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A multi-layer social force approach to model interactions in shared spaces using collision prediction

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Abstract

In shared space environments the movements of road users is not regulated by traffic rules, but is the result of spontaneous interaction between traffic users, who negotiate the priority according to social rules such as eye contact or courtesy behavior. However, appropriate micro simulation tools, which can reproduce the operation of shared spaces, are currently lacking. In this paper, a multi-layer approach for representing the movement of road users and their interaction, based on the Social Force Model, is developed. In a free-flow layer a realistic path is calculated for each user towards his destination, while a conflict layer is used for detecting possible conflict situations and computing an appropriate reaction. The novelty of this work in the field of shared space modeling is in the implementation of group dynamics and a SFM based approach for cyclists. The presented approach is qualitatively tested in different traffic situations involving cyclists, pedestrians and pedestrian groups, and shows realistic behavior.

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Keywords: Shared space; microscopic simulation; social force model; mixed traffic; traffic conflict.

1. Introduction

The coexistence of different transport modes within the same traffic area has always raised problems. In particular, the difficulty is how to conciliate motorized traffic with bicycle and pedestrian traffic, compensating for the physical and operational disparity by designing solutions which could preserve efficiency and safeness.

According to a classical approach, this issue is addressed by separating different transport modes spatially or temporally. When all modes of transport are traveling towards the same direction, spatially separated traffic areas for each type of user are established. In the case of crossings, traffic flows are separated on a temporal level by control devices such as markers, signs and signal devices.

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Nomenclature

CA	Conflict avoidance
CAR	Motorist
CONF	Conflict status
CU	Competitive User (opponent to the EGO)
CYC	Cyclist
EGO	Regarded user
FF	Free-Flow
FFT	Free-Flow Trajectory
LR	Long-Range
LRCA	Long-Range Conflict Avoidance
NO_CONF	No-Conflict status
PED	Pedestrian
SFM	Social Force Model
SR	Short-Range
TTC	Time-To-Collision

Opposingly, new road design approaches have been developed, which aim to let all transport modes share the same space and minimize road signs and regulations. The purpose is to force traffic to regulate itself as much as possible, encouraging user interaction and the use of the space for social activities, by giving the street a social function instead of simply a traffic function. According to this philosophy, the concept of living streets, namely residential streets, where motorists share the road with cyclists and pedestrians on a level surface has been developed.

The regulation of these streets, whose designation depends on local regulations, provides that motorists adjust their behavior according to pedestrians and cyclists, by adapting their speed and giving priority.

A more radical application is represented by the experiments of the Dutch traffic engineer Hans Monderman, and aims at making road users coexist by reducing road regulations, especially priority rules, totally or partially. In fact the minimization of markings and signs, as well as traffic regulations, acts as a traffic calming element in the *principle of risk compensation* (Adams, 1985). However, this principle can be achieved only by an adequate road traffic design, suggesting to the user how to behave, and addressing them towards a pleasant and safe coexistence. Many informal road design instruments, such as geometrical measurement, road markings, materials and pavement, could influence the behavior and speed of users (Marceau et al., 2007). Moreover, street material and color have an effect on their behavior and their propensity to behave carefully.

Currently the choice of a particular shared space design does not take into account performance indicators like Level of Service or safeness. Appropriate micro simulation tools, which can reproduce the operation of shared spaces, are currently insufficient. However, such tools would ideally provide precise performance outputs, namely their efficiency (average road user delays and road capacity), safety (initial time-to-collision) and environmental impacts (emissions based on instantaneous speed and acceleration of vehicles) (Anvari et al., 2015).

The challenge of the research project MODIS (**M**ulti-m**od**al **I**ntersection **S**imulation) is to build a micro simulation framework, capable of simulating the movements of road users in shared spaces and calculating performance indicators, in order to evaluate the suitability of a particular solution. In the previous work (Pascucci et al., 2015) the fundamentals of a three-layered social force based approach were described, and simple conflict situations involving pedestrians and motorists were modeled.

In this work, the framework is extended by improving vehicle-pedestrian interactions, and implementing models for cyclists and groups of pedestrians. In Section 2 the background on shared space modeling is provided. Section 3 describes the data survey and typical road user behavior, focusing on the strategies to avoid conflicts. In Section 4 the problems in modeling shared spaces with a SFM are presented and the required modifications and extensions are explained. A detailed explanation of the framework is given in Section 5, while the results of the simulation framework are then visually compared to three representative observed situations involving cyclists, pedestrians and pedestrian groups in Section 6.

2. Background on shared space modeling research

The issue of modeling the use of shared space environments has many difficulties relating to three main aspects. Firstly, there is no homogeneity in the characteristics of road users and all vehicle types have different operational and mechanical characteristics. Furthermore, distinctions can be made within the same transport mode. For example, a pedestrian could walk through the area as fast as possible, or may wish to make conversations or do window shopping.

Secondly, there is no homogeneity in the characteristics of the space. Although the common idea of shared spaces is the full sharing of the same street surface, they are actually formally or informally divided into areas in which vehicles or pedestrians are dominant (Kaparias et al., 2013). Therefore it must be determined how the space configuration influences the behavior of road users, or specifically, how road users react differently according to the area they find themselves in.

Thirdly, the greatest challenge is in understanding how road users interact with each other. Since no priority rules are formally established, interaction is based on social rules that need to be thoroughly investigated.

Many authors have already dealt with the issue of modeling interactions between different transport modes where priority rules are not fixed (Sun et al., 2002; Chen et al., 2016). However, only a few works directly addressed the issue of priority negotiation between different traffic modes in a 2-dimensional environment.

Gibb (2015) modelled pedestrian-motorist interaction in shared spaces using VISSIM microsimulation software by substituting cars with groups of moving pedestrians (called dummy pedestrians), and introducing specific priority rules for conflict areas between road users. Two other research projects have dealt with shared space modeling, namely the research at the Imperial College in London (Anvari et al., 2012, 2014, 2015) and the research project MixME carried out in Austria (Schönauer et al., 2012; Huang et al., 2012).

The common basis in these works is the Social Force Model (SFM), which was introduced by Helbing and Molnar (1995) for modeling pedestrian dynamic. This model is based on Newtons equation of dynamic, and has been modified and improved over time for different applications (Helbing et al., 2000; Johansson et al., 2007; Helbing et al., 2001; Moussaïd et al., 2010). These applications have only been modelled pedestrian behavior and have shown good capabilities in high-density environments (such as evacuation simulations or crowd dynamics).

However, shared spaces are low or medium density environments and concern multi-modal traffic. For this reason, the original formulation of the SFM has previously been modified and extended in literature for shared spaces. The main problems of the classical SFM for shared spaces are listed below along with some previously proposed solutions.

- The model has to be applied to non-pedestrian users. Huang et al. (2012) and Anvari et al. (2012) modeled car traffic by a SFM approach, with particular focus on the mechanical constraints in the steering behavior, while Anvari et al. (2012) implemented car specific patterns such as car-following. Cyclists have not previously been included in shared space simulations, but the SFM has been applied for representing bicycle movement, in lane-based applications, in order to evaluate bicycle infrastructure performance (Osowski and Waterson, 2014) or simulate bicycle-car mixed flows (Li et al., 2011). The SFM has also been applied to motorcycles for modeling mixed traffic flow at signalized intersections (Nguyen and Hanaoka, 2011; Huynh et al., 2013).
- There is the necessity to build a specific path for each road user towards the destination. Anvari et al. (2012) developed a distance map to compute the shortest path, while Schönauer et al. (2012) created an infrastructure model to provide a plausible trajectory for road users who are unaffected by other users.
- Interaction between road users in social spaces is the result of a complex human decision making process, requiring a sophisticated modelling approach. As explained by Helbing and Molnar (1995, see Table 1), a simple or standard situation with automatic user reactions, can be well modelled by a physical approach like the SFM, while complicated situations require other approaches such as a decision model. Schönauer et al. (2012) have developed a tactical model to solve conflicts based on game theory. Anvari et al. (2014) addressed conflict situations with a CDCR (Collision Detection and Conflict Resolution) strategy, in which conflicts were detected and solved using geometrical considerations.

Table 1. Classification of behaviors according to their complexity, from Helbing and Molnar (1995)

Stimulus	simple or standard situations	Complex or new situations
Reaction	automatic reaction, reflex	Results of evaluation, decision process
Characterization	well predictable	probabilistic
Modeling concept	Social force model, etc.	Decision theoretical model, etc.

3. Data survey and observed behavior

In comparison to streets and intersections with defined priority rules, the behavior of road users in shared spaces is more uncertain due to the reduction and the flexibility. For this reason, the operation of shared spaces must be observed and analyzed, in order to find some behavioral basic principle that can be implemented in software.

For this purpose, the motion of users in an intersection in the university district in Braunschweig (D) is analyzed. Although this intersection is regulated as bicycle priority street and a classical sidewalk is present, many shared space operational features are observed, namely the high level of interaction between different traffic modes, the negotiation of priority (including bicycles although they have right of way by law), and the presence of pedestrians standing still and interacting in groups. Table 2 shows the time period and duration of the on-site survey and the observed volumes of road users, with an indication of the traffic detected in a 15 min. peak period during a lessons break.

The aim of the on-site survey was to identify which factors influence road users in their motion towards the destination. Three main factors were identified:

- The type of user
- The infrastructure
- The interaction with other users.

Firstly, each user has different physical and mechanical characteristics, like operational speed and movement constraints.

Secondly, the infrastructure is giving information to road users about how to behave by direct (markers and signs) and indirect indications (the street layout). Regarding the last point, the space allocation is important to accommodate particular key activities or uses, such as pedestrian comfort space, vehicular movement, parking and loading, etc. (Department of Transport, 2011). Places are demarcated through structuring elements (poles, seats, greenery) and the type of pavement (color, material). This is a first indication for detecting which users are moving on a certain sub-area, and how they may behave. It is assumed that a shared space can be divided in three zones (traffic, safe and shared) as explained in Pascucci et al. (2015). As an example, a car approaching the shared zone from the traffic zone is usually adopting a more prudent behavior, e.g. reducing the speed.

Finally, the presence of other users forces a user to perform an adequate reaction in order to avoid a collision. The user reacts differently according to his transport mode and the intensity of the conflict. For the sake of clarity, a *conflict* is defined as "an observable situation in which two or more road users approach each other in time and space for such an extent that there is risk of collision if their movements remain unchanged", according to the definition of Gettman and Head (2003). Relevant behaviors in conflict situations have been observed in our video and are summarized in the following sections.

Table 2. Time period of the on-site survey and observed volumes of road users. (* relative to the 15-min peak, ** pedestrian involved in groups in the 15-min peak)

Location	Period	Road Users	Observed [n]	Observed [%]
Intersection Pockelstrasse - Katharinenstrasse - Konstantin Uhde Strasse Braunschweig, Germany	04.06.2014, Wednesday, 12:30 - 13:45 (15 min peak: 13:00 - 13:15)	Pedestrians	1936 (715*, 511**)	69.4 (69.5*, 49.7**)
		Cyclists	634 (239*)	22.7 (23.2*)
		Cars, Trucks	220 (75*)	7.9 (7.3*)

3.1. Pedestrian reaction to conflict

The intensity of pedestrian reactions are dependent on the involved transport modes, the environment and the density. In case of conflicts with other pedestrians (or groups), they tend to deviate from their spatial trajectory with constant walking pace; this behavior is usually taken in a certain advance – in relation to the possible collision – because it requires far less effort as that it would be required if the decision is postponed and the collision is imminent. In these cases, a deceleration is required to avoid the conflicting user, which implies an evasive action. Conflicts with higher transport modes, such as motorists and bicycles, possess a higher level of apprehension, and usually lead the pedestrian to a more prudent behavior, which could imply the complete stop, in order to wait till the conflict situation is cleared.

3.2. Group reaction to conflict

In crowd dynamics, the behavior of groups in crowded situations is part of recent research. In these situations groupings are mainly formed to avoid counter streams. In shared spaces the main intention of forming groups is for social reasons such as walking with colleagues or friends. Furthermore the number of these groups is much higher than in crowded situations. According to an empirical analysis of public areas conducted by Moussaïd et al. (2010), the proportion of pedestrians belonging to a group can be as high as 70%. This is coincident with our observations, described above.

When facing a conflict, pedestrian groups react in different ways than single pedestrians depending on the opponent. According to our observations, if a conflict with a single pedestrian is approaching, social groups show nearly no reaction to avoid the conflict. This is presumed to be due to social groups feeling more dominant than a single person and it being more complicated to achieve a joint reaction than to maintain current trajectory and speed. If the single person does not solve the conflict, an ad-hoc reaction is performed by each group user separately and the formation is reestablished afterwards. In conflict situations with other groups or users with a higher mode, e.g. cyclists or cars, the group members usually solve the conflict by adopting the avoidance strategy of the group leader(s). Observations show that groups try to react similarly to the leader while small groups may communicate via eye contact or speech to solve the conflict. The leader reacts in a similar way as a single person would, however he takes into account the spread of the group when calculating a safety distance for the new trajectory. Another possible reaction could be the group fractioning, however this has not yet been investigated in this research.

3.3. Cyclist reaction to conflict

In the case of cyclists, reactions to conflict situations are limited by physical constraints due to the bicycle. Firstly, deceleration rate is limited and it is related to the efficiency of the braking system. Furthermore, instability problems could be caused by extreme steering movements due to the trajectories curvature, which will be further explained in Section 5.3.

When other road users hinder the cyclist, he usually reacts by deviating from his current direction in order to avoid conflicts while maintaining speed. In some situations this behavior is not possible because the steering would be too abrupt and would compromise the stability of the cyclist. In this case a deceleration is needed, in order to safely perform the path deviation. When these options are not possible, a strong deceleration is needed in order to wait for the other users to pass.

3.4. Motorist reaction to conflict

When approaching a shared space, motorists assume a prudent behavior due to the higher likelihood of collisions. In case of conflicts, the tendency is to reduce speed to give priority, instead of leaving the trajectory, in order to make their future behavior predictable by others. Trajectory modifications are possible, but rare. In case of imminent conflicts they are able to stop nearly instantaneously if necessary.

4. A social force model approach

In the presented simulation framework the SFM structure of Helbing et al. (2000) with the modifications of Johansson et al. (2007) is adopted. According to this approach, the motion of a user is due to various types of social forces, which represent the effects of the environment on the behavior of the described user (Helbing and Molnar, 1995). The mathematical formulation is shown in Eq. 1.

$$m_i \frac{\mathrm{d}^2 \vec{x}_i(t)}{\mathrm{d}t^2} = \vec{f_i}(t) + \sum_{j,j\neq i} \vec{f_{ij}}(t) + \sum_b \vec{f_{ib}}(t) \tag{1}$$

Acceleration is controlled by the driving term $\vec{f_i}(t)$ as well as repulsive effects from other users $\vec{f_{ij}}(t)$ and fixed obstacles $f_{ib}(t)$. As mentioned in Section 2, this formulation has been used for representing pedestrian behavior, mostly in high density environment, and has shown good capabilities. In this work, a SFM-based approach is proposed for representing the use of shared spaces. This consists of both, solutions to solve specific problems such as free-flow and conflict detection issues, and also, the use of a transport mode-specific SFM formulation, which is represented by Equations 2 (for pedestrians) and 3 (for cars and bicycles).

$$m_i \frac{\mathrm{d}^2 \vec{x}_i(t)}{\mathrm{d}t^2} = \vec{f_i}(t) + \sum_{j,j\neq i} \vec{f_{ij}}(t) + \sum_b \vec{f_{ib}}(t) + \vec{f_{GROUP}}(t) + \vec{f_{SZ}}(t) + \vec{f_{DZ}}(t) \tag{2}$$

$$m_i \frac{\mathrm{d}^2 \vec{x}_i(t)}{\mathrm{d}t^2} = \vec{f_i}(t) + \vec{f_D}(t) \tag{3}$$

The weaknesses and limitations of the classic SFM are listed in the next subsections, together with proposed solutions.

4.1. A fixed trajectory for free-flow condition

The desire of reaching a destination is represented in the classical SFM by the driving term $\vec{f_i}(t)$, which leads the user to the objective along straight lines according to the shortest path principle. In shared spaces, pedestrians walk along smooth trajectories while avoiding sudden directional changes and try to minimize the time spent in zones that include cars. Furthermore, cars – and to a lesser extent bicycles – have a tendency to keep to the right side in right-driving countries. This leads to the necessity of assigning a free-flow trajectory (FFT) to each user.

The approach followed in this paper is based on the concept of *visibility graphs* (De Berg et al., 2000), to find the shortest path in an environment with obstacles, and the edgewise calculation of point-to-point clothoids (Bertolazzi and Frego, 2015), to smooth trajectories between points. This procedure has been explained in detail by Pascucci et al. (2015). The formulation of the FFT as a parametrized control point curve allows both orientation of the driving term in the direction of motion, and also, computation of future positions, which is especially useful for conflict detection.

4.2. A model for car and bicycle

In a SFM, cars and bicycles have to be handled differently than pedestrians due to their smoother movement. Notably, cars seldom leave their pre-calculated trajectory except to avoid obstacles. In order to achieve a continuously smooth trajectory the SFM is adapted for cars and bicycles (see Eq. 3).

When a pedestrian detects an imminent conflict with one of these modes, a repulsive force is applied to the pedestrian. In contrast, cars and bicycles are not influenced by any repulsive forces from other users. Instead, only an impulsive and a deceleration force are applied, controlling their speed along their trajectory. The deceleration force is directed in the opposite direction as the impulsive force and is dependent on the distance to the potential conflict. Its strength is controlled by a velocity-distance function and limited by the maximum possible deceleration.

In contrast to cars, bicycles can change their trajectory to avoid a conflict (see Section 5.3). Because cyclists lose a lot of energy by executing a full stop (which may include them putting their foot down), they often try to avoid conflicts by changing their trajectory. The magnitude of the reaction depends on their speed, since the curvature of a trajectory is limited by a maximal centripetal acceleration.

Similarly to cars, the behavior of cyclists is strongly limited by the physical characteristics of the bicycle. Firstly, a maximum deceleration value exists, based on the performance of the braking mechanism. Moreover, directional changes are limited by the cyclist's speed and resulting angle that the front wheel can safely turn (oversteering combined with high speed leads to loss of stability). For this reason, the presence of other repulsive forces in the SFM, which can be directed in any direction, would lead to unrealistic behavior for cyclists. In this model, a criterion for verifying the plausibility of cyclist movement based on the relation between speed and curvature is assumed and will be explained in Section 5.3.

4.3. Influence of the infrastructure on the behavior

As described in Section 3, the environment, in particular the space allocation, affects the behavior of road users. This means not only that some road users are excluded from various areas (cars are excluded for example from the safe zone), but also that their behavior differs depending on the zone they are in. For example, pedestrians commonly cross the shared zones at angles close to 90° (in order to minimize their time outside of a safe zone) and this behavior should be taken into account when designing the FFTs.

Moreover, cars, and to a lesser extent bicycles, typically decelerate when approaching shared zones, and accelerate when leaving it. This behavior has been implemented by assigning a desired speed dependent on the zone the user is in.

4.4. Influence in the direction of motion

The classical SFM considers repulsive forces from all other users depending on the distance to them. However, this distance-based principle is not realistic in shared space environments, where road users are only influenced by other users who they would collide with, if no reaction was taken.

Therefore it would be preferable to develop a conflict-based approach, in which conflicts between users are detected by considering their future positions and estimating the temporal and spatial proximity of users in a collision event.

In this paper, the conflict detection mechanism is based on a comparison of the FFT of the user under analysis with the trajectories of other users predicted by Lagrange polynomials (Pascucci et al., 2015). This allows for a calculation of all possible conflict situations, including which users are involved and the Time-To-Collision (TTC). Due to this mechanism, an appropriate reaction can be decided upon based on the users involved, and geometrical parameters.

4.5. Reaction in advance

Road users in shared spaces try to avoid collisions as soon as possible, enacting low-effort actions earlier to surely avoid the conflict, rather than high-effort actions later. Low-effort actions include smooth deviations from the planned trajectory and small decelerations, while high-effort actions include large deviations of the trajectory and severe decelerations. This long-range behavior contrasts the classical SFM, which is essentially a short-range model.

In order to model this long-range behavior in the model, all conflict situations are classified according to the TTC, using a fixed time threshold, as either Short-Range (SR) or Long-Range (LR). Initially this threshold has been assumed as 2 seconds. For SR conflicts, the classical SFM is used, while it is modified for LR conflicts in two possible ways:

- Change of direction to the driving term: a new trajectory is computed and the driving term is oriented according to it. A safe point is calculated according either to a spatial or a temporal criterion, and a new spatial trajectory is calculated through this point (Pascucci et al., 2015).
- Addition of a social force term: a new force is added to the equation, which represents a specific observed tendency. The list of forces and their related tendencies is presented in Table 3, with some explanations of the direction and the road users involved.

Table 3. Additional social forces for including LRCA mechanism in the classic SFM, with road user involved and short explanations.

Social force name	Math.	Road user	Tendency	Direction
Safe zone	$\vec{f}_{SZ}$	pedestrian	Tendency to remain in the safe zone when a motorist or cyclist is coming	From the border to the pedestrian
Danger zone	$\vec{f}_{DZ}$	pedestrian	Tendency to walk parallel to a motorist or a cyclist in the shared zone	From the car, perpendicular
Decelerating	$\vec{f}_{D}$	motorist, cyclist	Tendency to brake when a conflict with a pedestrian occurs	In opposition to the direction of the car

4.6. The pedestrian group dynamic

In the classical SFM pedestrians forming a (temporary) group have an attraction effect on each other. This usually leads to higher pedestrian speeds and an arrow-like formation. In shared spaces social groups tend to walk slower than single pedestrians to enable better communication between group members.

A social force model that represents this behavior must consider two factors. Firstly, in free-flow situations, group members try to stay in a certain formation to increase group interaction. In (Moussaïd et al., 2010) a V-like formation for groups with up to four members is postulated. To reproduce this behavior a group term was integrated into the social force equation. The group term consists of three forces each representing a unique group-organizing pattern.

$$\vec{f}_{GROUP}(t) = \vec{f}_{vis}(t) + \vec{f}_{att}(t) + \vec{f}_{rep}(t) \tag{4}$$

The visual force $\vec{f}_{vis}$ describes the desire of group members to be able to see all other group members. The necessity of turning the head towards the group center results in a decrease of speed proportional to the head rotation. The attraction force $\vec{f}_{att}$ ensures that group members are attracted to the group center if it is further away than a threshold value. To avoid physical contact between group members the repulsive force $\vec{f}_{rep}$ repels group members, which are closer to each other than a certain distance.

To avoid the group term interfering with the driving term it is necessary to achieve nearly parallel trajectories for each group member during preprocessing. Therefore the calculations of FFTs for all members of a group are coordinated, taking into account the distances between the members perpendicular to the moving direction (see Fig. 1).

Secondly, the reaction of social groups to an upcoming conflict is different to that of single pedestrians. While single pedestrians try to solve a conflict in an active manner, e.g. by changing trajectory or speed (see Section 5.1), social groups only solve conflicts actively when facing cars or groups of at least equal size. Otherwise social groups typically maintain their trajectory and speed. Solving conflicts actively requires a coordinated action of all group members. When one group member decides to avoid a conflict by choosing a new trajectory, the other group members must choose one of two options. The first option is that other members remain on their trajectory, leading to a splitting of the group. The second option is to change their trajectory based on the behavior of the group member who first reacts. Therefore this first reacting group member can be referred to as a group leader.

According to the observations previously explained, group members in shared spaces typically follow a temporary leader and adapt their trajectory when needed, in order to maintain the group formation. The multi-layer approach presented here has to include this mechanism, which will be part of future research.

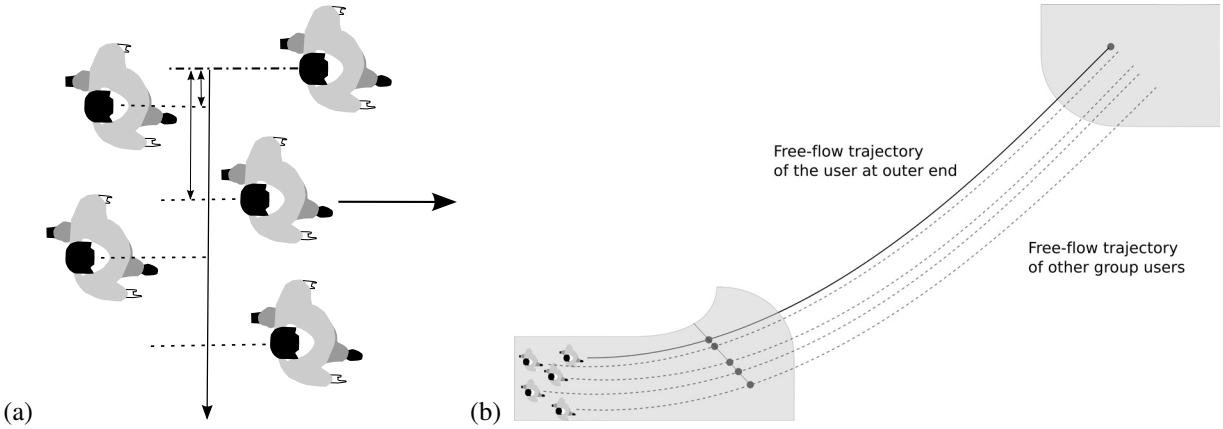

Fig. 1. Trajectory calculation for groups. (a) The outer group member is taken as reference to translate all other trajectories according to their distance perpendicular to the moving direction; (b) Resulting FFTs of all group members

5. A multi-layer approach

In this work a three-layered structure has been adopted, composing of a free-flow layer for computing FFTs, a conflict layer for conflict detection and computation of necessary SFM modifications, and the SFM layer itself.

The presented modeling approach incorporates the concept of a user-status, describing whether a user is facing a conflict (CONF status) or is unaffected by other users (NO_CONF status). This differentiation reflects the tendency of pedestrians and cyclists to continuously improve their conflict avoidance strategies (Fig. 2). As soon as a conflict is detected (*time of reaction*) the EGO user computes a new trajectory, which passes through a safe point. However this trajectory may later be improved if necessary, e.g. if the prediction of the movement of CUs changes. In this case the procedure is repeated until no further trajectory improvements are possible. The status of the EGO user is then set to NO_CONF and the improved trajectory is set as the FFT.

A conceptual flowchart of the proposed simulation is shown in Fig. 3. After users are initialized, they are each assigned a FFT as described in Section 4.1 and their status is set to NO_CONF.

At the beginning of a simulation step the status of each user is determined using the conflict detection mechanism described in Section 4.4. If a conflict is found, it is classified as LR or SR, depending on the time to collision. When a SR conflict is detected the SFM equation is applied, neglecting all LRCA terms, and the current trajectory remains unmodified. For LR conflicts one of several LRCA strategies (based on the behavior described in Sections 3.1-3.4) is

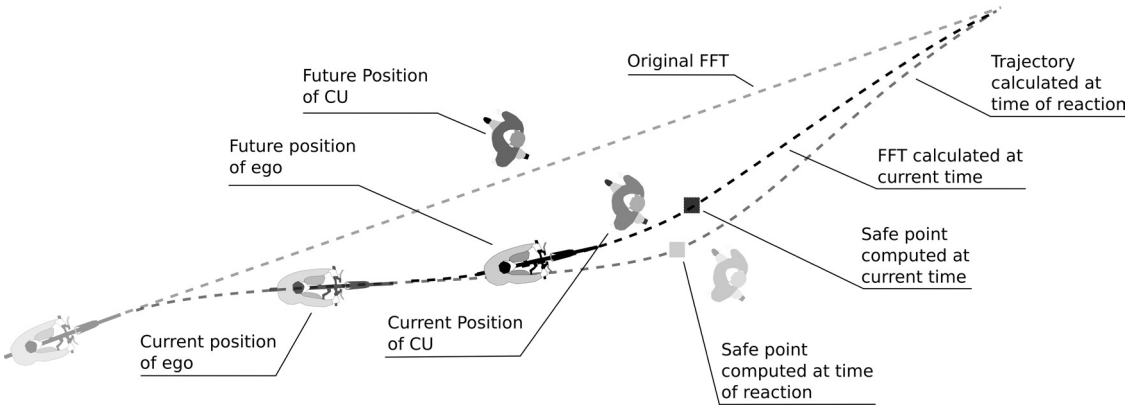

Fig. 2. Mechanism of trajectory improvement in CONF status, exemplarily shown for cyclists.

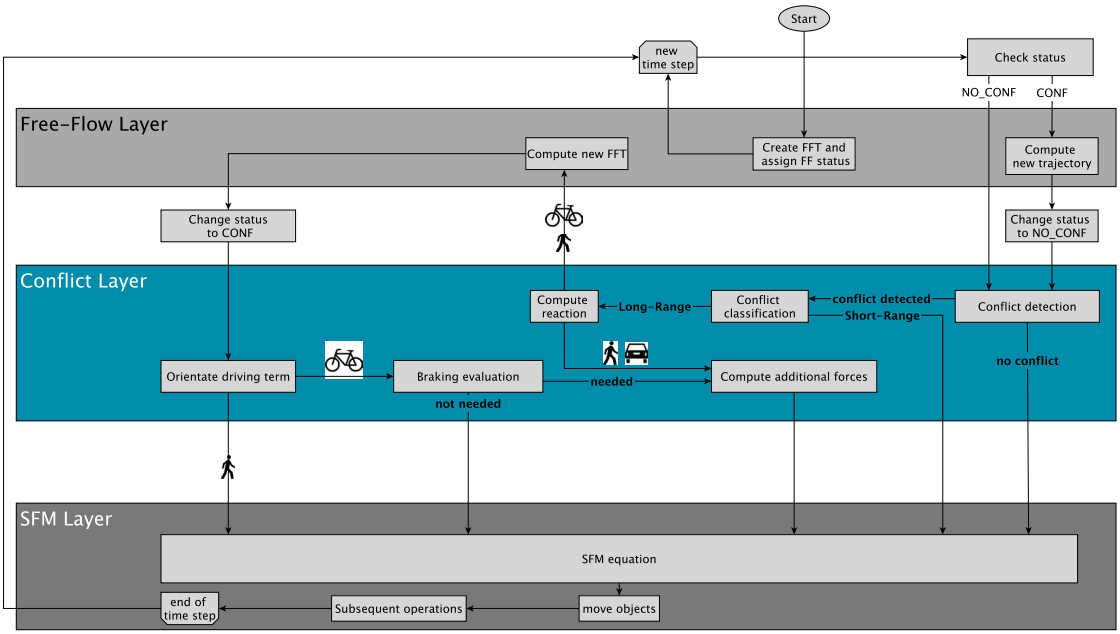

Fig. 3. Conceptual flowchart of the multi-layer-approach

applied, depending on the types of road users involved in the conflict, and a new trajectory may be calculated. Table 4 offers an overview of the conflict situations which may occur in a shared space, with an indication of whether a LRCA strategy has been implemented in this work. These implemented LRCA mechanisms are explained in the next sub-sections.

Table 4. List of LRCA behaviors implemented. Behaviors marked with ✓are already implemented, (✓) are assumed to be similar to other behaviors, * are covered in Pascucci et al. (2015), and ✗ will be part of future research.

EGO \ CU	pedestrian	group	cyclist	car
pedestrian	✓*	✓	(✓)	✓*
group	✓	✓	(✓)	(✓)
cyclist	✓	✓	✓	✗
car	✓*	(✓)	✗	✗

This research focuses on LRCA mechanisms of pedestrians as they are the most common transport mode in shared spaces. The implementation of avoidance mechanisms for conflicts with other pedestrians and cars (✓*) and the qualitative evaluation of specific conflict situations are presented in Pascucci et al. (2015). Currently no conflict situations involving single pedestrians and cyclists have been analyzed but it is assumed, that the behavior is similar to pedestrian-car conflicts. The behavior of pedestrians and pedestrian groups in conflict situations is described in Section 5.1. The reaction of cars with pedestrians is summarized in Section 5.2 since it is covered in detail in Pascucci et al. (2015). The avoidance mechanisms for cyclists are presented in Section 5.3, while conflict situations among cars and between cyclists and cars have not yet been integrated and will be part of future research.

5.1. Pedestrian and group reaction

LR conflicts between pedestrians are typically avoided by pedestrians changing their trajectory without any change in speed. This behavior has already been explained and simulated in Pascucci et al. (2015), and implies the computation of new trajectories passing through safe points.

When facing a conflict with a social group, pedestrians usually accept a longer path to their destination to avoid a group rather than interfering with the group members. Furthermore, pedestrians tend to pass behind a group instead of in front, even if a new trajectory passing in front of the group would be shorter. To represent this behavior a group avoidance mechanism has been implemented.

If a pedestrian detects a conflict with a group, the convex hull of the group is computed. Afterwards, a vector of all CUs, detected as shown above, from the actual to the predicted position is computed and used as a translation of the convex hull to approximate the predicted outline of the group. To include the preference to pass the group behind, the hull is stretched in walking direction. Finally the distance between all vertices of the hull and the current trajectory of the EGO user is computed. The farthest vertices of the hull in both directions are compared. The vertex with the shorter distance to the FFT plus a safety bias is chosen as a safe point and a new trajectory is calculated.

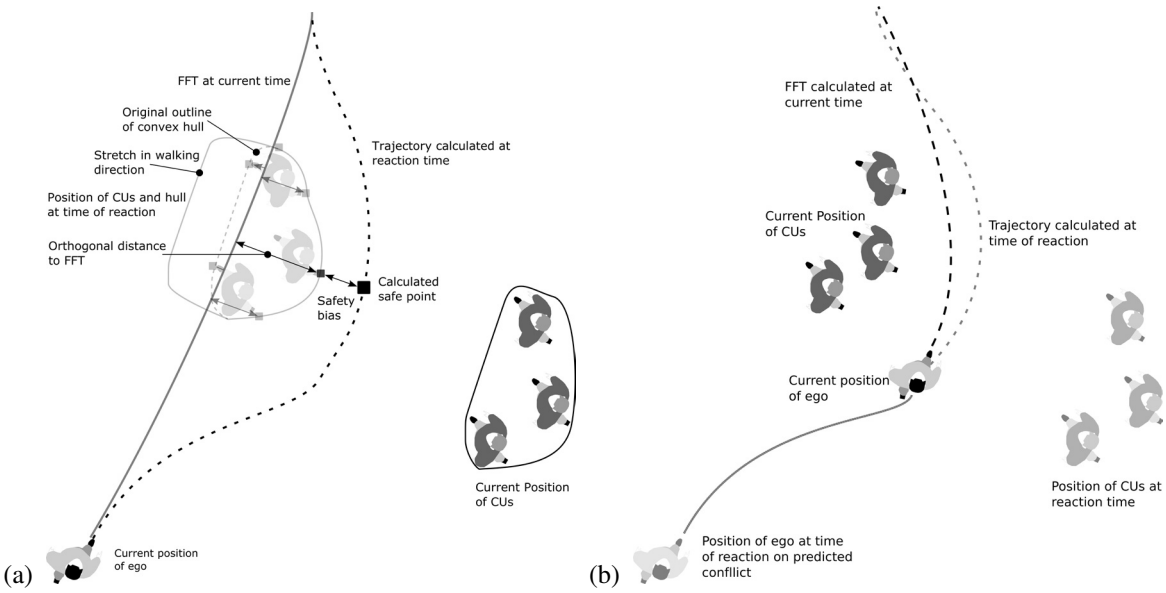

Fig. 4. Conflict solving strategy for a conflict between a pedestrian and a group. (a) Computation of a new trajectory on a safe point at reaction time. (b) Attempt to find a better FFT, in the following simulation steps.

When a pedestrian encounters a car or a bicycle in LR conflicts, he usually has to take a decision whether to give way or take priority. In this model, the decision is based on geometrical considerations. The pedestrian compares his current and predicted positions with the predicted trajectory of the CU. If he predicts that he will have passed more than the half of the car at conflict time he takes priority. Otherwise he tries to give way by decelerating or staying in a safe zone. This mechanism is based on prediction so it is necessary that the pedestrian reevaluates the situation continuously in order to slow down when the situation changes, e.g. the car accelerates.

Once the model declares a user to be prioritized, this user continues undisturbed along his trajectory, taking no reaction. If the pedestrian has to give way, he can react differently according to his position in the shared space. In this case, the model is modified by adding one of the forces already explained in Tab. 3.

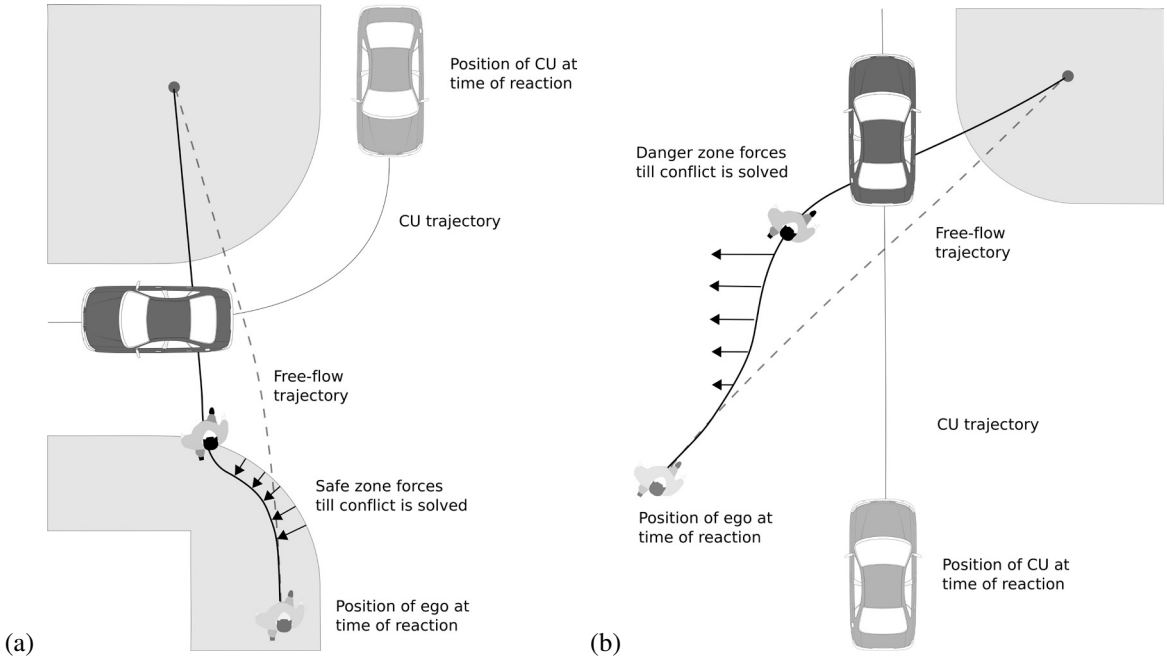

Fig. 5. Qualitative description of the safe zone force (a), and danger zone force (b).

- If he finds himself in the safe zone, his reaction is not to leave it. This behavior is implemented by a force which is pushing the pedestrian away from the border of the safe zone (*safe zone force*). The safe zone force $\vec{f}_{SZ}$ is calculated as:

$$\vec{f}_{SZ}(t) = \frac{d_{SZ}(t) - d(t)}{d_{SZ}(t) - d_{SZ,min}(t)} \cdot \vec{f}_{\perp}(t), \tag{5}$$

where $\vec{f}_{\perp}$ is a force directed perpendicular to the tangent to the nearest point of the border between the safe and shared zone. The magnitude of $\vec{f}_{\perp}$ is calculated to cancel out the movement of the user in the direction of the force. The safe zone force is only active when the current distance d is below d_{SZ}, while $d_{SZ,min}$ is the distance to the border of the safe zone where the pedestrian would wait until the conflict is solved. Effectively, the safe zone force leads the pedestrian approximately parallel to the border of the safe zone until the guiding force is perpendicular to the border. In this case, the safe zone force causes the user to completely stop until the car has passed or the outcome of the conflict situation is evident enough for the pedestrian to take another decision.

- If he finds himself in the shared zone, he proceeds parallel to the CU trajectory until the CU has passed. In this way, he can temporize and wait for the CU to give way. Therefore, a force perpendicular to the trajectory of CU scaled by the distance between the pedestrian and this trajectory is implemented (*danger zone force*). The calculation of the danger zone force $\vec{f}_{DZ}$ is very similar to that of the safe zone force:

$$\vec{f}_{DZ}(t) = \frac{d_{DZ}(t) - d(t)}{d_{DZ}(t) - d_{DZ,min}(t)} \cdot \vec{f}_{\perp}(t), \tag{6}$$

where $\vec{f}_{\perp}$ is the force needed to cancel out the part of the current sum of forces acting on the pedestrian that is perpendicular to the trajectory of CU, d_{DZ} is the distance to a car below which a pedestrian starts to feel uncomfortably close to the trajectory of a car and $d_{DZ,min}$ is the minimum distance $\vec{f}_{DZ}$ is intended to maintain. The danger zone force is only active when the current distance d is below d_{DZ}.

The respective distances used in the calculations need to be calibrated, with $d_{DZ,min}$ being currently assumed as half the width of a vehicle with an additional safety margin. In Fig. 5 qualitative examples are given. The behavior of pedestrians towards cyclists is assumed to be similar to that towards cars.

5.2. Car reaction

If the model assigns priority to the pedestrian in LR conflicts between pedestrians and cars, a reaction of the car is needed to avoid colliding. In this case, the reaction consists of a deceleration, which is implemented in the model by adding a braking force $\vec{f}_D$ (see Tab. 3). The structure of $\vec{f}_D$ is similar to the driving term:

$$\vec{f}_D(t) = \frac{v_i^D(t) \cdot \vec{e}_i^{\,0}(t) - \vec{v}_i(t)}{\tau_i}, \tag{7}$$

where the desired velocity v_i^D is calculated by a velocity-distance function. Due to the only restriction for this function being a monotonic increase, any approach from the area of micro traffic simulation can be used. The plausibility of this modification to the SFM has already been tested and compared to a real situation in Pascucci et al. (2015). Reactions of cars against groups, cyclists and other cars have not yet been investigated and will be part of future research.

5.3. Cyclist reaction

In the presented model, cyclists are the only users that can react both with the computation of a new trajectory and additional forces. The mechanism has some differences in comparison to the other cases. The reaction follows a two-step methodology. At first, a cyclist tries to compute a new trajectory while keeping their current speed. If no feasible trajectory can be found due to constraints described below, he will decrease speed. The mechanism is explained in Figure 7. Every time a cyclist detects a long-range conflict, possible safe points and new trajectories using these safe points are computed. The position of these safe points depends on the relative orientation of the CUs towards each other, as described in Pascucci et al. (2015) for PED-PED conflicts.

Currently lateral and frontal conflicts are regarded. Contrary to the behavior of pedestrians, two safe points, to the left and to the right in the direction of the ego user, are always calculated. Relative to the CU the safe points will be located behind and ahead of the conflict point in case of a lateral conflict (Fig. 6a) or left and right of the conflict point for frontal conflicts (Fig. 6b).

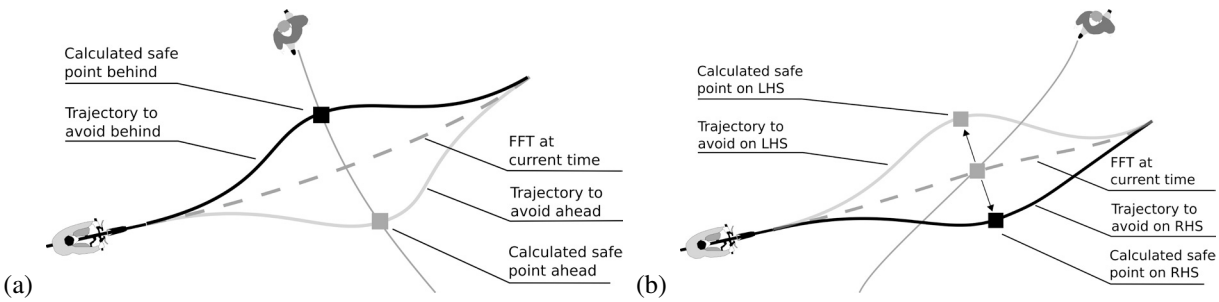

Fig. 6. Mechanism of conflict avoidance for cyclists, in case of lateral (a) or frontal conflicts (b).

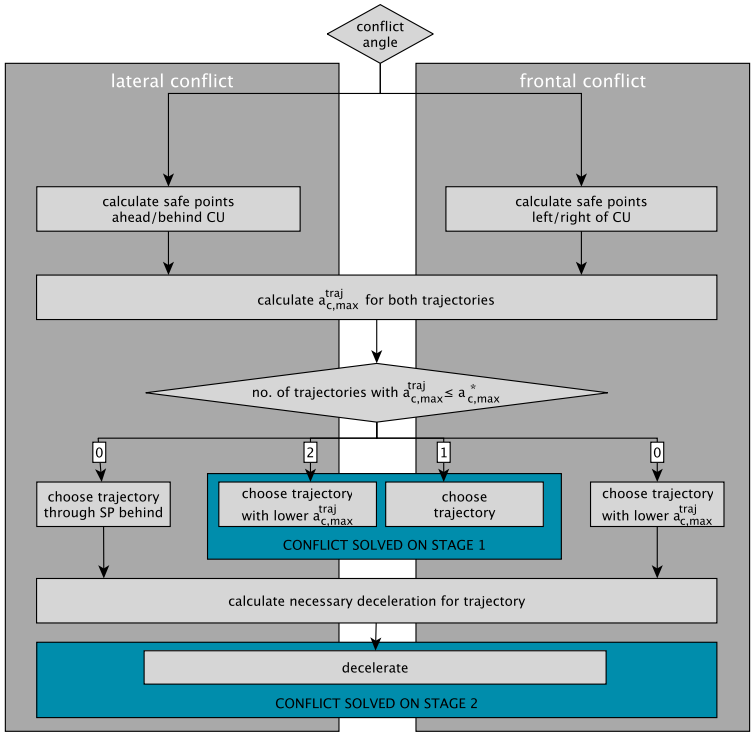

Fig. 7. Flowchart of conflict solving strategy for cyclists

New trajectories must be evaluated considering whether they can be followed at the current speed. The criterion of the centripetal acceleration is used as a ratio between speed and curvature at each point of the trajectory. The centripetal acceleration a_C can be calculated as

$$a_C = v^2 \cdot \kappa, \tag{8}$$

where κ is the curvature at a given point of the trajectory. In order to use the centripetal acceleration as a criterion, a threshold value $a^*_{C,max}$ is defined as the maximum comfortable centripetal acceleration for cyclists. For now $a^*_{C,max}$ is set to $2\,\mathrm{m/s^2}$, which is approximately equivalent to e.g. a curve radius of 15 m at a speed of 20 km/h. A field study investigating real world values for comfortable curve speed will be part of future research. For each possible safe point, the resulting new trajectory is analyzed for the maximum occurring centripetal acceleration $a^{traj}_{C,max}$, with the trajectory being able to be followed by the cyclist without braking if $a^{traj}_{C,max} \leq a^*_{C,max}$. The resulting three possible cases are:

1. Both trajectories can be used without braking. In this case, the trajectory with the lower $a^{traj}_{C,max}$ is chosen.
2. If only one trajectory can be used without braking, this one is chosen.
3. The cyclist has to decelerate for both trajectories. The decision process for this case is described below.

In the first two cases, a conflict can be solved by changing the trajectory and maintaining the current speed. This is defined as a stage 1 conflict solution. In the third case, additionally to the change of trajectory, a decrease of speed is required, which is regarded as stage 2. A third stage is possible for crowded situations or bottlenecks, where sufficient space for a change of trajectory is unavailable, resulting in strong deceleration while remaining on the current FFT.

In stage 2 situations, where the cyclist cannot reach any of the safe points without decelerating, the deceleration b that is required to satisfy $a^*_{c,max}$ at each point of the trajectory is calculated in an iterative process. Similarly to the initial determination of the two safe points, the actual choice of safe points depends on the conflict angle. For frontal

conflicts the trajectory requiring the lesser b is chosen, whereas for lateral conflicts it is not possible to use the safe point SP_{ahead} while decelerating, because decelerating would move the crossing point of the trajectories back towards the conflict point.

To achieve the desired behavior after a safe trajectory is chosen, the braking force resulting in the necessary deceleration is computed and applied against the direction of movement from the last simulation step.

It is important to note that, due to the constant shifting of the conflict point and possible safe points during a deceleration, the decision described above is reevaluated and adapted in each simulation step until the cyclist has passed the CU, that is, in terms of the social force model, the CU is no longer within the EGOs perception. Every time a cyclist solves a conflict using a stage 2 strategy, the EGO user is assigned the status of CONF, so that in the next time step a determination between a conflict-free FFT and a trajectory through a safe point is possible. If a user is in the CONF state, at the beginning of each time step a temporary trajectory (TT), starting at the current position and routing to the destination without regarding the safe point from the last time step, is calculated. The estimation of the conflict point, and consequently of the conflict classification and the possible safe points, is based on TT and the current speed during the following time step. The necessity of this step is explained in Fig. 2. Although the trajectory calculated at reaction time solves the conflict, an improved trajectory avoiding high curvatures could exist. This improvement can be achieved by repeating the calculation of TT at each time step.

6. Case studies and simulation results

The model has been implemented as part of the simulation software MODIS which is written in Java. This software contains mechanisms for preprocessing, simulation and postprocessing.

To represent infrastructure and to automatically compute the required geometrical and topological structures, such as the visibility graph for each transport mode, a digital information model is needed. For this reason CityGML (Open Geospatial Consortium, 2012) is used as data exchange format. CityGML is an information model to represent urban objects in certain levels of detail. The transportation sub-model focuses on geometrical as well as topological aspects when modeling traffic areas. Every traffic area has a specific function and a declaration of which traffic modes can use it. Furthermore related elements like kerbstones or road markings can also be modeled.

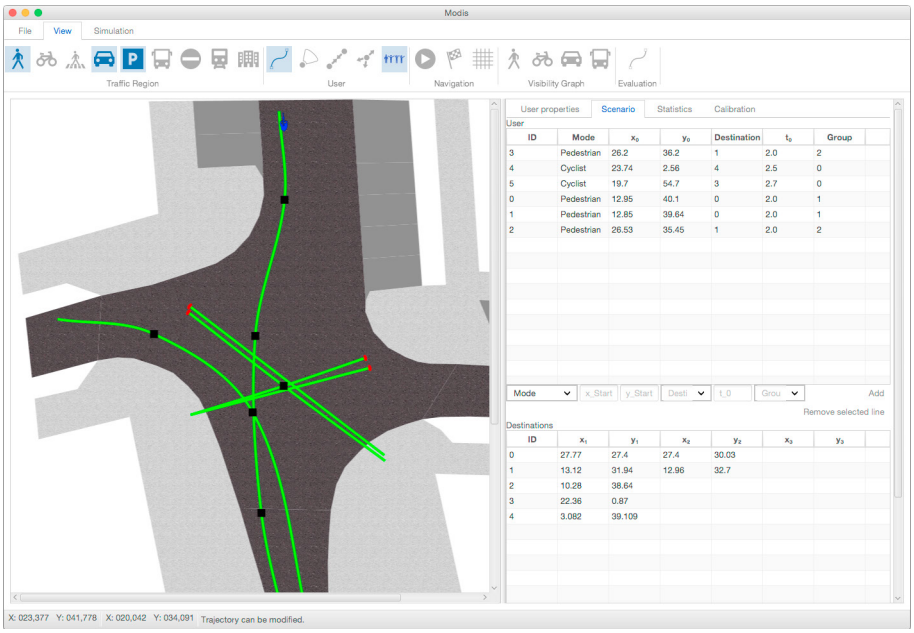

Fig. 8. View of the MODIS simulation software

After loading a CityGML infrastructure file the software user can create complex situations by defining sources, destinations and/or specific traffic users via the user interface. When a traffic user is integrated, a free-flow trajectory is computed automatically, which can be adjusted manually afterwards. Furthermore an XML-based input file can be used.

The multi-layer approach used when starting the simulation is described in Fig. 3. The user interface allows an observation of all mechanisms, e.g. conflict detection, state of conflict, avoidance strategy, and values of the social forces. For postprocessing the software integrates some evaluation methods. Real trajectory data can be loaded and compared to the simulated trajectories visually and via path-time and speed-time diagrams.

In Figure 8 a screenshot of the simulation software MODIS is given. In the main frame the scenario containing the infrastructure and the inserted traffic users with their FFTs is displayed. The path vertices of the FFTs can be edited manually by inserting and translating fix points of the clothoid. In the toolbar the present traffic areas according to their function defined in the CityGML-specification are highlighted. Further options to display model specific patterns, such as the visibility graph or the perception field, are also available. The control frame on the right side shows the configuration of the scenario and enables the integration of users and destinations with all relevant parameters.

In this section, some examples are given to demonstrate the conformity of the model to reality. The aim is to check the plausibility of the modification of the SFM approach, firstly qualitatively evaluating the behavior of road users, secondly by comparing the simulated and real trajectories.

As a first step, the necessity of the conflict layer is shown by a simple conflict situation between a pedestrian (A1) and a group (A2) (Fig. 9). According to the classical SFM (as formulated in Eq. 1), the pedestrian A1 would wait until the whole group has passed, by decelerating and finally stopping (Fig. 9b). However this is unrealistic behaviour, as pedestrians in this situation would change their trajectory rather than stop. Thanks to the implementation of the pedestrian-group CA strategy (Section 5.1), pedestrian A1 avoids the group by circumnavigating them, while maintaining their speed (Fig. 9c). The deviation between the observed (red) and simulated (green) trajectories is apparent, however better performance will be accomplished in future research by a parameter calibration.

In situations B and C, interactions involving cyclists are shown. Still images are presented in subfigures a, with an indication of the road users involved (by white ellipses) and the used trajectory. The same instant is shown in the simulation environment in subfigures b, with green continuous lines for used trajectories and dotted green lines for intended remaining trajectories. In subfigures c, observed (red) and simulated (green) trajectories are compared.

In situation B (Fig. 10) a conflict between two cyclists is presented. Cyclist B1 (coming from south) sees cyclist B2 (coming from east) and avoids him by deviating to the right, close to the road border, and by slightly decelerating. Cyclist B2 notices cyclist B1 only at a later stage, and decides to continue with an unmodified behavior.

The reaction of cyclist B1 is well represented in the simulation environment by a stage 2-mechanism, which implies a trajectory change associated with a deceleration (see Section 5.3). The conflict is not perceived by both cyclists at the same time in the simulation. Cyclist B1 is perceived by cyclist B2 some simulation steps later, when his perception field orientates to the left. In that moment, a CA strategy had been already taken by cyclist B1, so that additional CA reactions by cyclist B2 are unnecessary.

The distance between the observed and simulated trajectories of cyclist 2 is caused by the FFT computation not taking into account the right-side moving tendency, which will be part of future research.

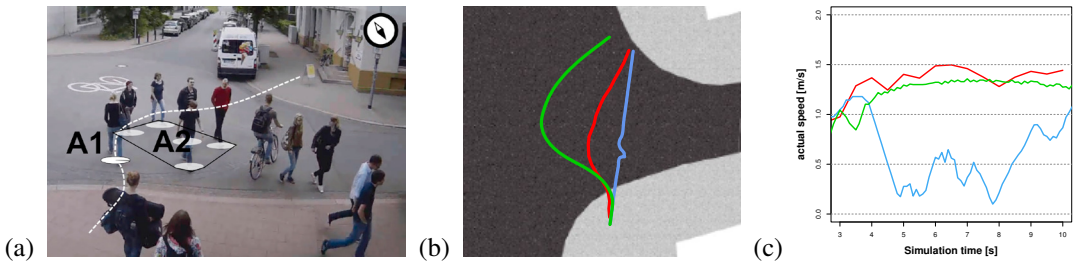

Fig. 9. Situation A. (a) Video footage, (b) Comparison of simulation with classical SFM formulation (blue) and our model (green) with observed trajectory (red), (c) related speed-time diagrams.

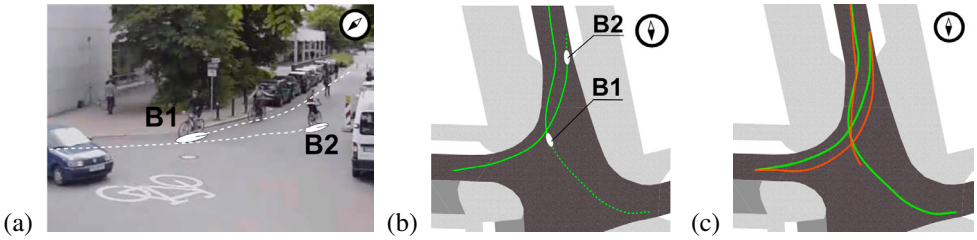

Fig. 10. Situation B. (a) Video footage, (b) Simulation according to our model, (c) Comparison between simulated (green) and observed (red) trajectory.

In situation C, two pedestrian groups (C1 from west and C2 from east) and two cyclists (C3 from north and C4 from south) are headed towards the shared zone in the center. Both groups detect a conflict with each other and react by swerving to their right hand sides respectively. The FFT of south headed cyclist C3 would result in a conflict with group C1. After the internal consideration of the two safe points SP_{left} is chosen. Due to the low curvature on the resulting trajectory, no braking is required. In the next time step, the new trajectory yields a conflict with group C2. This situation is resolved by correcting the trajectory slightly back towards C1. Because no deceleration is required for both conflict situation, the behavior of C3 can be classified as a stage 1 reaction according to Section 5.3.

Cyclist C4, who is coming from the south and intending to turn left towards west, does not notice the two groups until both have initiated their respective deviations. First a conflict with group C2 is detected, while the expected trajectory of C1 does not interfere with the FFT of C4 at this point. The conflict with C2 is solved by choosing a safe point to the right, which can be reached safely without decelerating. Shortly afterwards, C4 has observed the other cyclist C3 long enough to detect an upcoming conflict. The trajectory is corrected further to the right according to the strategy for a conflict between two cyclists. The curvature of the resulting trajectory requires a deceleration, which classifies the reaction of C4 to the whole situation as a stage 2 reaction.

The sequential reactions of C4 to several immediately consecutive conflict situations show the necessity for a modified CA strategy for situations where multiple conflicts have to be considered in advance. In the current stage of development only the closest conflict is regarded. With only slight modifications to the initial configuration of situation C it is possible to create artificial cases in which this mechanism comes into effect. The result is a continuous shift between detected conflicts and thus avoidance strategies, which can lead to unsteady and unrealistic trajectories. Appropriate CA strategies for such situations will be part of future research.

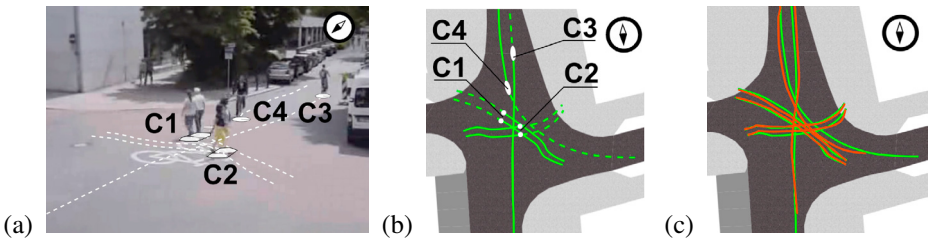

Fig. 11. Situation C. (a) Video footage, (b) Simulation according to our model, (c) Comparison between simulated (green) and observed (red) trajectory.

7. Conclusions

In this paper, a multi-layer framework to simulate multi-modal traffic in a shared space was presented. The first layer is dedicated to the computation of a pre-calculated free-flow trajectory for each road user towards his destination. In the second layer, conflicts with other users are detected and an appropriate reaction is computed. Firstly, conflicts are identified in a predictive manner for each user by the estimation of other trajectories using Lagrange polynomials successively. Afterwards an avoidance strategy is applied depending on the transport modes involved and their geometrical position. The model has been implemented in a simulation software and the avoidance strategies were qualitatively tested in three situations. In all situations the computed reaction conforms to reality. However the magnitude of the reaction differs from reality. To achieve a better performance, a parameter calibration using genetic algorithms will be part of future research.

Furthermore this approach integrates a model for the movement of bicycles and integrates realizable avoidance strategies considering the possible maximum centripetal acceleration. The problem of the maximum centripetal acceleration for bicycles is solved by evaluating possible trajectories constantly. Future research should focus on the computation of plausible trajectories for bicycles with respect to speed and the investigation of a functional relationship between speed and maximum centripetal acceleration.

A conflict avoidance strategy for each transport mode has been implemented. In the presented model, pedestrians and cyclists react to conflicts by modifying the direction of motion and/or their speed, while motorists only decelerate or accelerate, since they drive on a fixed trajectory. The model is capable of solving single conflicts between road users, but it does not implement multiple conflict strategies. For this reason a new mechanism, which handles all occurring conflicts with a single reaction, has to be developed.

Lastly, in shared spaces pedestrians can also linger due to attractions or social activities. Therefore the model has to be extended to allow a potential halt and, as a consequence thereof, a forming of temporary groups which is especially relevant for modeling bus stops.

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