. 2, pp. 196-208.

Smale, S. (1972), "On the mathematical foundation of electrical circuit theory", *Journal of Differential Geometry*, Vol. 7 Nos 1-2, pp. 193-210.

Tchizawa, K. (1984), "An analysis of nonlinear systems with respect to jump", *Yokohama Mathematical Journal*, Vol. 32, pp. 203-14.

Thiessen, T., Gutschke, M., Blanke, P., Mathis, W. and Wolter, F.-E. (2010), "Numerical analysis of relaxation oscillators based on a differential geometric approach", *International Conference on Signals and Electronic Systems* (*ICSES*), pp. 209-12.

Corresponding author
Tina Thiessen can be contacted at: thiessen@tet.uni-hannover.de

This article has been cited by:

1. Martin Gutschke, Alexander Vais, Franz-Erich Wolter. 2015. Differential geometric methods for examining the dynamics of slow-fast vector fields. *The Visual Computer* **31**:2, 169-186. [CrossRef]
2. Tina Thiessen, Soren Plonnigs, Wolfgang MathisTransient solution of fast switching systems without regularization 578-581. [CrossRef]