Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 199 (2017) 669–674

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Modelling friction characteristics in turbine blade vibrations using a fourier series expansion of a real friction hysteresis

Thomas Hoffmann[a,*], Lars Panning-von Scheidt[a], Jörg Wallaschek[a]

[a] *Institute for Dynamics and Vibration Research, Leibniz University Hannover, Appelstraße 11, 30167 Hannover, Germany*

Abstract

A new contact model is proposed to simulate the forced response of a two degree of freedom mechanical model which resembles the simplest form of a friction damped turbine blade. The model scales a measured friction hysteresis to the current state of the mechanical model to obtain the contact force. The equations of motion are solved iteratively with a Newton-Raphson method using an analytical *Jacobian*. Furthermore the nonlinear system is linearised by using the *Monoharmonic Balance Method*. Based on experiments functions are found that scale a single hysteresis in such a way that a wide range of contact states can be predicted sufficiently well. The forced response of the mechanical model using the proposed contact model shows good agreement with the forced response using an *Elastic Coulomb Friction Model*.

Keywords: turbine blades, friction damping, hysteresis

1. Introduction

Turbine blades of industrial gas turbines have to withstand harsh operating conditions. These include high temperatures and structural stresses due to vibrations. Vibrations are excited by an inhomogeneous circumferential pressure field which is for example formed by the wake of stator blades. The blades are forced to vibrations as they pass through this pressure field. During run-up and run-down the blades are typically passing their resonance frequencies. Especially in resonance the vibration amplitude and the structural stresses are high and can lead to failure of the blade. Therefore, it is necessary to reduce the forced vibration amplitudes of the turbine blades [1]. To accomplish this friction damping is widely used [2]. It utilizes the relative motion between adjacent blades to dissipate energy using coupling elements like shrouds or underplatform dampers. Especially vibration amplitudes in resonance can be effectively lowered with this approach. During the design process of a gas turbine it is important to be able to simulate the vibrational behaviour of the blades. That is to reduce costly and potentially unsafe engine tests. The key to accurately simulate the forced response of the blades is how the contact forces between coupling elements are calculated. One of the most commonly used friction models is the *Elastic Coulomb Friction Model* [3]. The main drawback of this model is that the model parameters contact stiffness and friction coefficient have to be determined experimentally [4]

* Corresponding author. Tel.: +495117624181 ; fax: +495117624164.
E-mail address: hoffmann@ids.uni-hannover.de

1877-7058
10.1016/j.proeng.2017.09.586

[5] [6] or through mathematical models [7] [8] [9] [10] [11]. In general, the determination of the contact parameters is highly uncertain.

In this study a new approach to determine the friction forces acting on the turbine blades due to their forced response is presented. This approach doesn't suffer from the high model uncertainties that are introduced by the use of contact parameters such as friction coefficient and contact stiffness, as it doesn't use those parameters directly. A measured friction hysteresis is used to determine the status and resulting friction force of a simulated contact pair. The hysteresis is measured using a friction hysteresis test rig which is available at the institution. The friction hysteresis is decomposed into its Fourier series to be better accessible by the algorithm. This approach makes it possible to simulate a gradual change between the contact states. First, the forced response of a simple two degree of freedom slider on a rough surface is simulated using the new approach. As the friction damped system is non-linear, the *Monoharmonic Balance Method* (HBM) is used to reduce the equations of motion to their first harmonic. The results are compared against a calculated forced response using the *Elastic Coulomb Friction Model*.

Nomenclature

Lower case letters denote time domain, capitals denote frequency domain. Bold symbolizes vectors or matrices.

f	Force
w	Displacement
τ	Normalized time
ψ	Phase shift
α	Scaling factor
J	Jacobian
H	Dynamic stiffness

2. Mechanical Model

Figure 1 (a) shows the mechanical model used in this study. It resembles a system of two damped vibrating masses m_1, m_2. The mass vibrates due to the linear stiffnesses c_1, c_2 and the forcing f_l. Furthermore, linear viscous dampers d introduce damping to the system. At mass m_1 a contact force is applied to the system which simluates friction at mass m_1. This system can be seen as the simplest section model of a turbine blading, where m_1 is the turbine blade root and m_2 resembles the turbine blade.

2.1. Equations of Motion

To form the equations of motion, Newtons second law is applied to the system. It yields for the displacement vector $\mathbf{w} = [w_1 \;\; w_2]^T$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{w}_1 \\ \ddot{w}_2 \end{bmatrix} + \begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix} \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} f_{\text{Contact}} \\ f_l \end{bmatrix}. \tag{1}$$

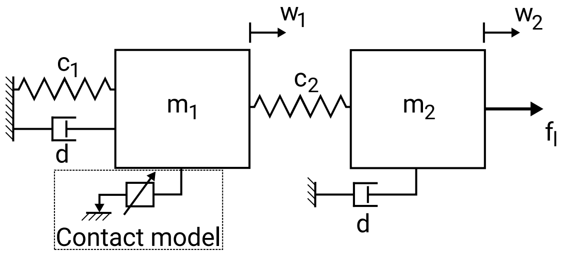

Fig. 1. Schematic model

Here the forces from the stiffness c and the viscous damper d have been replaced by their respective material laws. The linear forcing is assumed to be harmonic, so

$$f_l = \hat{f}\cos(\tau), \tag{2}$$

where τ is the dimensionless time. The non-linear contact force f_{Contact} shows in general a dependency on displacement and velocity. Contact models have to provide this dependency.

2.2. Solving strategy

To avoid a costly time integration, equations of motions for linear systems are solved in the frequency domain using a Fourier transformation of displacement and forces in eq. (1):

$$w_j = \sum_{k=-N}^{N} W_{j,k}\, \mathrm{e}^{\mathrm{i}k\tau} = \sum_{k=-N}^{N} |W_{j,k}|\, \mathrm{e}^{\mathrm{i}(k\tau+\psi_{\mathrm{W},j,k})}, \tag{3}$$

where for an exact approximation $N = \infty$ and $j = [1, 2]$. The forces are transformed accordingly. Because the displacement is periodic, the contact force f_{Contact} is also periodic and can be constructed using a Fourier series. The equation of motion for each harmonic k in the frequency domain is

$$\mathbf{H}_k \mathbf{W}_k = \begin{bmatrix} F_{1,k} \\ F_{\text{Contact},k}(w, \dot{w}) \end{bmatrix}. \tag{4}$$

$\mathbf{H}_k$ denotes the dynamic stiffness matrix. To solve this system of non-linear equations, the HBM is used. The HBM means linearising the system through using only the first harmonic of all Fourier series, so $N = 1$ in eq. (3). The remaining system of equations is iteratively solved using a damped Newton-Raphson method due to the nonlinearity of the contact force. For faster convergence, the *Jacobian* can provided numerically or analytically [12].

The solving strategy is shown in figure 2. Starting with an initial displacement of the mass m_1, W_1^0, for a given excitation, eq. (2), the contact model calculates the friction force for the actual iteration step, $F^i_{\text{Contact},1}$. For this friction force the residual of eq. (4) is evaluated. A Newton-Raphson step gives the new displacement W_1^{i+1}. This process is repeated until a convergence criteria is fulfilled. Then the algorithm proceeds to the next frequency step.

3. Contact Model

The general concept of the proposed contact model is to avoid the need of derived contact parameters by using a measured friction hysteresis directly to determine the status of the friction contact. This approach is a step back from contact models like the *Elastic Coulomb friction model* as it makes use of the stiffening and dissipating characteristics of a friction contact without the need to quantify them. As it is shown in [13] or [14], the actual value apart from the rough magnitude of the contact stiffness doesn't influence the forced response in a significant way. So the abstraction

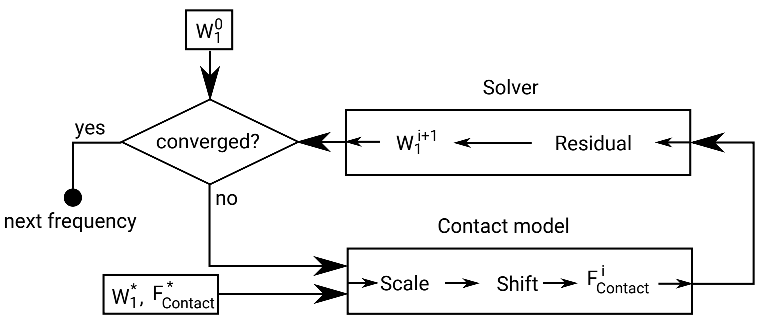

Fig. 2. Iterative process for solving the model

of the contact model seems suitable. Theoretically, for every contact status of a structure (relative displacement, temperature, stress state,...) a friction hysteresis would have to be measured to derive the contact force from the hysteresis. As this is not applicable, a scaling approach is pursued. Starting from just one measured friction hysteresis $w^*, f^*_{\text{Contact}}$ and its monoharmonic Fourier series $W^*, F^*_{\text{Contact}}$, all other hysteresis are calculated through a scaling law in magnitude and phase angle.

3.1. Scaling and shifting

For each iteration step, the hysteresis has to be scaled in a way that the current displacement W matches the maximum displacement of the hysteresis. This is to obtain a valid hysteresis for each current sinus-like displacement in the time domain with amplitude W. A scaling factor between the current displacement and the displacement of the measured friction hysteresis is used as

$$W = \alpha W^* . \tag{5}$$

With the scaling factor, a function has been found through a least square approximation of all hysteresis turning points, fig. 3(a), depicted as the dotted line:

$$F^{\text{ms}}_{\text{Contact}} = F^*_{\text{Contact}}(0.98 + 0.21 \log(\alpha + 0.01)), \tag{6}$$

where $\log(\cdot)$ is the logarithmus naturalis. In a similar way the phase shift ψ_{C} between hysteresis displacement and contact force is scaled. At small displacements, the contact has a spring like behaviour resulting in a phase shift of $\psi_{\text{C}} = -\pi$ between contact displacement and contact force. As high displacements, the contact acts like a damper, resulting in a phase shift of $\psi_{\text{C}} = -\frac{\pi}{2}$. The scaling function for the phase shift has also been found as a least square approximation of the phase shift of each hysteresis as

$$\psi_{\text{C}} = -\frac{\pi}{2} - \frac{7}{15}\pi \mathrm{e}^{-4\alpha} . \tag{7}$$

As the last step, the phase shift ψ_{W} of the monoharmonic displacement has also to be applied to the monoharmonic contact force to match the time signals of displacement and force. This phase shift can be calculated as

$$\psi_{\text{W}} = \text{Arg}(W) . \tag{8}$$

The combined phase shift is then applied to the monoharmonic contact force using the shift theorem [15]:

$$F_{\text{Contact}} = F^{\text{ms}}_{\text{Contact}} \mathrm{e}^{\mathrm{i}(\psi_{\text{C}} + \psi_{\text{W}})} \tag{9}$$

Eq. (9) gives then the first harmonic Fourier coefficient of the friction force to be used in eq. (4). For faster convergence, the *Jacobian* is analytically calculated.

4. Comparison of Results

4.1. Measured vs. predicted hysteresis

Figure 3(b) shows the predicted friction hysteresis based on eq. (6). Comparing with fig. 3(a), only little differences can be seen. Especially comparing the inner most hysteresis, orange solid in fig. 3(a) and blue dashed in fig. 3(b), even the deviation of the general form is very small showing that the scaling approach is viable for the measured contact hysteresis.

4.2. Forced response

Figure 4 shows the forced response of the system depicted in fig. 1. As reference, the dash-dotted lines show the response of the linear system where the contact model generates no force at all. The solid lines show the forced response calculated with the proposed Hysteresis Contact model. For comparison, the response using the *Elastic*

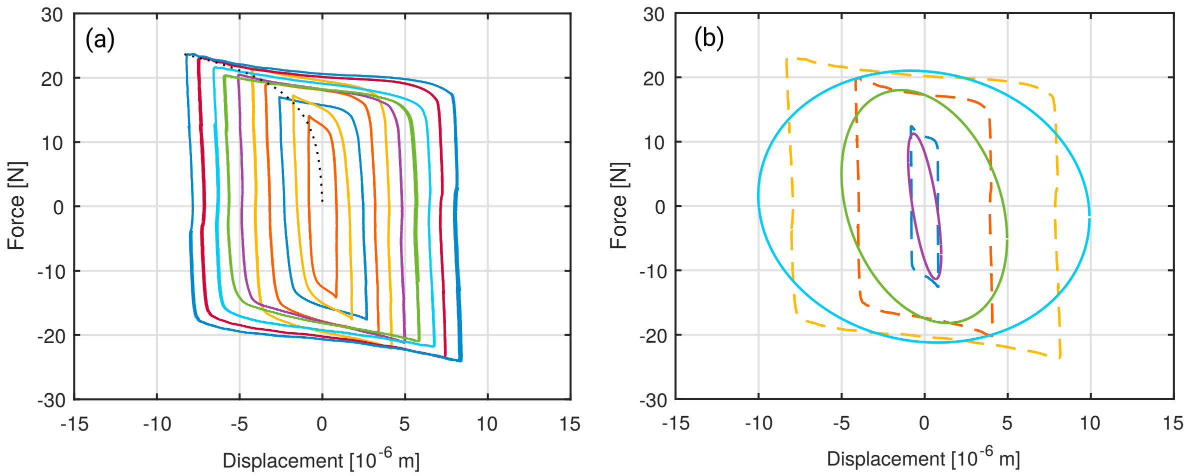

Fig. 3. (a) Solid: Measured friction hysteresis. Relative displacement from 0.8μm to 8μm. Normal force: 22N. Dotted: Scaling function, eq. (6) (b) Dashed: Predicted hysteresis for displacements 0.8μm and 4μm based on the 8μm hysteresis. Solid: Monoharmonic approximations.

Coulomb Friction Model is shown dotted which was tuned to fit the static response of m_2 using the proposed contact model.

Looking at the linear response, two resonance frequencies at 43 Hz and 108 Hz can be observed. By applying the Hysteresis Contact Model, the first resonance amplitude of m_2 is reduced by 96% and just barely visible. Also a frequency shift by 15 Hz of the first resonance can be seen. The second resonance amplitude of m_2 is reduced by 72% without a frequency shift. Most interestingly, the forced response using the proposed contact model shows qualitatively the same behaviour as the *Elastic Coulomb Friction Model*. Between 60 Hz and 120 Hz both models also agree quantitatively well. The amplitude variations in this frequency range can be explained by the essential modelling differences between both models and are not of further interest. In the frequency ranges 0 Hz - 60 Hz and 120 Hz - 200 Hz large differences between both contact models exist for m_1. The fact that the *Elastic Coulomb Friction Model* is unsteady at the stick-slip transition leads to the steep amplitude growth and fall of the forced response as here the large effective stiffness added by the contact stiffness reduces the amplitude. The dotted lines also show the discontinuity as a sharp bend at 55 Hz and 118 Hz. The proposed contact model shows a much smoother transition between the stick-like and the slip-like behaviour. It rather doesn't divide the contact status in stick or slip, but always has properties of both states which is expected to be a more realistic behaviour.

5. Conclusion

A new contact model was proposed that calculates the contact force by means of scaling a measured friction hysteresis to fit the numerical problem. It was shown that this model predicts the forced response of a simple system as good as the widely accepted *Elastic Coulomb Friction Model*. Moreover the proposed model has a steady behaviour over the whole amplitude range which resembles a real contact better. For the shown mechanical model the approach seems viable.

Acknowledgements

The investigations were conducted as part of the joint research programme COOREFlex-turbo in the frame of AG Turbo. The work was supported by the Bundesministerium für Wirtschaft und Technologie (BMWi) as per resolution of the German Federal Parliament under grant number 03ET7041L. The authors gratefully acknowledge Siemens AG for their support and permission to publish this paper. The responsibility for the content lies solely with its authors.

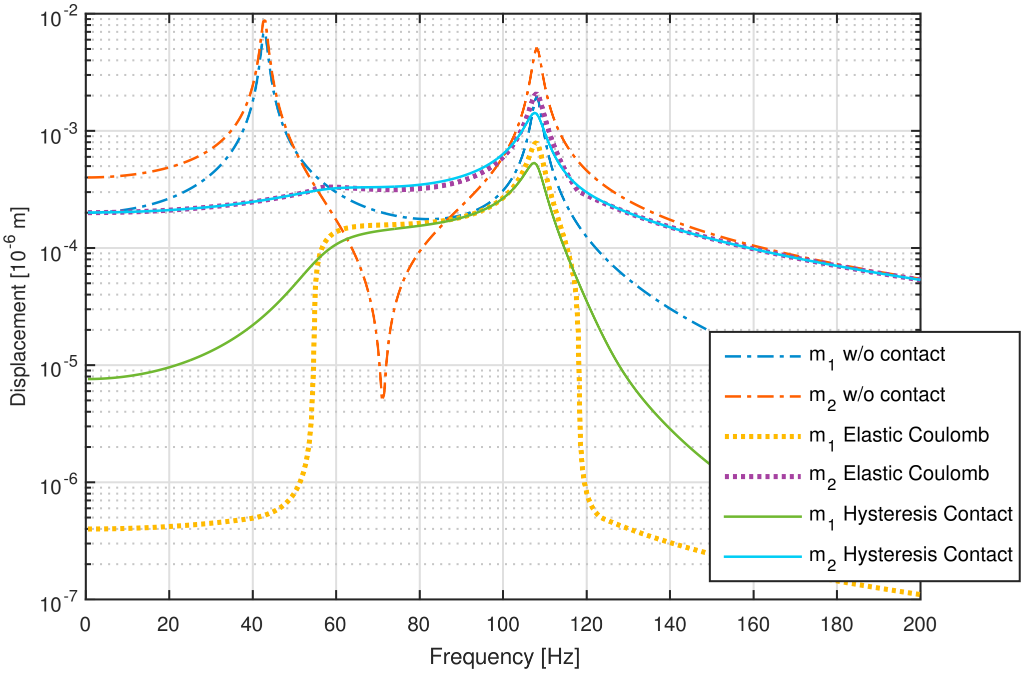

Fig. 4. Forced response. Solid: Hysteresis contact model. Dotted: Elastic Coulomb friction model. Dash-dotted: Linear system without contact

References

[1] L. Panning, Auslegung von Reibelementen zur Schwingungsdämpfung von Turbinenschaufeln, Ph.D. thesis, VDI-Verlag, Düsseldorf, 2005.

[2] M. Krack, L. Salles, F. Thouverez, Vibration Prediction of Bladed Disks Coupled by Friction Joints, Archives of Computational Methods in Engineering (2016).

[3] C. Menq, B. Yang, Non-linear Spring Resistance and Friction Damping of Frictional Constraint having Two-Dimensional Motion, Journal of Sound and Vibration 217 (1998) 127–143.

[4] C. Schwingshackl, E. Petrov, D. Ewins, Measured and estimated friction interface parameters in a nonlinear dynamic analysis, Mechanical Systems and Signal Processing 28 (2012) 574–584.

[5] J. Wojewoda, A. Stefanski, M. Wiercigroch, T. Kapitaniak, Hysteretic effects of dry friction: Modelling and experimental studies, Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 366 (2008) 747–765.

[6] C. Gastaldi, E. Grossi, T. Berruti, On the Choice of Contact Parameters for the Forces Response Calculation of a Bladed Disk with Underplatform Dampers, in: Proceedings of the 1st Global Power and Propulsion Forum, Zürich, 2017.

[7] E. Cigeroglu, N. An, C.-H. Menq, Forced Response Prediction of Constrained and Unconstrained Structures Coupled Through Frictional Contacts, Journal of Engineering for Gas Turbines and Power Copyrigh (2009).

[8] K. L. Johnson, Contact Mechanics, Cambridge University Press, Cambridge ; New York, 1985.

[9] C. M. Firrone, M. Allara, M. M. Gola, A Contact Model for Nonlinear Forced Response Prediction of Turbine Blades: Calculation Techniques and Experimental Comparison, in: Proceedings of GT2008: ASME Turbo Expo 2008: Power for Land, Sea and Air, Berlin, 2008.

[10] C. Siewert, L. Panning, A. Schmidt-Fellner, A. Kayser, The Estimation of the Contact Stiffness for Directly and Indirectly Coupled Turbine Blading, volume 2006, ASME, 2006, pp. 841–853. doi:10.1115/GT2006-90473.

[11] C. Gastaldi, M. M. Gola, Estimation Accuracy vs. Engineering Significance of Contact Parameters for Solid Dampers, in: Proceedings of the 1st Global Power and Propulsion Forum, Zürich, 2017.

[12] J. Borrajo, S. Zucca, M. Gola, Analytical formulation of the Jacobian matrix for non-linear calculation of the forced response of turbine blade assemblies with wedge friction dampers, International Journal of Non-Linear Mechanics 41 (2006) 1118–1127.

[13] L. Pesaresi, L. Salles, A. Jones, J. Green, C. Schwingshackl, Modelling the nonlinear behaviour of an underplatform damper test rig for turbine applications, Mechanical Systems and Signal Processing 85 (2017) 662–679.

[14] I. A. Sever, E. E. Petrov, D. J. Ewins, Experimental and Numerical Investigation of Rotating Bladed Disk Forced Response using Under-Platform Friction Dampers, in: Proceedings of GT2007 ASME Turbo Expo 2007: Power for Land, Sea and Air, Montreal, 2007.

[15] A. V. Oppenheim, R. W. Schafer, Discrete-time signal processing, 3. ed., internat. ed ed., Pearson, Upper Saddle River, 2010.