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Effects of patent privateering on settlements and R&D under sequential market entry ☆

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ABSTRACT

We investigate patent litigation, settlements and R&D incentives on a market where two firms develop technologies in order to obtain patents and produce goods. Firms may sell IP rights to a Patent Assertion Entity (PAE) that acts as intermediary for patent monetization. We find that compared to simultaneous market entry, the effect of this so-called patent privateering is mitigated if firms enter sequentially. Furthermore, we show that privateering may decrease industry profits by distortion of R&D incentives even when there is no rent extraction by the PAE.

1. Introduction

Over the last few decades, patents have become increasingly important, especially in high-tech industries. This development has given rise to a new business model in which firms strategically build patent portfolios. Often, patents are not used for production but instead to license their technology to other firms under the threat of litigation. By refraining from selling products, these companies are not exposed to infringing other companies' patents and, therefore, are shielded from counterlawsuits when enforcing their patents. The report by the Federal Trade Commission (2016) differentiates between firms that develop their own technologies and patents—non-producing entities (NPEs)—and firms that do not invest in R&D but acquire patents to monetize them—patent assertion entities (PAEs).

In this paper, we focus on patent privateering, which describes a patent transaction between a producing firm and a PAE—the patent privateer (or IP privateer)—that will then accuse competitors of the original firm of patent infringement. Following Ewing (2012), patent privateering can be formally characterized as the beneficial application of third-party intellectual property rights for a sponsoring entity against a competitor to achieve a corporate goal of the sponsor.

The topic of IP privateering has been investigated from a legal point of view (see, for instance, Popofsky and Laufert (2014), Ewing (2012), Sipe (2016), Thumm (2018), Harris (2014), and Sokol (2017)). There is also an ongoing debate in research and the media on whether privateering is desirable from the perspective of social welfare.[1] On the one hand, privateering may decrease overall R&D incentives by skimming the rents of competing innovative firms and partly destroying the defensive values of patent portfolios. In addition, it may lead to excessive litigation and legal costs, which is another source of inefficiency. On the other hand, privateers could play the role of a financial intermediary that helps monetize patents and increases incentives to invest in R&D. If firms underinvest in R&D relative to the socially efficient level, privateering can induce firms to choose investments closer to the efficient outcome due to a rent-seeking incentive. Higher R&D investments accelerate the development of innovative technologies, which is ben-

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[1] See, for instance, the Washington Post article written by Fung (2014).

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eficial for society. Lemus and Temnyalov (2017) provide a theoretical framework to investigate the effects of privateering on R&D investments in a setting where firms simultaneously enter the market. They emphasize the possibly positive effects that patent privateering may have on R&D efforts and social welfare.

We contribute to this debate by providing an argument suggesting that privateering may be even less desirable than previously assumed. Specifically, we investigate the effects of privateering in the context of *sequential* market entry. There are many reasons why one firm may be forced to delay its market entry. For instance, there could be operational constraints (e.g., manufacturing delays or capacity constraints), organizational constraints (e.g., financial constraints), or technological constraints (e.g., product differentiation corresponding to the need for access to future technologies).[2] We show that sequential rather than simultaneous entry has several implications for the structure of licensing agreements and the role of privateering.

In our setting, two firms make simultaneous R&D investments on multiple components. After all the components necessary for production have been developed, firms can enter the market. Crucially, firms do not enter at the same time. We assume that one firm enters first, and the other firm is constrained to enter only after a certain amount of delay. Firms can generate profits from selling in the product market and from enforcing their patents through licensing agreements or litigation. Additionally, firms can sell their patents to a privateer while retaining a license for themselves. The privateer would then monetize the acquired patents against one of the competitors of the original patent owner. To investigate the effects of privateering on R&D and welfare, we compare licensing and product market outcomes in environments with and without privateering.

We provide insights into how privateering affects licensing agreements and, therefore, R&D incentives when firms enter the product market sequentially. Our results suggest that the positive effects of privateering, i.e., enhancing R&D efforts, are mitigated by sequential entry. The intuition for this result is as follows: The last firm to enter the product market cannot be a target of an infringement counter-lawsuit litigation during the period when the other firm has already entered the market. This immunity to litigation, even if it is only temporary, provides the same strategic advantage to the privateer in a setting with simultaneous entry. Consequently, a single firm may achieve the same outcomes whether it sells patents to a privateer or not. Thus, sequential market entry reduces the impact of a privateer since the laggard firm itself acts as a privateer. Furthermore, we provide evidence that privateering can reduce industry profits because it leads to over-investment in R&D.

The results of our paper may apply to oligopolistic markets in the high-tech industry in which firms need access to new technologies to enter. More specifically, we focus on markets in which competitors enter sequentially.

We contribute to the economic literature on innovation and patents. More specifically, we focus on the impact of privateering on incentives and welfare. Our model closely follows the work of Lemus and Temnyalov (2017). The crucial difference is that we relax the assumption of simultaneous entry, demonstrating that it qualitatively changes the results. Our main contribution is to study sequential entry in a licensing model with asymmetric bargaining partners (where the asymmetry arises from delayed market entry) and a PAE as an intermediary. Other articles focusing on privateering include Geradin (2019), who investigates how IP privateering leads to downstream market exclusion. Kesan et al. (2019) investigate privateering from an empirical perspective.

More broadly, there is a larger literature on PAEs. For example, in their empirical work, Cotropia et al. (2014) present arguments for and against PAEs. Another seminal paper in this literature is Choi and Gerlach (2017), who investigate PAE's litigation strategies for exogenously given patent portfolios. More recently, He (2020) investigates PAE's settlement strategies in a setting with asymmetric information, and Bergin (2022) investigates how PAE activity affects R&D incentives, focusing on the exclusive value of patents. Other papers in this area include Hovenkamp (2013) and Choi and Gerlach (2018). To the best of our knowledge, the present paper is the first study investigating the effects of privateers on markets with sequential entry.

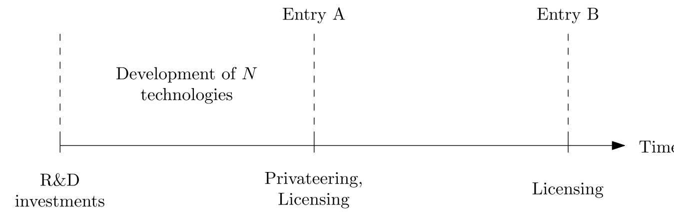


Fig. 1. Timing of the game.

The remainder of this paper is organized as follows. In Section 2, we describe our model in detail. In Section 3, we provide bargaining solutions for different patent allocations and derive equilibrium transfer outcomes. Using these results, we discuss the effect that privateering may have on R&D incentives in Section 4. Section 5 concludes.

2. Model setup

We investigate a multi-stage game with complete information. Firms A and B exert R&D efforts to develop technologies necessary for the production of goods, where the production of one's goods may potentially require technologies developed by the rival firm. Both firms need access to the same set of technologies $\mathcal{L} = \{1, \dots, N\}$ to be able to produce. Before the goods can be sold on the product market, all associated technologies have to be developed. We assume that one firm enters the product market before the rival does. Without loss of generality, suppose that firm A enters before B.

The timing of the events is illustrated in Fig. 1. First, firms invest in R&D, start research and immediately obtain patents for their respective inventions. Afterwards, firm A enters product market. At that time, firms may engage in privateering and make licensing agreements. Then, firm B enters product market and firms may engage in licensing again.

At the start of the game, firms A and B simultaneously make lump-sum R&D investments $x \geq 0$ and $y \geq 0$.[3] The costs associated with R&D are $C(\cdot)$ with the respective effort x or y as argument. We make the following standard assumptions: $C(\cdot)$ is increasing, convex and differentiable and $C(0) = 0$, $C'(0) = 0$. We assume that lump-sum R&D expenditures at the start of the game determine the probability of discovering each technology. If a firm chooses zero R&D effort, it will never develop any technology. After investments are sunk, N parallel R&D races start. We assume that all technologies are equally valuable for production and difficult to develop. Thus, each firm allocates the same amount of effort to every technology.[4] Times at which firms A and B discover technology $i \in \mathcal{L}$ are exponentially distributed, i.e., $\tau_{A,i} \sim \exp(x)$ and $\tau_{B,i} \sim \exp(y)$. The probability that A wins race i is $\mathbb{P}(\tau_{A,i} \leq \tau_{B,i}) = \int_0^\infty \int_0^t x e^{-xs} y e^{-yt} \mathrm{d}s\mathrm{d}t = x/(x+y)$. So, basically firms

[2] In the quantitative analysis of patent infringement claims by Kesan et al. (2019), it was found that IP privateers tend to acquire patents in the information technology sector, aligning with the scope of our work.

[3] This is in line with Lemus and Temnyalov (2017) and Loury (1979).

[4] These assumptions are critical because relaxing them results in two problems. First, if technologies differ in ex ante values or difficulties, both firms decide about individual effort levels for each component. Thus, firms do not only face a level but also allocation and project choice problems, which changes the structure of the model and is beyond the scope of this paper. Second, patent portfolios could not be characterized by the number of patents they consist of. Instead, the value of a portfolio would be a function of patent combinations, which makes our licensing negotiation approach not applicable.

compete in N identical Tullock contests with the respective technology as single prize.[5] Accordingly, the probability that A wins exactly k out of N races is then

$$P(k;x,y)=\binom{N}{k}\cdot\left(\frac{x}{x+y}\right)^{k}\cdot\left(\frac{y}{x+y}\right)^{N-k}. \tag{1}$$

Each technology $i\in\mathcal{L}$ is developed at time $\hat{\tau}_i(x,y)=\min[\tau_{A,i}(x),\tau_{B,i}(y)]$. The firm that discovers a technology first obtains a patent on it at no cost. We assume that patenting is publicly observable so that firms can freely imitate any technology discovered by any other firm without delay.[6] At the end of this stage, all N technologies are developed. The outcome of the stage is a realization of patent portfolios over $\mathcal{L}$. As a consequence of the assumption that all technologies are equally important for production, a portfolio can be characterized by the number of patents it includes. Denote the time at which all technologies are discovered as $t_A=\max_{i\in\mathcal{L}}[\hat{\tau}_i(x,y)]$.

At time t_A, firm A enters product market. Note that entry is possible even without owning all related patents or licenses because of observability of technologies and cost-less imitation.[7] We assume that entry is free and product market profits are high enough to make it a strict dominant strategy, such that A always enters. For the time span at which A is a monopolist on the product market, it realizes the flow profit $\pi_m>0$ per unit of time. Profits are discounted with rate $r>0$.

When firm A enters, firms A and B may engage in privateering, where a single PAE acts as a bargaining partner for both firms. Each firm may sell a subset of its patents to the PAE, where subsets and prices are determined by simultaneous bilateral and symmetric Nash bargaining. The inventor of a technology retains a free usage license. All patent holders and producers may engage in licensing. A already uses technologies for production and B will use them later. Licensing agreements are defined by simultaneous bilateral and symmetric Nash bargaining with patent litigation trials as outside option. We employ a simplistic approach to determine expected litigation outcomes. If a firm uses a technology without owning the related patent or permission of the patent holder, it can be subject to a patent infringement lawsuit. In the context of the process, the validity of each involved intellectual property right is checked. If a patent has been validated by court, courts will not re-evaluate its validity. As a consequence, a patent that has been involved in an infringement claim cannot be used in a second one. We assume that a license cannot be retracted, such that the same is true in case of bargaining and licensing. Denote $D>0$ the expected lump-sum damage payment per patent a firm has to pay if it infringes on it. A lawsuit incurs costs $L>0$ for each involved party, regardless of the number of involved patents. Counter-litigation incurs no additional costs if it is initiated immediately. As a consequence, a producing firm that is sued for patent infringement will always counter-litigate, i.e., sue the plaintiff for patent infringement at the time at which it is impeached. Since market entries occur sequentially in our setting, there are two reasonable times at which patents may be used - entry of A and entry of B (after B enters, it may be subject of infringement claims, too). Firms always use their whole patent portfolio. Thus, expected litigation outcomes depend on the allocation of patents across the involved parties and the time span between market entries. Licensing agreements include the specification of transfer payments, which are determined by symmetric and bilateral Nash bargaining.

We assume that firm B enters the product market after firm A, where the time span between entries is exogenous and denoted as $\Delta t>0$. From time $t_B=t_A+\Delta t$ on, A and B generate product market duopoly flow profits $\pi_d>0$ per unit of time, respectively. Again, we assume profits are high enough to make entry a strict dominant strategy.[8] Firms and patent owners may engage in licensing at t_B again. We formulate the game with two separate licensing stages for the sake of completeness. The outcome of licensing and privateering stages, namely, the determination of overall transfer payments, will be summarized in one term capturing potential t_A and t_B payments.

Ex ante expected payoffs of firms A and B are

$$\Pi_A(x,y)=\delta_{t_A}(x,y)\cdot\left\{(1-\delta)\cdot\frac{\pi_m}{r}+\delta\cdot\frac{\pi_d}{r}+\sum_{k=0}^{N}\left[P(k;x,y)\cdot\hat{T}(k,\delta)\right]\right\} - C(x) \tag{2}$$

and

$$\Pi_B(x,y)=\delta_{t_A}(x,y)\cdot\left\{\delta\cdot\frac{\pi_d}{r}-\sum_{k=0}^{N}\left[P(k;x,y)\cdot\hat{T}(k,\delta)\right]\right\}-C(y), \tag{3}$$

respectively, where we denote the expected discount factor between time of R&D investments and A's entry as $\delta_{t_A}(x,y)$ and the discount factor between market entries as $\delta\in(0,1)$. The expected time at which A enters is endogenously determined by R&D investments, such that the corresponding discount factor is a function of x and y:

$$\delta_{t_A}(x,y)=\frac{N!\,(x+y)^N}{\prod_{j=1}^{N}[r+j(x+y)]}. \tag{4}$$

The derivation of $\delta_{t_A}(x,y)$ is presented in Appendix A. $\delta_{t_A}(x,y)$ is strictly increasing in the R&D efforts as the expected time to develop each technology decreases as the total R&D effort increases. It is strictly decreasing in the number of technologies N, as a higher number of technologies is expected to take longer to develop and therefore leads to stronger discounting, or equivalently, a lower discount factor. From the perspective of t_A, the time span in which A is a monopolist is independent of R&D investments, which allows us to separate discount factors and treat the expected discount factor between entries $\delta=\mathrm{e}^{-r\cdot\Delta t}$ as exogenous parameter. Without a delay, $\Delta t=0$ and $\delta=1$, so the firms enter simultaneously, which corresponds to Lemus and Temnyalov (2017) with equal bargaining power. For any given r, a lower δ corresponds to a higher Δt. From t_A on, the net present value (NPV) of A's product market profits is $(1-\delta)\cdot\pi_m/r+\delta\cdot\pi_d/r$. The NPV of B's product market profits is $\delta\cdot\pi_d/r$. Total transfers vary in the discount factor between entries and the allocation of patents, represented by the argument k. They are defined positive from B to A and denoted as $\hat{T}(k,\delta)$, such that the sums describe expected transfers over all possible allocations of patents in $\mathcal{L}$. In what follows, we solve the game via backward induction and focus on pure strategies. First, we determine the outcome of licensing negotiations without and with the possibility of privateering. Then, we go one step back and investigate R&D investment decisions.

[5] We employ a simplified approach for the sake of manageability. The primary objective of this study is to examine the effects of patent privateering within a sequential framework. Privateering affects the outcomes of licensing negotiations, contingent upon the patent portfolios held by firms. Because licensing starts after all technologies are discovered, the specific configuration of the innovation process itself is immaterial to our findings, as long as higher R&D investments correspond to earlier access (in expectation) and an increased likelihood of patent ownership.

[6] For tractability, we abstract from strategic technological secrecy or delay of patenting and assume immediate publication of patents. In reality, patents and the related technologies are published after a legally defined period of time. As long as companies need access to all new technologies to be developed and have not developed all of them themselves, they must wait for publication. The earliest possible time of market entry would then be either t_A or $t_A+\epsilon$, where ϵ represents the exogenous time between patenting and publication. Since $\epsilon>0$ does not change the structure of the payoffs with respect to R&D investments, our results are robust. We assume that patenting costs are low such that it is always profitable to develop a technology. This avoids the problem of project choice, which would be beyond the scope of this paper.

[7] Note that production of the good is possible for A without owning all related patents. An extreme case, in which firm B owns all patents and A enters product market is within the scope of our model.

[8] In fact, we assume $(1-\delta)\pi_m+\delta\pi_d>rND$ and $\delta\pi_d>rND$ in order to ensure entries, where δ is a symmetric discount factor between entries.

3. Licensing in the shadow of patent litigation

3.1. Licensing without PAE and simultaneous entry

First, we analyze the game without PAE, taking the sizes of patent portfolios as given. That is, firms A and B bilaterally bargain over licensing agreements which result in transfer payments. Starting at the end of the game, we investigate a situation in which both firms are on the product market. So, both companies may be targets of patent lawsuits. Lemus and Temnyalov (2017) analyze this symmetric setting. In their model, firms enter product market simultaneously. In what follows, we briefly provide their analysis and refer to the situation as simultaneous entry setting, which is basically characterized by vulnerability to litigation by both producers.

Consider a scenario in which firms A and B hold n and m patents, respectively, that have not been used in licensing negotiations previously, where $n, m \in \mathbb{N}$ and $n + m = N$.[9] Firm A's expected payoff from litigation is $nD - mD - L$. Similarly, firm B's expected payoff from litigation is $mD - nD - L$. If either firm A or B has a credible litigation-threat (i.e., a positive expected payoff from litigation), firms enter into negotiations in order to avoid the joint costs of a trial, where the outcomes of the litigation process are the disagreement payoffs. Under equal bargaining power, the Nash bargaining solution leads to the transfer $T(n,m)$ paid by B to A. We have

$$T(n,m) = \begin{cases} (n-m)D & , n \leq m - \frac{L}{D} \\ 0 & , m - \frac{L}{D} < n < m + \frac{L}{D} \\ (n-m)D & , n \geq m + \frac{L}{D}. \end{cases} \quad (5)$$

Notice that this transfer can be negative. In this case, firm B receives a positive payment. We assume that $N > 3L/D$, which ensures that the number of new technologies is sufficiently high, such that $T(n, N-n)$ has three cases. Furthermore, the assumption captures the fact that innovative products, which are the scope of this paper, usually involve a very high number of new technologies.[10] In the first case of (5), firm B holds a credible litigation threat against A, i.e., $(m-n)D - L \geq 0$. In the second case, neither firm holds a credible threat because the gains from litigation do not exceed the costs. Following Lemus and Temnyalov (2017), we refer to this case as *patent truce*. In the last case, firm A holds a threat against B. Firms agree on a transfer payment equal to the outcome of trial plus the equally divided surplus from bargaining, instead of bringing an action at law. This surplus is L for each firm. If any firm holds a credible litigation threat, the resulting transfer is $(n-m)D - L + 2L/2 = (n-m)D$. Otherwise, it is zero. If a firm holds a credible litigation threat, its patents are used offensively. The other firm uses its patents to counter-litigate in order to protect itself in a defensive manner. So, obtaining an additional patent has two effects. It means not only owning one more but also the opponent owning one less patent. Thus, it is accompanied with an increase in transfers of $D + D = 2D$. From the perspective of the defensive firm, the value of an additional patent is also $2D$ - one patent less is used to threaten it and one patent more for defensive use. If an additional patent does not generate a credible threat for neither firm in the area of patent truce, its marginal value equals zero.

3.2. Licensing without a PAE and sequential market entry

We now introduce sequential market entry and investigate how firms agree on licensing transfers. Sequential entries cause a significant change in the structure of transfers. We define equilibrium transfers which we assume to be paid in t_A and are determined under anticipation of t_B bargaining outcomes.[11]

At t_A, only firm A can be subject to infringement claims, which changes the bargaining positions of both firms. Recall that courts check the validity of patents and licensing agreements cannot be retracted. Thus, firm B is not able to use its patents twice, so that it has no patents left for defensive use in t_B if it uses them in t_A. At t_B, both firms are active in the market, such that both firms are potential litigation targets. Here, two scenarios are possible. First, consider a situation in which B has not used its patents at t_A and, as a consequence, is fully armed with a portfolio consisting of m patents. As stated in (5), the corresponding transfer is $T(n,m)$. Now consider a situation in which B has already used its patents to enter bargaining at the time at which A has entered the market. Then, it has no patents left to defend itself later on. The resulting transfer that is negotiated in t_B equals $T(n,0)$. $T(n,0)$ is either nD if $n \geq L/D$ which ensures that A holds a credible threat without the possibility of counter-litigation by B, or otherwise $T(n,0) = 0$.

Going backwards to the time of A's entry, only firm B faces the possibility of either bringing its patents into negotiations at t_A, or holding them back for utilization in t_B. Holding back leads to the transfer $T(n,m)$ in t_B, which is discounted by δ. If B initiates bargaining in t_A, it may be able to exploit its advantage of not being a possible target of counter-litigation. Using the notation from before, this results in $T(0,m)$, where A cannot use any of its patents defensively. Nevertheless, A may hold a portfolio strong enough to credibly threaten B with litigation once both firms are active on the product market. As described above, the corresponding transfer payment at this time equals $T(n,0)$. So, if firm B decides to use its patents in t_A, the overall transfer payment is given by $T(0,m) + \delta T(n,0)$. Thus, B has an incentive to use its patents in t_A if

$$T(0,m) + \delta T(n,0) \leq \delta T(n,m). \quad (6)$$

The following proposition summarizes the analysis from above:

Proposition 1. *Consider a duopoly market with sequential entry without PAE. Then, symmetric Nash-bargaining at time t_A leads to the transfer*

$$\overline{T}(n,m,\delta) = \begin{cases} T(0,m) + \delta T(n,0) & , \phi(n,m,\delta) \geq 0 \\ \delta T(n,m) & , \phi(n,m,\delta) < 0 \end{cases} \quad (7)$$

paid by firm B to firm A, where $\phi(n,m,\delta) = \delta[T(n,m) - T(n,0)] - T(0,m)$ indicates whether B prefers using its patent portfolio in t_A or not.

Substitution of (5) into $\overline{T}(n,m,\delta)$ yields

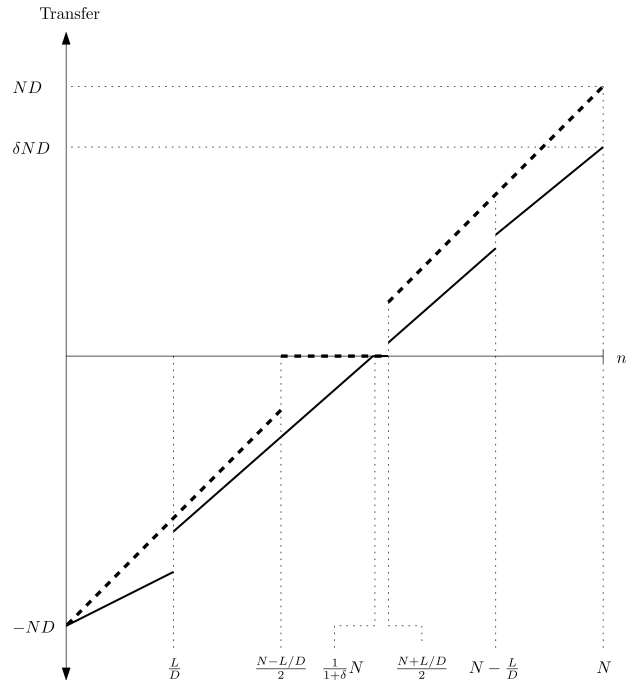
$$\overline{T}(n,m,\delta) = \begin{cases} \delta(n-m)D & , m < \frac{L}{D} \\ -mD & , m \geq \frac{L}{D} \wedge n < \frac{L}{D} \\ 0 & , m \geq \frac{L}{D} \wedge m - \frac{L}{D} < n < m + \frac{L}{D} \wedge m < \delta n \\ (\delta n - m)D & , \text{otherwise.} \end{cases} \quad (8)$$

Fig. 2 depicts the transfers defined in (5) and (7), i.e., transfers for simultaneous and sequential market entries. Obviously, sequential market entries change the structure of transfers. $\bar{T}(n, N-n, \delta)$ is illustrated

[9] For simplicity, assume that there has been no licensing before such that both firms hold their whole portfolios and may use them for negotiations.

[10] An extreme example for the high number of technologies a product can involve is presented by Drummond (2011). In 2011, a smartphone has already involved around 250,000 patents. Given the well-known fact that technological complexity increases over time, we can only imagine how high this number is in 2024.

[11] Because of sequential rationality, it is not relevant for the expected payoffs whether firms A and B negotiate at t_A and t_B, or only in t_A while anticipating t_B-outcomes. Both scenarios lead to transfers equivalent to the combination of both outcomes in consideration of the discount factor δ.



This figure illustrates transfers for simultaneous and sequential market entries without the presence of a PAE from a t_A-perspective. The dashed line shows transfers in case of simultaneous entries, $T(n, N-n)$. Transfers under sequential entry, $\overline{T}(n, N-n, \delta)$, are depicted by the solid line for $\delta \in \mathcal{P}$.

Fig. 2. Transfers without patent privateering: simultaneous vs. sequential entry.

by the solid line. In the first case of (8), firm B's portfolio is so weak that it never holds a credible litigation threat. However, its patents have a positive value since they can be used defensively in t_B, which leads to the t_A-transfer $\delta(n-m)D$. In the second case, firm A holds such a low number of patents that it is not able to generate any litigation threat - even if B holds no patents to defend itself. B uses its patents in t_A and generates the transfer $-mD$ without facing the possibility of litigation in t_B. The only value of A's patents is that they cannot be used against A itself. The third case covers situations in which the patent portfolios have a similar size, and, at the same time, t_B-transfers are not discounted massively. Here, it is profitable for B to hold its patents back in t_A which leads to patent truce later on. We call this *inter-temporal patent truce* with a resulting transfer of zero. For every other possible allocation of patents, B prefers early use of IP rights. The resulting transfer is $(\delta n - m)D$.

While patent truce can always be achieved in the simultaneous setting for patent portfolios of similar size, this does not necessarily hold if firms enter sequentially. We can show the following proposition:

Proposition 2. *Consider a duopoly market with sequential entry and without PAE. Then, there exist patent allocations that result in inter-temporal patent truce for* $\delta \in \mathcal{P} = ((ND-L)/(ND+L), 1)$. *The number of patent allocations that lead to patent truce decreases when firm B's entry delay is larger, i.e.,* δ *decreases. Inter-temporal patent truce is impossible for patent allocations in which the laggard firm B holds more patents than the firm that enters first, A, i.e.,* $n \le m$.

Proof. See Appendix B.

Note that the interval of δ in which inter-temporal patent truce is possible, $\mathcal{P}$, gets wider with higher litigation cost, L, and narrower with higher damage payment per patent, D.

To see this, differentiate the lower bound of $\mathcal{P}$ with respect to L, which gives $\mathrm{d}\left[(ND-L)/(ND+L)\right]/\mathrm{d}L = -2ND/(ND+L)^2 < 0$. With higher L, the lower bound decreases so that inter-temporal patent truce is possible for a broader range of δ. Differentiating the lower bound with respect to D, we obtain $\mathrm{d}\left[(ND-L)/(ND+L)\right]/\mathrm{d}D = 2NL/(ND+L)^2 > 0$. By contrast, the interval becomes narrower with higher D.

Talking about transfers, intuitively, the temporal structure favors the firm that enters last. Under sequential entry, firm B is able to generate a temporary advantage of not being target to counter-litigation. The wider the time gap between entries, the higher B's advantage. It helps B to generate threats and influences maximal amount of transfers it can achieve. For illustration, consider two extreme situations: one in which A holds all patents and one in which B holds all patents. Obviously, holding all relevant patents, the maximal payment A receives is $\overline{T}(N, 0, \delta) = \delta ND$, which is lower than $-\overline{T}(0, N, \delta) = ND$, the maximal payment B may generate. The difference between these extreme outcomes decreases in δ. The following lemma summarizes the analysis:

Lemma 1. *Consider a duopoly market in which firm A enters product market before B. Then,*

(i.) *the temporal structure results in an advantage with regard to patent monetization for firm B, independent of the allocation of patents, i.e.,* $T(n, N-n) - \overline{T}(n, N-n, \delta) \ge 0$, $\forall\, n \in [0, N]$, $\delta \in (0, 1)$*;*
(ii.) *this advantage increases when firm B's entry delay is larger, namely, lower* δ.

Proof. See Appendix C.

In summary, transfer payments are determined through bargaining in the context of patent litigation, meaning they are the result of a cooperative game in which litigation defines the disagreement outcome. Transfers equal the expected litigation payoffs plus the equally divided settlement surplus. The possibility of sequential market entry changes the structure of transfer payments compared to the case of simultaneous entry. Specifically, the temporal structure reduces the number of patent allocations that result in inter-temporal patent truce and favors the firm that enters last. In what follows, we will repeat our analysis in an environment where patent privateering is an option in order to investigate the effects of a PAE.

3.3. Licensing with a PAE and simultaneous entry

Now, firms A and B face the possibility of selling their patents to a PAE, which does not produce any good. This part of the game does not differ from the one Lemus and Temnyalov (2017) present. Throughout, we use their bargaining model which goes back to Horn and Wolinsky (1988). In what follows, we briefly present an equilibrium outcome of privateering negotiations and describe the resulting transfers for the case of simultaneous entries. Afterwards, we use it in order to determine equilibrium transfers for our sequential market entries setting.

Firms A and B simultaneously and bilaterally bargain with the PAE over the redistribution of their patents. Suppose the PAE buys n' and m' patents from firms A and B for total prices p_A and p_B, respectively. Denote the payoffs from licensing, dependent on the patent allocation after making deals with the PAE, by $S_i(n', m')$, $i \in \{A, B, PAE\}$. Every equilibrium outcome of this bargaining game maximizes the joint surplus of the bilateral bargaining partners, taking the deal of the other pair of partners as given. Thus, they solve

$$(n', p_A) \in \arg\max_{(z,p)} \left(\left[S_{PAE}(z, m') - p - S_{PAE}(0, m')\right]^{\frac{1}{2}} \cdot \left[S_A(z, m') + p - S_A(0, m')\right]^{\frac{1}{2}} \right),$$

$$(m', p_B) \in \arg\max_{(z,p)} \Big(\big[S_{PAE}(n',z) - p - S_{PAE}(n',0)\big]^{\frac{1}{2}} \cdot \big[S_B(n',z) + p - S_B(n',0)\big]^{\frac{1}{2}} \Big).$$

Lemus and Temnyalov (2017) prove that it is an equilibrium if each producing firm sells its whole portfolio to the PAE for prices equal to the transfer payments the PAE receives from the other firm, respectively. The intuition behind this is as follows: Since a PAE does not produce or sell any product, it can never be subject to patent infringement claims. If a PAE that has acquired patents from one firm sues the respective rival firm for patent infringement, the defendant is not able to initiate counter-litigation. Thus, patents do not have a defensive value if a PAE is involved. Suppose firm A has sold all its patents to the PAE, such that it obtains usage licenses for its 'own' $n = n'$ technologies and infringes on the m patents developed by its competitor. The PAE then may sue firm B for patent infringement if it holds a credible threat. Since firm B is no longer able to use its patents for counter-litigation, the PAE holds a credible threat against it if $n \geq L/D$. In this case, the resulting transfer from B to the PAE is nD or zero otherwise. Thus, the transfer can be described by $T(n, 0)$. Nevertheless, B's portfolio is of offensive value and can be used against A that is now defenseless. Here, B's threat is credible if $m \geq L/D$ which results in a transfer in the amount of mD between A and B, or zero otherwise. In our notation that is $T(0, m)$.

Firm B is indifferent between keeping or selling an arbitrary subset of its patent portfolio to the PAE if the PAE offers a total price equivalent to the licensing revenue that B could generate by bargaining with firm A directly. Suppose the PAE offers this price and B sells all its patents to the PAE. The same reasoning can be applied to firm A if firm B sells all its patents to the PAE. In consequence, there is no profitable deviation for neither A, B, nor the PAE if both producers sell all patents to the PAE for prices equal to the licensing revenues they can generate without counter-litigation by themselves. Therefore, this constitutes an equilibrium. Note that in this equilibrium, the PAE does not extract rents and the total payments firms A and B receive (or pay) are combinations of two 'regular' transfer payments without counter-litigation.

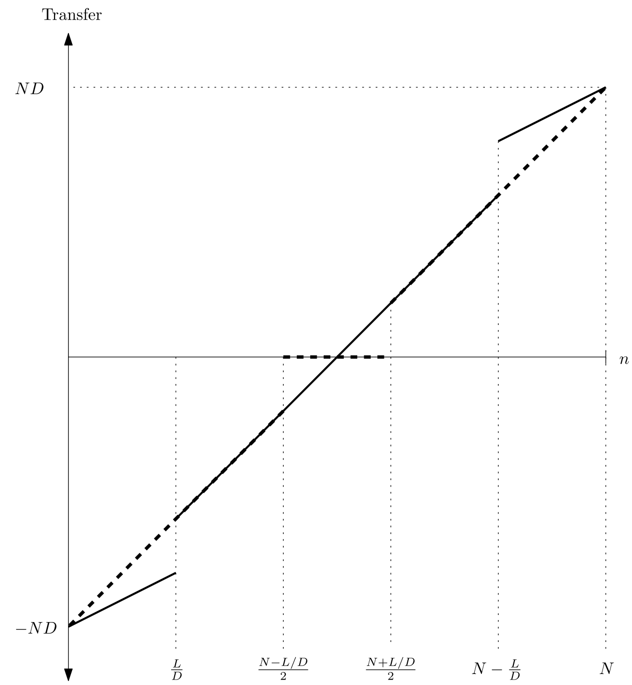
All payments are aggregated in a transfer function under privateering, $T_{PAE}(n, m)$. Taking the representation in (5) into account, the total transfer with two active producers from B to A under privateering is

$$T_{PAE}(n, m) = T(n, 0) + T(0, m), \tag{9}$$

where transfers are not paid directly from producing firm to producing firm.[12] Here, the PAE acts as intermediary that receives revenues from licensing agreements and pays prices for patent portfolio acquisition. Fig. 3 illustrates the transfers in this equilibrium for all possible patent allocations.

The availability of a privateer changes the marginal value of patents for patent allocations in which either one firm holds the majority, or in which both firms hold a similar amount of patents. If one firm holds a very large number of patents, such that its opponent is never able to create a credible threat, the weaker portfolio loses its defensive value. If the number of patents a firm holds is not sufficient to create a credible threat of litigation, they cannot be used offensively. Since privateering destroys any defensive value of patents, the portfolio is worthless in context of patent monetization. Therefore, the value of an additional patent equals D - one patent more the weaker firm holds is one patent less that can be used against it; one patent more the stronger firm holds is one patent more that can be used in an offensive manner. If patent portfolios are of similar size, the fear of counter-litigation leads to patent truce if privateering is not available. The PAE eliminates the threat of counter-litigation, such that patent truce will not arise if a privateer is present. The value of an additional patent a firm develops always equals $2D$, even if the portfolios are of the same size.

[12] Besides this equilibrium, Lemus and Temnyalov (2017) derive other kinds of equilibria in which the PAE may extract rents from the product market. In the present paper, we abstract away from the rent-extraction effect privateering may have and investigate the equilibrium that provides the strongest incentives for investing in R&D.



This figure illustrates transfers for simultaneous market entries with and without the presence of a PAE. The dashed line shows transfers in case of simultaneous entries without privateering, $T(n, N-n)$. Transfers with privateering, $T_{PAE}(n, N-n)$, are depicted by the solid line.

Fig. 3. Transfers for simultaneous market entries: with vs. without privateering.

3.4. Licensing with a PAE and sequential entry

Finally, we consider our setting with sequential entry and apply the aforementioned results. All payments taken together result in a combination of transfers as presented in (9). Applying this result to (7) in Proposition 1, we can express transfers for sequential market entries under patent privateering as a combination of sequential transfers without a PAE, that is, $\overline{T}(n, 0, \delta) + \overline{T}(0, m, \delta)$, where

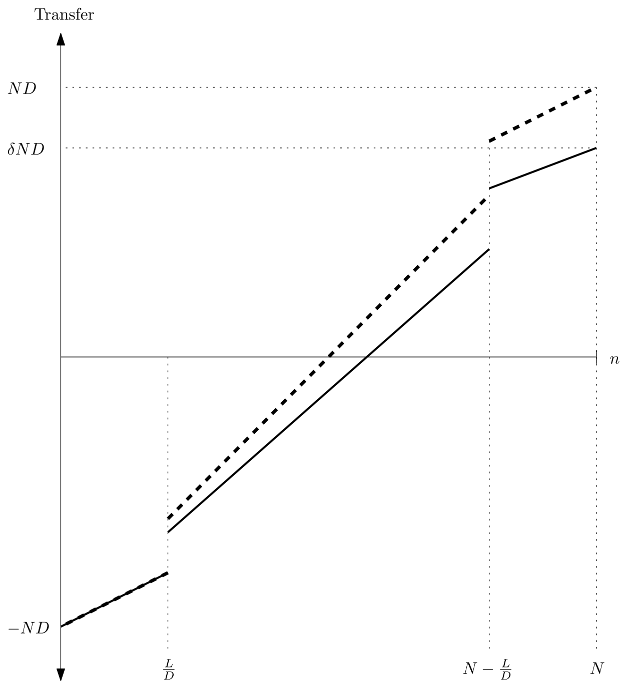
$$\overline{T}(n, 0, \delta) = \begin{cases} 0 & , n < \frac{L}{D} \\ \delta n D & , n \geq \frac{L}{D} \end{cases} \quad \text{and}$$

$$\overline{T}(0, m, \delta) = \begin{cases} 0 & , m < \frac{L}{D} \\ mD & , m \geq \frac{L}{D} \end{cases}$$

characterize transfers for situations in which patents are never used for counter-litigation. The following proposition summarizes:

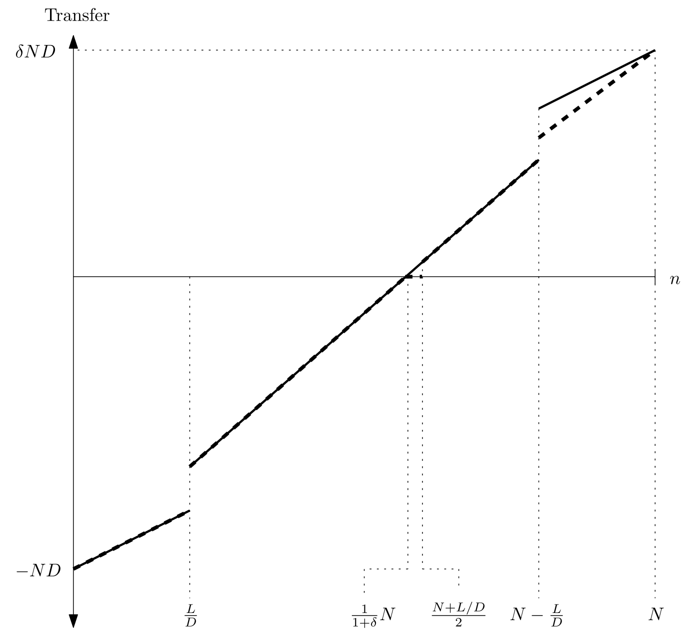
Proposition 3. *Consider a duopoly market with sequential entry and patent privateering. Then, symmetric Nash-bargaining leads to the transfer*

$$\overline{T}_{PAE}(n, m, \delta) = \begin{cases} -mD & , n < \frac{L}{D} \wedge m \geq \frac{L}{D} \\ (\delta n - m)D & , n \geq \frac{L}{D} \wedge m \geq \frac{L}{D} \\ \delta n D & , n \geq \frac{L}{D} \wedge m < \frac{L}{D} \end{cases}. \tag{10}$$



This figure illustrates transfers for simultaneous and sequential market entries with the presence of a PAE. The dashed line shows transfers in case of simultaneous entries, $T_{PAE}(n, N-n)$. Transfers in case of sequential entry, $\overline{T}_{PAE}(n, N-n, \delta)$, are depicted by the solid line.

Fig. 4. Transfers with patent privateering: simultaneous vs. sequential entry.



This figure illustrates transfers for sequential market entries with and without the presence of a PAE. The dashed line shows transfers without privateering, $\overline{T}(n, N-n, \delta)$. Transfers with privateering in case of sequential entry, $\overline{T}_{PAE}(n, N-n, \delta)$, are depicted by the solid line.

Fig. 5. Transfers for sequential market entries: with vs. without privateering.

Consider the first case of (10) in which A has developed a very low number of patents so that it is not sufficient to credibly threaten B with litigation, $n < L/D$. By $n + m = N > 3L/D$, there does not exist a case in which both firms generate portfolios that are too weak to generate credible threats at the same time. So, the first case captures a situation in which A has developed a very weak portfolio and B a rather strong one. Here, only B's portfolio is of value and is used as early as possible, i.e., at time t_A. The resulting transfer is $-mD$. In the second case, patents are allocated in a way that provides credible litigation threats, $n, m \geq L/D$. All patents are used as early as possible. Because of sequential entry, A would be vulnerable to litigation already in t_A and B would not be vulnerable until t_B, such that the resulting transfer is $\delta nD - mD = (\delta n - m)D$. Third, only A has developed enough patents to generate a credible litigation threat, $n \geq L/D$. Nevertheless, A's patents are only usable for litigation in t_B. This leads to a transfer δnD from the perspective of time t_A.

Lemma 2. *Consider a duopoly market in which firm A enters product market before B and the firms face the possibility of patent privateering. Then,*

(i.) *sequential market entries result in an advantage for firm B with regard to patent monetization, i.e.,*

$$T_{PAE}(n, N-n) - \overline{T}_{PAE}(n, N-n, \delta) \geq 0, \quad \forall n \in [0, N], \delta \in (0, 1),$$

with equality if A develops a very small patent portfolio, $n < L/D$;

(ii.) *the effect increases the weaker B's patent portfolio is, i.e., $T_{PAE}(n, N-n) - \overline{T}_{PAE}(n, N-n, \delta)$ (weakly) increases in n;*

(iii.) *the effect increases with lower δ, i.e., larger entry delay of firm B.*

Proof. See Appendix D.

Fig. 4 illustrates the differences in transfers with PAE for simultaneous and sequential market entries. Clearly, firm B benefits from the sequential structure. The transfers from B to A are either strictly lower in the sequential entry setting or the same as those in the simultaneous entry setting. If A holds a weak patent portfolio that cannot generate a credible litigation threat, the temporal structure does not affect transfers under privateering. If firm A possesses a sufficiently large number of patents to pose a litigation threat against a defenseless opponent, firm B benefits more from the sequential structure, especially when its own patent portfolio is weaker. In case of simultaneous entries, the n patents of A and the m patents of B are monetized at the same time. In case of sequential entry, only B can use its patents offensively in t_A. The portfolio of A can only be monetized later at t_B, resulting in a discounted value. The more patents A holds, the higher is the share of total patent value that is discounted. As n increases, the effect of the temporal structure becomes more pronounced. Overall, B benefits from stronger discounting with respect to transfers, i.e., lower δ which corresponds to a larger entry delay.

3.5. Effects of privateering on licensing under sequential entry

Comparing sequential entry transfers with and without the presence of a PAE yields a main result of our paper, as follows:

Proposition 4. *Suppose firms enter the market sequentially. Then, patent privateering has no effect on equilibrium transfers if $n < m$. Otherwise:*

(i.) *If $n \in (1/((1+\delta)N), \frac{1}{2}(N + L/D))$ and $\delta \in \mathcal{P}$, then $\overline{T}_{PAE}(n, m, \delta) \neq 0$. That is, privateering eliminates the possibility of inter-temporal patent truce for these patent allocations.*

(ii.) *If $m < L/D$, then $\overline{T}(n, m, \delta) \leq \overline{T}_{PAE}(n, m, \delta)$.*

Proof. See Appendix E.

Transfers with and without PAE and sequential market entries are depicted in Fig. 5. In order to understand the effects of privateering under sequential entry, consider Proposition 2, Lemma 1 and Lemma 2. The advantage of a PAE over a producing firm in terms of patent monetization lies in its invulnerability to counter-litigation. The sequential structure temporarily bestows this advantage upon the firm that enters the product market last. For the period of time between t_A and t_B, firm B cannot be target to litigation. Litigation of A (or the PAE) is not possible until t_B when B produces its own product and therefore may infringe on some patents. As a consequence, the resulting transfers closely resemble those achieved with privateering, significantly mitigating the overall impact of patent privateering in comparison to the simultaneous setting presented by Lemus and Temnyalov (2017). For illustration, compare Figs. 3 and 5.

If A holds a very weak portfolio, B uses its portfolio offensively in t_A and A cannot use its own patents for counter-litigation. Later in t_B, A does not hold enough patents to credibly threaten B. In these cases, the usage of a PAE does not offer any additional advantage - neither for A, nor for B, such that the presence of a privateer has no effect on transfers. If both firms hold portfolios large enough to justify threats, $n, m \geq L/D$, B will almost always use its patents offensively in t_A and A threatens B with litigation in t_B. If this is the case, the resulting transfer is a combination of t_A- and discounted t_B-payments, each of them determined without defensive use of opponent's patents. Again, the PAE does not offer any additional advantage, such that sequential entry transfers with and without PAE are equivalent. Exceptions are patent allocations such that $n \in (1/((1+\delta)N), \frac{1}{2}(N + L/D))$. For these allocations, it would be profitable for firm B to reserve its patents in t_A for defensive use in t_B, aiming to achieve an inter-temporal patent truce as described in detail in Proposition 2. Here, the privateer offers A the possibility to eliminate the defensive value of B's portfolio, such that the resulting transfer A receives is positive (rather than zero under patent truce). Clearly, A benefits from the presence of a PAE in this case. Last, consider a situation in which B's portfolio is weak, $m < L/D$. Without privateering, firm B, which is not in a position to hold a credible threat, retains its patents in t_A to use them defensively in t_B. In this scenario, the sequential game structure offers no advantage to B — except that it has to pay later. Under privateering, this defensive use is no longer possible and B's portfolio is neither of offensive, nor of defensive value. As a consequence, the presence of a PAE favors firm A again. Overall, patent privateering has either no effect on transfers if firms enter sequentially, or it changes total transfers in a way that is beneficial for the firm that enters first.

Comparing the equilibrium transfers between the simultaneous and the sequential setting leads to the following proposition:

Proposition 5. *For all $n \in \mathcal{L}$, the absolute effect of patent privateering on equilibrium transfers is (weakly) lower if firms enter sequentially than if they enter simultaneously, i.e., for $\delta \in (0, 1)$,*

$$\left|\overline{T}(n, N-n, \delta) - \overline{T}_{PAE}(n, N-n, \delta)\right| \leq \left|T(n, N-n) - T_{PAE}(n, N-n)\right|. \tag{11}$$

Proof. See Appendix F.

The LHS of (11) displays the absolute effect of privateering in a setting with transfer bargaining in anticipation of sequential market entries. The RHS is the absolute effect of privateering in a symmetric setting in which both firms enter product market at the same time. Thus, Proposition 5 states that the effect of privateering on equilibrium transfers is mitigated in a setting with sequential market entries. Furthermore:

Proposition 6. *For all $n \in \mathcal{L}$ and $\delta \in (0, 1)$, the absolute effect of patent privateering on equilibrium transfers under sequential entry, $|\overline{T}(n, N-n, \delta) - \overline{T}_{PAE}(n, N-n, \delta)|$, increases (weakly) in δ, i.e., decreases with larger entry delay.*

Proof. See Appendix G.

The proposition states that the effect of privateering on equilibrium transfers decreases weakly with stronger discounting. A smaller value of δ indicates a stronger time preference given a fixed time interval between market entries or a larger time interval with a fixed time preference. In the sequential setting, firm B temporarily exhibits the characteristics of a PAE with respect to licensing, with an increasing advantage as the discount factor becomes smaller. For more information, see Lemma 1, Lemma 2, and Proposition 4. As a result, the smaller δ is, the less impact privateering has on licensing arrangements, and the overall effect is weakly reduced for all patent allocations.

4. Investments in R&D

Firms maximize the expected NPV of their respective profits (2) and (3) by choosing R&D efforts in anticipation of transfer payments. As presented in the previous chapter, equilibrium transfers with and without PAE differ. For $\delta \in (0, 1)$, we have transfers $\hat{T}(k, \delta) = \overline{T}(k, N-k, \delta)$ without privateering and $\hat{T}(k, \delta) = \overline{T}_{PAE}(k, N-k, \delta)$ with PAE, specified in Proposition 1 and Proposition 3, respectively. The situation with simultaneous entry is analogue to the one presented by Lemus and Temnyalov (2017). Corresponding transfers are (5) and (9), such that, for $\delta = 1$, $\hat{T}(k, \delta) = T(k, N-k)$ and $\hat{T}(k, \delta) = T_{PAE}(k, N-k)$ without and with PAE, respectively. Similarly, denote profits without privateering as $\Pi_A(x, y)$ and $\Pi_B(x, y)$, and profits with privateering as $\Pi_{A,PAE}(x, y)$ and $\Pi_{B,PAE}(x, y)$. We are searching for a pure-strategy subgame-perfect Nash equilibrium that incorporates bargaining solutions. In order to find the equilibrium R&D efforts, we aim to find a Nash equilibrium on the first stage while considering the equilibrium outcomes on following stages. We define $x^R(y)$, $y^R(x)$, $x^R_{PAE}(y)$ and $y^R_{PAE}(x)$ as the reaction functions for firms A and B, respectively. As before, the subscript 'PAE' indicates the possibility of patent privateering:

$$x^R(y) = \arg\max_{x \geq 0} \Pi_A(x, y), \qquad x^R_{PAE}(y) = \arg\max_{x \geq 0} \Pi_{A,PAE}(x, y),$$
$$y^R(x) = \arg\max_{y \geq 0} \Pi_B(x, y), \qquad y^R_{PAE}(x) = \arg\max_{y \geq 0} \Pi_{B,PAE}(x, y).$$

We provide illustrations in Appendix H, with examples of expected profits shown in Fig. H.6 and reaction functions in Fig. H.7. Every intersection of reaction functions characterizes a Nash equilibrium. Denote x^*, y^* and x^*_{PAE}, y^*_{PAE} as the equilibrium efforts of firms A and B without and with the possibility of patent privateering, respectively. Applying the theorem of Debreu (1952), Glicksberg (1952) and Fan (1952) leads to the following lemma:

Lemma 3. *There exists a pure-strategy Nash equilibrium in the R&D game.*

Proof. See Appendix I.

A full analytical representation of general results is beyond the scope of this paper. In contrast to Lemus and Temnyalov (2017), assuming symmetric equilibria is not reasonable in our context, and as a result, we are unable to exploit such structures here. Nevertheless, we present numerical results in which Nash equilibria are approximated using best-response dynamics to provide an intuition of the effects at play in our setting.[13]

Welfare. To establish a benchmark, we consider social welfare. The allocation of patents and resulting transfer payments are not relevant

[13] Firm $i \in \{A, B\}$ maximizes expected payoffs $\Pi_i(x_i, x_{-i})$ by choosing its R&D effort x_i. Firm i's reaction function is $x^R_i(x_{-i}) = \arg\max_{x_i} \Pi_i(x_i, x_{-i})$. We denote the equilibrium efforts of i as x^*_i. In every Nash equilibrium, it holds that $x^*_i = x^R_i(x^R_{-i}(x^*_i)), \forall i \in \{A, B\}$. We exploit this property in order to approximate Nash equilibrium R&D efforts by using an iterative approach during numerical computation. Details are provided upon request.

since we focus on the cooperative equilibrium in which the PAE does not extract rents. Once firms enter the product market, they generate a flow of profits and provide their respective product, generating consumers' surplus. The presence of a patent privateer does not affect this in a direct manner. In our model, there are two firms that produce one product each, and both firms enter sequentially. Recall that firm A temporarily is a monopolist and generates profits π_m per unit of time. Once firm B enters, firms A and B gain duopoly profits π_d per unit of time. While firm A is a monopolist, consumers receive a surplus of w_m per unit of time. In duopoly, consumer surplus per unit of time is w_d, where $w_d > w_m \geq 0$. Expected welfare can be expressed as a function of R&D efforts, i.e.,

$$W(x, y) = \delta_{t_A}(x, y) \cdot \left((1-\delta) \cdot \frac{\pi_m + w_m}{r} + \delta \cdot \frac{2\pi_d + w_d}{r} \right) - C(x) - C(y).$$

A social planner maximizes welfare. Denote the social optimal R&D efforts of firms A and B as x^S and y^S, respectively. They are the solution to the planner's maximization problem, namely,

$$(x^S, y^S) = \underset{x \geq 0,\, y \geq 0}{\arg\max}\, W(x, y).$$

As mentioned before, privateering affects transfers but does not have a direct effect on economic welfare. Nevertheless, the presence of a PAE shapes firms' R&D incentives and may distort them away from the optimal level. We will investigate this effect.

Numerical results. In the following, we vary parameter constellations in order to illuminate the effects of privateering under sequential entry in various settings, i.e., different extents of discounting between entries. A discount factor $\delta \in (0, 1)$ represents sequential entry. $\delta = 1$ indicates a situation with simultaneous entries as in Lemus and Temnyalov (2017). Selected numerical results are listed in Appendix J. We present the following findings:

Result 1. Patent privateering increases firms' overall investments in R&D, i.e., $x^*_{PAE} + y^*_{PAE} \geq x^* + y^*$ for all $\delta \in (0, 1]$.

For all parameter constellations in which patent privateering affects equilibrium R&D incentives, it holds that the presence of a PAE increases the sum of equilibrium R&D efforts.

Denote the effect of privateering on equilibrium R&D investments as $\Delta x^*_\delta = x^*_{PAE} - x^*$ and $\Delta y^*_\delta = y^*_{PAE} - y^*$ for firm A and firm B, respectively, where the index δ represents the corresponding discount factor between entries.

Result 2. The positive effect of privateering on overall equilibrium R&D investments decreases with decreasing δ, i.e., a larger entry delay of firm B. It holds that $\Delta x^*_{\delta_1} \leq \Delta x^*_{\delta_2}$ and $\Delta y^*_{\delta_1} \leq \Delta y^*_{\delta_2}$ for any $\delta_1 \leq \delta_2$.

As stated in Proposition 4, patent privateering has a limited impact on equilibrium transfers if firm A enters the market before firm B. This temporal structure provides firm B with a temporary advantage similar to what a PAE would typically offer. As a result, the use of a PAE is not as beneficial for producing firms in the case of sequential entry compared to simultaneous entry. Therefore, the presence of a privateer does not provide as much additional R&D incentives in this setting, particularly for firm B, and the overall effect of patent privateering is reduced. A larger extent of discounting, either caused by a stronger time preference or a longer time span between entries, is represented by a smaller δ. As stated in Proposition 5, the effect of privateering on licensing outcomes is reduced in the sequential setting. It becomes weaker with stronger discounting, as shown in Proposition 6. Lemus and Temnyalov (2017) do not consider this effect because they study patent privateering in a symmetric framework.

Result 3. Patent privateering increases equilibrium over-investment in R&D. Over-investment decreases under sequential entry.

In our setting, firms tend to over-invest in R&D relative to the optimal solution of the social planner, even in the absence of patent privateering. However, patent privateering does influence the equilibrium R&D efforts of producing firms by enhancing the monetization of their patent portfolios, resulting in an overall increase in firms' equilibrium efforts and a corresponding rise in over-investment. Note that the social optimal investment is unchanged by the presence of a PAE. Denote the equilibrium profits of firm A and firm B without privateering as $\Pi^*_A = \Pi_A(x^*, y^*)$ and $\Pi^*_B = \Pi_B(x^*, y^*)$, respectively. $\Pi^*_{A,PAE} = \Pi_A(x^*_{PAE}, y^*_{PAE})$ and $\Pi^*_{B,PAE} = \Pi_B(x^*_{PAE}, y^*_{PAE})$ are profits with PAE.

Result 4. The presence of a patent privateer can decrease equilibrium industry profits, that is $\Pi^*_{A,PAE} + \Pi^*_{B,PAE} \leq \Pi^*_A + \Pi^*_B$ for all $\delta \in (0, 1]$.

Furthermore, patent privateering has a negative effect on industry profits, even in the bargaining equilibrium without rent extraction. We observe the decrease of industry profits throughout all considered parameter constellations. This negative effect on profits can be attributed to the incentives for increased R&D efforts that patent privateering creates, leading to excessive R&D costs and contributing to the observed over-investment. This suggests that the over-investment identified in Result 3 is not only caused by the delayed provision of final goods, but also by excessively high R&D costs.

Result 5. If firms enter sequentially, privateering is less favorable for firm B than for firm A. For all $\delta \in (0, 1)$, it holds that $\Pi^*_{A,PAE} - \Pi^*_A \geq \Pi^*_{B,PAE} - \Pi^*_B$.

We find that $\Pi^*_{A,PAE} - \Pi^*_A \geq 0$ and $\Pi^*_{B,PAE} - \Pi^*_B \leq 0$, indicating that only firm A benefits from patent privateering if the firms enter the market sequentially. This highlights the asymmetric effects of patent privateering on the profits of firms in differentiated markets.

5. Conclusion

In this paper, we present a theoretical model to analyze the effects of patent privateering in a market in which firms enter sequentially. Our main contribution is the development of a licensing game and its equilibrium outcome with sequential market entries and a PAE as an intermediary, as well as the modeling of the corresponding R&D game. We build upon the model of Lemus and Temnyalov (2017) by incorporating the feature of sequential entry. While they examine the welfare effects of patent privateering in a fully symmetric setting and argue in favor of patent privateering, we investigate its effects in an asymmetric duopoly. We examine the implications of this temporal structure on licensing agreements. If firms enter the market sequentially, the structure of licensing agreements changes. This results in a decrease of the overall effect of PAEs on patent monetization and R&D investments.

In our model, patent disputes are always resolved through out-of-court settlements to avoid the inefficiencies and costs of litigation. This means that transfers equivalent to the expected litigation outcome are paid to resolve the disputes. PAEs cannot be sued for patent infringement, so the threat of patent litigation cannot be used against them in negotiations. This gives PAEs a stronger bargaining position in patent licensing negotiations compared to producing firms, as they can eliminate the defensive value of patent portfolios and help monetize otherwise useless patents.

Under sequential entry, a significant portion of the advantage that a patent privateer could offer to producers is already held by the firm that enters the product market last. After the entry of the first firm, the firm that is not yet active in the product market can use its patents to sue the producer for infringement without the risk of immediate counter-litigation. During the period in which the first entrant acts as a monopolist, the opponent firm benefits from not being a potential target of patent litigation, even though it holds relevant patents itself. It is worth noting that this firm may be sued later on, but during the monopoly phase, the patent privateer does not offer any advantage for it.

It is important to acknowledge the limitations of our work. While we prove the existence of equilibrium, we do not provide a full analytical characterization of the Nash equilibrium of the R&D game due to the complexity of the payoffs and reaction functions. However, we present numerical examples and use them to deduce the implications of privateering on R&D incentives and equilibrium profits in the sequential setting. These findings match the theoretical results we obtain for the licensing game.

There are several directions for future research that could address open questions in this area. It could be interesting to analyze how product differentiation is affected by privateering, which would require the addition of a game stage in which firms make choices about the quality or complexity of their respective products. Additionally, we assume that firms exert R&D efforts to develop a fully overlapping set of technologies, which are equally difficult to discover and equally valuable for production. Relaxing these assumptions could lead to insights about the effects of privateering regarding R&D project choice and innovation paths. We leave these questions open for future research.

CRediT authorship contribution statement

Felix B. Klapper: Writing – review & editing, Writing – original draft, Visualization, Software, Methodology, Formal analysis, Conceptualization. **Christian Siemering:** Writing – review & editing, Writing – original draft, Methodology, Formal analysis, Conceptualization.

Data availability

No data was used for the research described in the article.

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Appendix A. Expected discount factor R&D

For the time when all N technologies are developed, the expected discount factor is denoted as $\delta_{t_A}(x,y)$. We have

$$\delta_{t_A}(x,y) = \int_0^\infty \mathrm{e}^{-rt} \cdot f_{t_A}(t)\,\mathrm{d}t,$$

where $f_{t_A}(t)$ is the probability density function (PDF) of $t_A = \max\{\hat{\tau}_1, \dots, \hat{\tau}_N\}$. Recall that $\hat{\tau}_i = \min\{\tau_{A,i}, \tau_{B,i}\}$, $i \in \{1,\dots,N\}$ is the time at which technology i is discovered by the first firm. Each $\hat{\tau}_i$ is distributed exponentially according to the cumulative distribution function (CDF) $F_{\hat{\tau}_i}(t) = 1 - \mathrm{e}^{-(x+y)t}$. From this, the CDF of t_A can be expressed as

$$F_{t_A}(t) = P(\hat{\tau}_1 \le t \wedge \cdots \wedge \hat{\tau}_N \le t) = \prod_{i=1}^N P(\hat{\tau}_i \le t) = \prod_{i=1}^N F_{\hat{\tau}_i}(t) = \left(1 - \mathrm{e}^{-(x+y)t}\right)^N$$

and the corresponding PDF is $f_{t_A}(t) = \frac{\mathrm{d}F_{t_A}(t)}{\mathrm{d}t} = N\left(1-\mathrm{e}^{-(x+y)t}\right)^{N-1}(x+y)\mathrm{e}^{-(x+y)t}$. Accordingly, the expected discount factor is given by

$$\delta_{t_A}(x,y) = \int_0^\infty N\left(1-\mathrm{e}^{-(x+y)t}\right)^{N-1}(x+y)\mathrm{e}^{-(x+y+r)t}\,\mathrm{d}t = \frac{N!\,(x+y)^N}{\prod_{j=1}^N [r + j(x+y)]}.$$

This is equivalent to (4).

Appendix B. Proof of Proposition 2

Consider the case of inter-temporal patent truce. The condition that has to be satisfied such that $\overline{T} = 0$ is a combination of evenly distributed patents across firms A and B and time preference of B. Substitution of $m = N - n$ into (8) yields

$$\overline{T}(n, N-n, \delta) = \begin{cases} -(N-n)D & , n \in [0, \frac{L}{D}) \\ ((1+\delta)n - N)D & , n \in [\frac{L}{D}, \frac{1}{2}(N - \frac{L}{D})] \\ ((1+\delta)n - N)D & , n \in (\frac{1}{2}(N-\frac{L}{D}), \frac{1}{2}(N+\frac{L}{D})) \wedge n \le \frac{1}{1+\delta}N \\ 0 & , n \in (\frac{1}{2}(N-\frac{L}{D}), \frac{1}{2}(N+\frac{L}{D})) \wedge n > \frac{1}{1+\delta}N \\ ((1+\delta)n - N)D & , n \in [\frac{1}{2}(N+\frac{L}{D}), N - \frac{L}{D}] \\ \delta(2n - N)D & , n \in (N - \frac{L}{D}, N]. \end{cases}$$

The number of patents firm A holds, n, is located within the interval $[0,N]$. For $N > 3\frac{L}{D}$, it holds that $0 < \frac{L}{D} < \frac{1}{2}(N - \frac{L}{D}) < \frac{1}{2}N < \frac{1}{2}(N+\frac{L}{D}) < N - \frac{L}{D} < N$. Then, for inter-temporal patent truce to be possible, there has to exist a non-empty interval $\mathcal{T} = (\frac{1}{1+\delta}N, \frac{1}{2}(N+\frac{L}{D}))$ for $\frac{1}{1+\delta}N > \frac{1}{2}(N - \frac{L}{D})$.

For sequential market entries, $\delta \in (0,1)$. From this, one can conclude that $\frac{1}{1+\delta}N \in [\frac{1}{2}N, N]$, with $\lim_{\delta\to 0}\frac{1}{1+\delta}N = N$, $\lim_{\delta\to 1}\frac{1}{1+\delta}N = \frac{N}{2}$. Then, $\frac{1}{1+\delta}N > \frac{1}{2}(N - \frac{L}{D}), \forall \delta \in (0,1)$.

- In order for $\mathcal{T}$ to be non-empty, it has to hold that $\frac{1}{1+\delta}N < \frac{1}{2}(N + \frac{L}{D})$. It follows that inter-temporal patent truce is only possible for $\delta \in \mathcal{P}$, with $\mathcal{P} = \left(\frac{ND-L}{ND+L}, 1\right)$.
- $\frac{\mathrm{d}}{\mathrm{d}\delta}[\frac{1}{1+\delta}N] = -(1+\delta)^{-2}N < 0$. With higher δ, the left border of the interval moves left. This means, the interval gets broader, i.e., there exist more patent allocations that lead to inter-temporal patent truce.
- It follows that for sequential market entries, inter-temporal patent truce is never possible if $n < \frac{1}{2}N$ or equivalently $n < m$. □

Appendix C. Proof of Lemma 1

Here, transfers are defined positive as payments from B to A. In order to prove the lemma, it suffices to show that $-\overline{T}(n, N-n, \delta) - (-T(n, N-n)) \ge 0$ for all discount factors and patent allocations, or equivalently

$$T(n, N-n) - \overline{T}(n, N-n, \delta) \ge 0 \quad , \forall n \in [0,N],\ \delta \in (0,1).$$

Displaying this difference in an explicit manner, we get $T(n, N-n) - \overline{T}(n, N-n, \delta)$

$$= \begin{cases} nD & , n \in [0, \frac{L}{D}) \\ (1-\delta)nD & , n \in [\frac{L}{D}, \frac{1}{2}(N - \frac{L}{D})] \\ (N-(1+\delta)n)D & , n \in (\frac{1}{2}(N-\frac{L}{D}), \frac{1}{1+\delta}N] \\ 0 & , n \in (\frac{1}{1+\delta}N, \frac{1}{2}(N+\frac{L}{D})) \\ (1-\delta)nD & , n \in [\frac{1}{2}(N+\frac{L}{D}), N-\frac{L}{D}] \\ (1-\delta)(2n-N)D & , n \in (N-\frac{L}{D}, N]. \end{cases}$$

(i.) Starting with the first case, it is easy to see that $nD \geq 0, \forall n \in [0, \frac{L}{D})$. In the second case, it holds that $(1-\delta)nD > 0, \forall n \in [\frac{L}{D}, \frac{1}{2}(N-\frac{L}{D}), \delta \in (0,1)$. Consider the third case. $\frac{d}{dn}[(N-(1+\delta)n)D] < 0$. For the highest n included in this case, $n = \frac{1}{1+\delta}N$, $(N-(1+\delta)n)D = 0$, such that $(N-(1+\delta)n)D \geq 0, \forall n \in [\frac{1}{2}(N-\frac{L}{D}), \frac{1}{1+\delta}N], \delta \in (0,1)$. For case four, $n \in (\frac{1}{1+\delta}N, \frac{1}{2}(N+\frac{L}{D})), 0 = 0$. Case five is similar to case two, i.e., $(1-\delta)nD > 0, \forall n \in [\frac{1}{2}(N+\frac{L}{D}), N-\frac{L}{D}]\,\delta \in (0,1)$. Last, consider case six. $1-\delta > 0, \forall \delta \in (0,1)$, such it suffices to show that $2n - N \geq 0$. This is true for $n \geq \frac{N}{2}$. Taking into account that $N > 3\frac{L}{D}$ by assumption, it holds that the lowest n included in this case, $N - \frac{L}{D} > \frac{N}{2}$ and therefore $(1-\delta)(2n-N)D > 0, \forall n \in (N-\frac{L}{D}, N], \delta \in (0,1)$. Thus, the condition is satisfied for all cases.

(ii.) In order to investigate the effect of discounting on B's advantage, differentiate all parts of the transfer function with respect to δ: $\frac{d}{d\delta}[nD] = 0, \forall n \in [0, \frac{L}{D})$, $\frac{d}{d\delta}[(1-\delta)nD] = -nD < 0, \forall n \in [\frac{L}{D}, \frac{1}{2}(N-\frac{L}{D})]$, $\frac{d}{d\delta}[(N-(1+\delta)n)D] = -nD < 0, \forall n \in [\frac{1}{2}(N-\frac{L}{D}), \frac{1}{1+\delta}N]$, $\frac{d}{d\delta}[0] = 0, \forall n \in (\frac{1}{1+\delta}N, \frac{1}{2}(N+\frac{L}{D}))$, $\frac{d}{d\delta}[(1-\delta)nD] = -nD < 0, \forall n \in [\frac{1}{2}(N+\frac{L}{D}), N-\frac{L}{D}]$ and $\frac{d}{d\delta}[(1-\delta)(2n-N)D] = -(2n-N)D < 0, \forall n \in (N-\frac{L}{D}, N]$. The effect of δ on the difference between transfers is weakly negative, i.e., a higher δ corresponds to a smaller difference. □

Appendix D. Proof of Lemma 2

In order to prove the lemma, it suffices to show that

$$T_{PAE}(n, N-n) - \overline{T}_{PAE}(n, N-n, \delta) \geq 0 \quad , \forall n \in [0, N], \delta \in (0,1).$$

Displaying this difference explicitly, we get

$$T_{PAE}(n, N-n) - \overline{T}_{PAE}(n, N-n, \delta) = \begin{cases} 0 & , n \in [0, \frac{L}{D}) \\ (1-\delta)nD & , n \in [\frac{L}{D}, N]. \end{cases}$$

(i.) Obviously, $0 = 0, \forall n \in [0, \frac{L}{D})$ and $(1-\delta)nD > 0, \forall n \in [\frac{L}{D}, N], \delta \in (0,1)$, such that $T_{PAE}(n, N-n) - \overline{T}_{PAE}(n, N-n, \delta) \geq 0, \forall n \in [0, N], \delta \in (0,1)$.

(ii.) $\frac{d}{dn}[0] = 0, \forall n \in [0, \frac{L}{D})$ and $\frac{d}{dn}[(1-\delta)nD] = (1-\delta)D > 0, \forall n \in [\frac{L}{D}, N], \delta \in (0,1)$.

(iii.) $\frac{d}{d\delta}[0] = 0, \forall n \in [0, \frac{L}{D})$ and $\frac{d}{d\delta}[(1-\delta)nD] = -nD < 0, \forall n \in [\frac{L}{D}, N]$. □

Appendix E. Proof of Proposition 4

Consider sequential market entries transfers without and with privateering, as described in Propositions 1 and 3, and substitute $m = N - n$ to get $\overline{T}(n, N-n, \delta)$ and $\overline{T}_{PAE}(n, N-n, \delta)$. Then, the effect of privateering is the difference of these transfers. We have

$$\overline{T}(n, N-n, \delta) - \overline{T}_{PAE}(n, N-n, \delta)$$

$$= \begin{cases} 0 & , n \in [0, \frac{1}{1+\delta}N] \\ (N-(1+\delta)n)D & , n \in (\frac{1}{1+\delta}N, \frac{1}{2}(N+\frac{L}{D})) \\ 0 & , n \in [\frac{1}{2}(N+\frac{L}{D}), N-\frac{L}{D}] \\ \delta(n-N)D & , n \in (N-\frac{L}{D}, N]. \end{cases}$$

Taking a look at the first case, the difference between transfers equals zero for portfolios in which B holds the majority of patents, i.e., $n \in [0, \frac{1}{1+\delta}N]$, where $\lim_{\delta \to 1} \frac{1}{1+\delta}N = \frac{1}{2}N$. Patent privateering affects transfers in the area of inter-temporal patent truce, $n \in (\frac{1}{1+\delta}N, \frac{1}{2}(N+\frac{L}{D}))$. For $n > \frac{1}{1+\delta}N$ it holds that $(N-(1+\delta)n)D < 0$, such that $\overline{T}(n, N-n, \delta) < \overline{T}_{PAE}(n, N-n, \delta)$ and firm A benefits from privateering *(i.)*. For $n \in [\frac{1}{2}(N+\frac{L}{D}), N-\frac{L}{D}]$ the presence of a PAE has no effect on transfers if firms enter sequentially. The difference is zero. In the last case, B holds a very low number of patents $m < \frac{L}{D}$, or equivalently $n \in (N-\frac{L}{D}, N]$. Here, the difference between transfers without and with privateering equals $\delta(n-N)D \leq 0$, such that $\overline{T}(n, N-n, \delta) < \overline{T}_{PAE}(n, N-n, \delta)$. Since transfers are defined as payments from B to A, A benefits from privateering *(ii.)*. □

Appendix F. Proof of Proposition 5

In order to prove the proposition, we show that, for all $n \in [0, N]$ and $\delta \in (0,1)$, the inequality (11),

$$\left|\overline{T}(n, N-n, \delta) - \overline{T}_{PAE}(n, N-n, \delta)\right| \leq \left|T(n, N-n) - T_{PAE}(n, N-n)\right|,$$

holds. Taking into consideration (5) and (9) and substituting $m = N - n$ yields the following expression used for the RHS:

$$T(n, N-n) - T_{PAE}(n, N-n)$$

$$= \begin{cases} nD & , n \in [0, \frac{L}{D}) \\ 0 & , n \in [\frac{L}{D}, \frac{1}{2}(N-\frac{L}{D})] \\ (N-2n)D & , n \in (\frac{1}{2}(N-\frac{L}{D}), \frac{1}{2}(N+\frac{L}{D})) \\ 0 & , n \in [\frac{1}{2}(N+\frac{L}{D}), N-\frac{L}{D}] \\ (n-N)D & , n \in (N-\frac{L}{D}, N] \end{cases}$$

In Appendix E, we present $\overline{T}(n, N-n, \delta) - \overline{T}_{PAE}(n, N-n, \delta)$ which is used for the LHS of the inequality. Recall Proposition 2. Inter-temporal patent truce is possible for $\delta \in \mathcal{P}$. In order to show that the inequality holds for all $\delta \in (0,1)$, split cases:

- $\delta \in \mathcal{P}$:

$$\overline{T}(n, N-n, \delta) - \overline{T}_{PAE}(n, N-n, \delta)$$

$$= \begin{cases} 0 & , n \in [0, \frac{1}{1+\delta}N] \\ (N-(1+\delta)n)D & , n \in (\frac{1}{1+\delta}N, \frac{1}{2}(N+\frac{L}{D})) \\ 0 & , n \in [\frac{1}{2}(N+\frac{L}{D}), N-\frac{L}{D}] \\ \delta(n-N)D & , n \in (N-\frac{L}{D}, N] \end{cases}$$

  - $n \in [0, \frac{L}{D})$: (11) holds with strict inequality for $n \neq 0$;
  - $n \in [\frac{L}{D}, \frac{1}{2}(N-\frac{L}{D})]$: (11) holds with weak inequality;
  - $n \in (\frac{1}{2}(N-\frac{L}{D}), \frac{1}{1+\delta}N)$: (11) holds with strict inequality;
  - $n \in (\frac{1}{1+\delta}N, \frac{1}{2}(N+\frac{L}{D}))$: since $\lim_{\delta \to 1} \frac{1}{1+\delta}N = \frac{1}{2}N$, we have to consider only $n > \frac{1}{2}N$. Thus, $(N-(1+\delta)n)D < 0$ and $(N-2n)D < 0$. (11) holds if $(N-(1+\delta)n)D \geq (N-2n)D$ which is true with strict inequality;
  - $n \in [\frac{1}{2}(N+\frac{L}{D}), N-\frac{L}{D}]$: (11) holds with weak inequality;

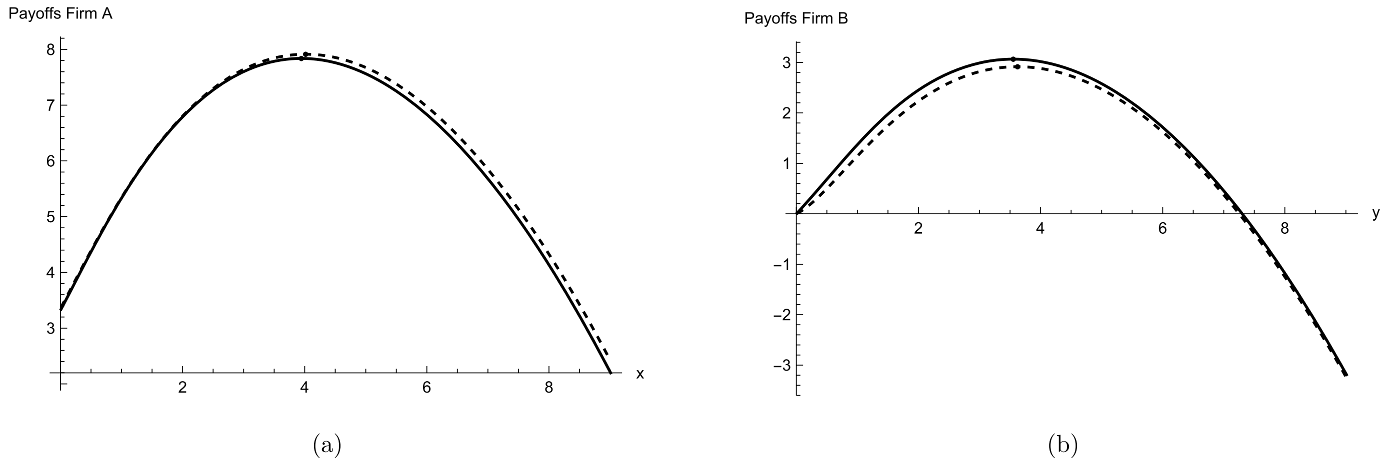

Figure (a) illustrates $\Pi_A(x, y^*)$ and $\Pi_{A,PAE}(x, y^*_{PAE})$, figure (b) illustrates $\Pi_B(x^*, y)$ and $\Pi_{B,PAE}(x^*_{PAE}, y)$ for sequential entry. Payoffs with PAE are depicted by the dashed lines. Cost functions and parameter values: $N = 5,\ \delta = 0.4,\ D = 2,\ L = 3,\ r = 1,\ \pi_d = 10,\ \pi_m = 20,\ C(x) = 1/7x^2,\ C(y) = 1/7y^1$.

Fig. H.6. Expected payoffs of firms A and B.

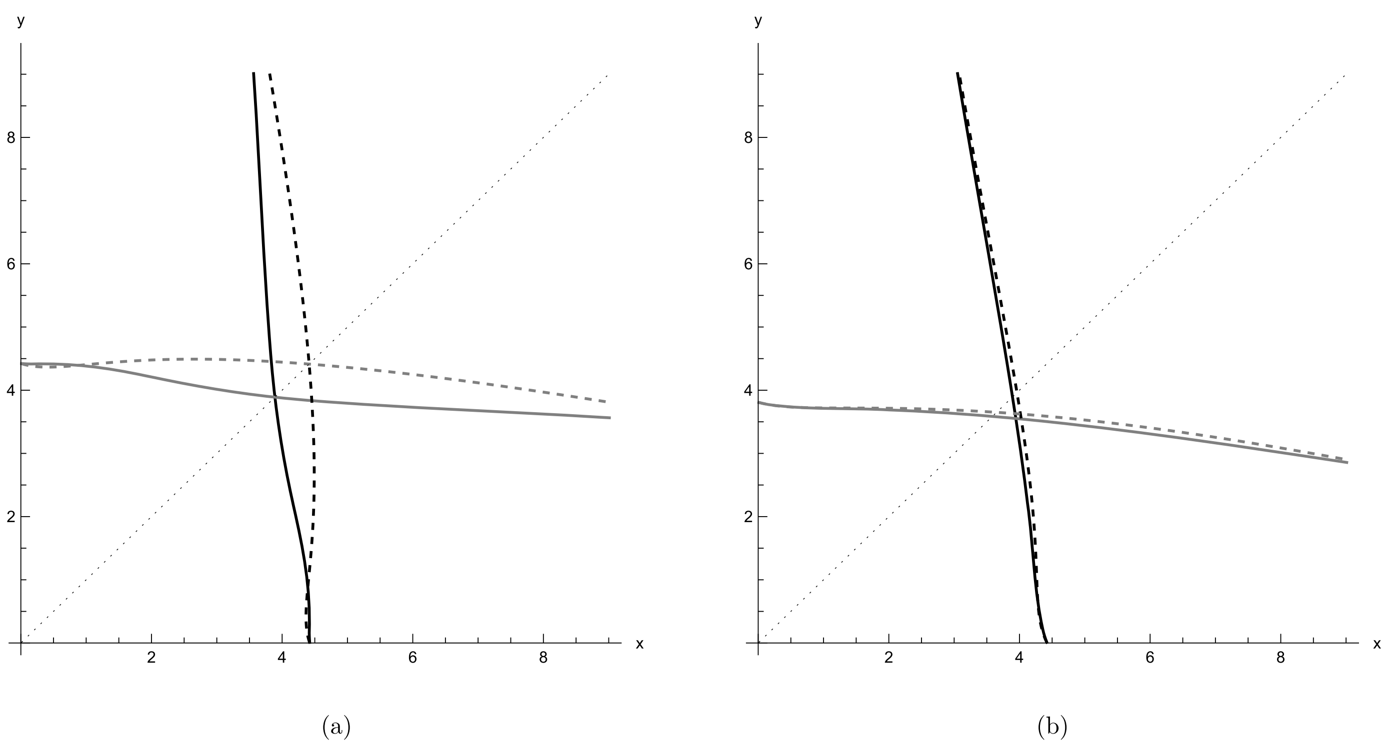

Figure (a) illustrates reaction functions for simultaneous entries ($\delta = 1$). Figure (b) depicts reaction functions for sequential entry ($\delta = 0.4$). The black and gray graphs are reaction functions of firm A, $x^R(y)$ and $x^R_{PAE}(y)$, and B, $y^R(x)$ and $y^R_{PAE}(x)$, respectively, with a dashed style indicating privateering. Cost function and parameter values: $N = 5, D = 2,\ L = 3,\ r = 1, \pi_d = \pi_m = 20,\ C(x) = 1/7x^2,\ C(y) = 1/7y^2$.

Fig. H.7. Reaction functions.

   – $n \in (N - \frac{L}{D}, N]$: (11) holds with strict inequality for $n \neq N$;
- $\delta \in (0,1) \setminus \mathcal{P}$:

$$\overline{T}(n, N-n, \delta) - \overline{T}_{PAE}(n, N-n, \delta) = \begin{cases} 0 & , n \in [0, N - \frac{L}{D}] \\ \delta(n-N)D & , n \in (N - \frac{L}{D}, N] \end{cases}$$

   – $n \in [0, \frac{L}{D})$: (11) holds with strict inequality for $n \neq 0$;
   – $n \in [\frac{L}{D}, \frac{1}{2}(N - \frac{L}{D})]$: (11) holds with weak inequality;
   – $n \in (\frac{1}{2}(N - \frac{L}{D}), \frac{1}{2}(N + \frac{L}{D}))$: (11) holds with strict inequality for $n \neq \frac{1}{2}N$;
   – $n \in [\frac{1}{2}(N + \frac{L}{D}), N - \frac{L}{D}]$: (11) holds with weak inequality;
   – $n \in (N - \frac{L}{D}, N]$: (11) holds with strict inequality for $n \neq N$.

The inequality (11) holds for all discount factors δ and patent allocations $(n, N-n)$. □

Appendix G. Proof of Proposition 6

In order to prove the proposition, consider the LHS of (11). Let $\delta_1, \delta_2 \in (0,1)$ be two values of δ. For all $n \in [0, N]$ and $\delta_1 < \delta_2$, it has to hold that

Table J.1
Numerical Results.

δ	x^*	y^*	x^*_{PAE}	y^*_{PAE}	x^S	y^S	Δx^*	Δy^*	$\Delta(x^*+y^*)$	$\Delta\Pi^*_A$	$\Delta\Pi^*_B$	$\Delta\Pi^*_{A+B}$	$\Delta\tilde{x}^S$	$\Delta\tilde{y}^S$
Benchmark Case with $N=5$, $r=1$, $L=3$, $D=2$, $\pi_A=\pi_B=10$ and $\pi_m=20$.														
1	3.8900	3.8900	4.4135	4.4135	3.0310	3.0310	0.5235	0.5235	1.0470	-0.3730	-0.3730	-0.7460	0.5235	0.5235
0.7	4.0706	3.9236	4.2049	4.0373	3.0149	3.0149	0.1344	0.1137	0.2481	0.1517	-0.3182	-0.1665	0.1344	0.1137
0.4	3.9443	3.5525	4.0149	3.6228	2.9985	2.9985	0.0707	0.0703	0.1409	0.0753	-0.1500	-0.0747	0.0707	0.0703
0.1	3.8429	3.1365	3.8601	3.1574	2.9820	2.9820	0.0172	0.0208	0.0381	0.0245	-0.0383	-0.0139	0.0172	0.0208
1: $L=2.6$.														
1	3.8900	3.8900	4.4135	4.4135	3.0310	3.0310	0.5235	0.5235	1.0470	-0.3730	-0.3730	-0.7460	0.5235	0.5235
0.7	4.0706	3.9236	4.2049	4.0373	3.0149	3.0149	0.1344	0.1137	0.2481	0.1517	-0.3182	-0.1665	0.1344	0.1137
0.4	3.9443	3.5525	4.0149	3.6228	2.9985	2.9985	0.0707	0.0703	0.1409	0.0753	-0.1500	-0.0747	0.0707	0.0703
0.1	3.8429	3.1365	3.8601	3.1574	2.9820	2.9820	0.0172	0.0208	0.0381	0.0245	-0.0383	-0.0139	0.0172	0.0208
2: $L=2.2$.														
1	3.8900	3.8900	4.4135	4.4135	3.0310	3.0310	0.5235	0.5235	1.0470	-0.3730	-0.3730	-0.7460	0.5235	0.5235
0.7	4.0706	3.9236	4.2049	4.0373	3.0149	3.0149	0.1344	0.1137	0.2481	0.1517	-0.3182	-0.1665	0.1344	0.1137
0.4	3.9443	3.5525	4.0149	3.6228	2.9985	2.9985	0.0707	0.0703	0.1409	0.0753	-0.1500	-0.0747	0.0707	0.0703
0.1	3.8429	3.1365	3.8601	3.1574	2.9820	2.9820	0.0172	0.0208	0.0381	0.0245	-0.0383	-0.0139	0.0172	0.0208
3: $L=1$.														
1	4.1043	4.1043	4.1043	4.1043	3.0310	3.0310	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.7	3.9372	3.7554	3.9372	3.7554	3.0149	3.0149	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.4	3.7899	3.3667	3.7899	3.3667	2.9985	2.9985	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	3.6779	2.9224	3.6779	2.9224	2.9820	2.9820	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4: $N=7$.														
1	4.5489	4.5489	4.8782	4.8782	3.1353	3.1353	0.3293	0.3293	0.6586	-0.3046	-0.3046	-0.6092	0.3293	0.3293
0.7	4.5197	4.4335	4.5837	4.4924	3.1184	3.1184	0.0640	0.0589	0.1229	0.0125	-0.1150	-0.1025	0.0640	0.0589
0.4	4.2522	4.0378	4.2916	4.0762	3.1013	3.1013	0.0394	0.0384	0.0778	0.0121	-0.0644	-0.0523	0.0394	0.0384
0.1	4.0024	3.6116	4.0130	3.6230	3.0841	3.0841	0.0106	0.0115	0.0221	0.0051	-0.0159	-0.0108	0.0106	0.0115
5: $N=7$, $L=3.5$.														
1	4.5489	4.5489	4.8782	4.8782	3.1353	3.1353	0.3293	0.3293	0.6586	-0.3046	-0.3046	-0.6092	0.3293	0.3293
0.7	4.5197	4.4335	4.5837	4.4924	3.1184	3.1184	0.0640	0.0589	0.1229	0.0125	-0.1150	-0.1025	0.0640	0.0589
0.4	4.2522	4.0378	4.2916	4.0762	3.1013	3.1013	0.0394	0.0384	0.0778	0.0121	-0.0644	-0.0523	0.0394	0.0384
0.1	4.0024	3.6116	4.0130	3.6230	3.0841	3.0841	0.0106	0.0115	0.0221	0.0051	-0.0159	-0.0108	0.0106	0.0115
6: $N=7$, $L=4.2$.														
1	4.5489	4.5489	5.4351	5.4351	3.1353	3.1353	0.8863	0.8863	1.7725	-0.9233	-0.9233	-1.8467	0.8863	0.8863
0.7	4.8028	4.7504	5.0817	4.9919	3.1184	3.1184	0.2789	0.2415	0.5204	0.1835	-0.7085	-0.5250	0.2789	0.2415
0.4	4.5580	4.3598	4.7254	4.5119	3.1013	3.1013	0.1674	0.1520	0.3194	0.1253	-0.3965	-0.2713	0.1674	0.1520
0.1	4.3342	3.9434	4.3775	3.9860	3.0841	3.0841	0.0433	0.0427	0.0860	0.0394	-0.0971	-0.0576	0.0433	0.0427
7: $N=7$, $L=4.6$.														
1	4.5489	4.5489	5.4351	5.4351	3.1353	3.1353	0.8863	0.8863	1.7725	-0.9233	-0.9233	-1.8467	0.8863	0.8863
0.7	4.8028	4.7504	5.0817	4.9919	3.1184	3.1184	0.2789	0.2415	0.5204	0.1835	-0.7085	-0.5250	0.2789	0.2415
0.4	4.5580	4.3598	4.7254	4.5119	3.1013	3.1013	0.1674	0.1520	0.3194	0.1253	-0.3965	-0.2713	0.1674	0.1520
0.1	4.3342	3.9434	4.3775	3.9860	3.0841	3.0841	0.0433	0.0427	0.0860	0.0394	-0.0971	-0.0576	0.0433	0.0427

(*continued on next page*)

$$\left|\overline{T}(n, N-n, \delta_1) - \overline{T}_{PAE}(n, N-n, \delta_1)\right| \leq \left|\overline{T}(n, N-n, \delta_2) - \overline{T}_{PAE}(n, N-n, \delta_2)\right|. \quad \text{(G.1)}$$

Recall Appendix F and consider two cases:

- $\delta \in \mathcal{P}$:
  $\overline{T}(n, N-n, \delta) - \overline{T}_{PAE}(n, N-n, \delta)$ can take two functional forms other than zero.
  - For $n \in (\frac{1}{1+\delta}N, \frac{1}{2}(N + \frac{L}{D}))$, $(N-(1+\delta)n)D < 0$. Since $\frac{\mathrm{d}}{\mathrm{d}\delta}[(N-(1+\delta)n)D] < 0$, $\frac{\mathrm{d}}{\mathrm{d}\delta}[|(N-(1+\delta)n)D|] > 0$ and (G.1) holds.
  - For $n \in (N - \frac{L}{D}, N]$, $\delta(n-N)D \geq 0$ and $\frac{\mathrm{d}}{\mathrm{d}\delta}[\delta(n-N)D] \geq 0$ with equality for $n = N$. Thus, (G.1) holds.
- $\delta \in (0,1) \setminus \mathcal{P}$:
  $\overline{T}(n, N-n, \delta) - \overline{T}_{PAE}(n, N-n, \delta)$ is either zero, or $\delta(n-N)D$. As stated above, (G.1) holds.

Thus, (G.1) holds for all $n \in [0, N]$ and $\delta \in (0,1)$. □

Appendix H. Examples for profits and reaction functions

Examples of payoff functions are presented in Fig. H.6. See Fig. H.7 for examples of reaction functions.

Appendix I. Proof of Lemma 3

The assumptions made regarding $C(\cdot)$ imply that there exist $(\overline{x}, \overline{y}) > 0$ so that R&D efforts $x' > \overline{x}$, $y' > \overline{y}$ are strictly dominated by providing zero effort and, therefore, can be eliminated. In the resulting game, the strategy space is a nonempty, convex and compact subset of the Euclidean space $\mathbb{R}^2_+$. Observe that the payoff functions Π_A, Π_B, $\Pi_{A,PAE}$ and $\Pi_{B,PAE}$ are continuous in (x, y) and, furthermore, are (quasi-)concave in the respective own strategy variable if R&D investments are high enough. According to the theorem of Debreu (1952), Glicksberg (1952) and Fan (1952), then there exists a Nash equilibrium in pure strategies. □

Appendix J. (Selected) numerical results

In what follows, we provide Table J.1 with selected numerical results. $\Delta x^* = x^*_{PAE} - x^*$ and $\Delta y^* = y^*_{PAE} - y^*$ denote the effect of privateering on equilibrium R&D investments of individual firms. $\Delta(x^* + y^*)$ is the sum and captures the overall effect. Equilibrium investments may deviate from social optimal investments. Deviations of firms A and B without privateering are $\tilde{x}^S = x^* - x^S$ and $\tilde{y}^S = y^* - y^S$, respectively, where a positive value indicates over-investment. $\tilde{x}^S_{PAE}$ and $\tilde{y}^S_{PAE}$ are deviations under privateering. The effect of patent privateering on deviations in equilibrium are $\Delta\tilde{x}^S = \tilde{x}^S_{PAE} - \tilde{x}^S$ and $\Delta\tilde{y}^S = \tilde{y}^S_{PAE} - \tilde{y}^S$.

Table J.1 (*continued*)

δ	x^*	y^*	x^*_{PAE}	y^*_{PAE}	x^S	y^S	Δx^*	Δy^*	$\Delta(x^*+y^*)$	$\Delta\Pi^*_A$	$\Delta\Pi^*_B$	$\Delta\Pi^*_{A+B}$	$\Delta\tilde{x}^S$	$\Delta\tilde{y}^S$
8: $D = 2.2$.														
1	4.0407	4.0407	4.6022	4.6022	3.0310	3.0310	0.5615	0.5615	1.1230	-0.4446	-0.4446	-0.8891	0.5615	0.5615
0.7	4.2152	4.0970	4.3593	4.2184	3.0149	3.0149	0.1441	0.1214	0.2655	0.1593	-0.3581	-0.1988	0.1441	0.1214
0.4	4.0522	3.7240	4.1288	3.7977	2.9985	2.9985	0.0766	0.0737	0.1503	0.0764	-0.1672	-0.0908	0.0766	0.0737
0.1	3.9066	3.3069	3.9257	3.3284	2.9820	2.9820	0.0191	0.0215	0.0407	0.0244	-0.0421	-0.0177	0.0191	0.0215
9: $D = 2.9$.														
1	4.5432	4.5432	5.2239	5.2239	3.0310	3.0310	0.6808	0.6808	1.3615	-0.7049	-0.7049	-1.4099	0.6808	0.6808
0.7	4.7071	4.6622	4.8813	4.8084	3.0149	3.0149	0.1743	0.1462	0.3205	0.1854	-0.5020	-0.3167	0.1743	0.1462
0.4	4.4370	4.2735	4.5319	4.3587	2.9985	2.9985	0.0949	0.0852	0.1801	0.0807	-0.2302	-0.1495	0.0949	0.0852
0.1	4.1586	3.8415	4.1835	3.8654	2.9820	2.9820	0.0249	0.0239	0.0488	0.0243	-0.0559	-0.0316	0.0249	0.0239
10: $r = 0.7$.														
1	4.0614	4.0614	4.5864	4.5864	3.1830	3.1830	0.5250	0.5250	1.0501	-0.3932	-0.3932	-0.7864	0.5250	0.5250
0.7	4.2690	4.0681	4.3980	4.1890	3.1669	3.1669	0.1289	0.1209	0.2498	0.1670	-0.3432	-0.1763	0.1289	0.1209
0.4	4.1683	3.6702	4.2354	3.7461	3.1506	3.1506	0.0671	0.0759	0.1430	0.0854	-0.1651	-0.0797	0.0671	0.0759
0.1	4.1035	3.2183	4.1192	3.2414	3.1341	3.1341	0.0157	0.0230	0.0387	0.0287	-0.0433	-0.0146	0.0157	0.0230
11: $r = 0.3$.														
1	4.2852	4.2852	4.8126	4.8126	3.3821	3.3821	0.5274	0.5274	1.0548	-0.4201	-0.4201	-0.8402	0.5274	0.5274
0.7	4.5281	4.2570	4.6501	4.3872	3.3659	3.3659	0.1220	0.1301	0.2521	0.1881	-0.3771	-0.1891	0.1220	0.1301
0.4	4.4605	3.8238	4.5229	3.9069	3.3497	3.3497	0.0624	0.0831	0.1455	0.0996	-0.1856	-0.0861	0.0624	0.0831
0.1	4.4432	3.3236	4.4568	3.3496	3.3332	3.3332	0.0136	0.0259	0.0395	0.0347	-0.0500	-0.0154	0.0136	0.0259
12: $r = 0.1$.														
1	4.3953	4.3953	4.9240	4.9240	3.4803	3.4803	0.5287	0.5287	1.0574	-0.4336	-0.4336	-0.8672	0.5287	0.5287
0.7	4.6556	4.3501	4.7741	4.4848	3.4642	3.4642	0.1186	0.1347	0.2533	0.1989	-0.3943	-0.1954	0.1186	0.1347
0.4	4.6042	3.8992	4.6642	3.9860	3.4479	3.4479	0.0600	0.0867	0.1467	0.1070	-0.1960	-0.0891	0.0600	0.0867
0.1	4.6103	3.3749	4.6228	3.4022	3.4315	3.4315	0.0125	0.0273	0.0399	0.0378	-0.0534	-0.0156	0.0125	0.0273
13: $\pi_m = 30$.														
1	3.8900	3.8900	4.4135	4.4135	3.0310	3.0310	0.5235	0.5235	1.0470	-0.3730	-0.3730	-0.7460	0.5235	0.5235
0.7	4.2173	3.9082	4.3452	4.0257	3.1703	3.1703	0.1279	0.1175	0.2454	0.1681	-0.3229	-0.1548	0.1279	0.1175
0.4	4.2624	3.5203	4.3252	3.5963	3.2998	3.2998	0.0628	0.0760	0.1388	0.0965	-0.1572	-0.0607	0.0628	0.0760
0.1	4.3593	3.0891	4.3725	3.1121	3.4210	3.4210	0.0133	0.0230	0.0362	0.0346	-0.0419	-0.0073	0.0133	0.0230
14: $\pi_m = 50$.														
1	3.8900	3.8900	4.4135	4.4135	3.0310	3.0310	0.5235	0.5235	1.0470	-0.3730	-0.3730	-0.7460	0.5235	0.5235
0.7	4.4951	3.8779	4.6108	4.0019	3.4469	3.4469	0.1157	0.1240	0.2397	0.2003	-0.3323	-0.1320	0.1157	0.1240
0.4	4.8279	3.4574	4.8766	3.5418	3.7898	3.7898	0.0486	0.0843	0.1330	0.1373	-0.1709	-0.0336	0.0486	0.0843
0.1	5.2153	2.9904	5.2226	3.0153	4.0852	4.0852	0.0073	0.0249	0.0322	0.0521	-0.0480	0.0041	0.0073	0.0249

$\Delta\Pi^*_A = \Pi^*_{A,PAE} - \Pi^*_A$ and $\Delta\Pi^*_B = \Pi^*_{B,PAE} - \Pi^*_B$ denote the effect of privateering on the expected equilibrium profits of firms A and B, respectively. The total effect of privateering on industry profits is $\Delta\Pi^*_{A+B} = \Delta\Pi^*_A + \Delta\Pi^*_B$.

We assume cost functions $C(x) = \frac{1}{7}x^2$ for A and $C(y) = \frac{1}{7}y^2$ for B. Starting with our benchmark case with $N = 5$, $L = 3$, $D = 2$, $r = 1$, $\pi_d = 10$, $\pi_m = 20$, $w_m = 1$ and $w_d = 2$, we variate single parameters in order to investigate their effects, ceteris paribus. Parameter constellations are named using a number and the change in comparison to the benchmark case. For example, the first constellation in which litigation costs L are varied from $L = 3$ to $L = 2.6$ is specified as '1: $L = 2.6$'.

References

Bergin, J., 2022. Patent licensing, non-practising entities, and investment in r&d. J. Ind. Econ. 70, 396–462.

Choi, J.P., Gerlach, H., 2017. A theory of patent portfolios. Am. Econ. J. Microecon. 9, 315–351. https://doi.org/10.1257/mic.20150003.

Choi, J.P., Gerlach, H., 2018. A model of patent trolls. Int. Econ. Rev. 59, 2075–2106. https://doi.org/10.1111/iere.12330.

Cotropia, C.A., Kesan, J.P., Schwartz, D.L., 2014. Unpacking patent assertion entities (paes). Minnesota Law Rev. 99, 649–703.

Debreu, G., 1952. A social equilibrium existence theorem. Proc. Natl. Acad. Sci. 38, 886–893.

Drummond, D., 2011. When Patents Attack Android. https://googleblog.blogspot.com/2011/08/when-patents-attack-android.html.

Ewing, T., 2012. Indirect exploitation of intellectual property rights by corporations and investors: IP privateering and modern letters of marque and reprisal. Hastings Sci. Technol. Law J. 4, 1–108.

Fan, K., 1952. Fixed-point and minimax theorems in locally convex topological linear spaces. Proc. Natl. Acad. Sci. 38, 121–126.

Federal Trade Commission, 2016. Patent Assertion Entity Activity: An FTC Study. Federal Trade Commission.

Fung, B., 2014. Patent trolls now account for 67 percent of all new patent lawsuits. https://www.washingtonpost.com/news/the-switch/wp/2014/07/15/patent-trolls-now-account-for-67-percent-of-all-new-patent-lawsuits/.

Geradin, D., 2019. Patent assertion entities and EU competition law. J. Compet. Law Econ. 15, 204–236. https://doi.org/10.1093/joclec/nhz012.

Glicksberg, I.L., 1952. A further generalization of the Kakutani fixed point theorem, with application to Nash equilibrium points. Proc. Am. Math. Soc. 3, 170–174.

Harris, R.G., 2014. Patent assertion entities & privateers: economic harms to innovation and competition. Antitrust Bull. 59, 281–325.

He, L., 2020. A theory of pre-filing settlement and patent assertion entities. Int. J. Ind. Organ. 70, 1–21. https://doi.org/10.1016/j.ijindorg.2020.102618.

Horn, H., Wolinsky, A., 1988. Bilateral monopolies and incentives for merger. Rand J. Econ. 19, 408–419. https://doi.org/10.2307/2555664.

Hovenkamp, E., 2013. Predatory patent litigation: how patent assertion entities use reputation to monetize bad patents. SSRN Working Paper, https://doi.org/10.2139/ssrn.23081150.

Kesan, J.P., Layne-Farrar, A., Schwartz, D.L., 2019. Understanding patent "privateering": a quantitative assessment. J. Empir. Leg. Stud. 16, 343–380. https://doi.org/10.1111/jels.12217.

Lemus, J., Temnyalov, E., 2017. Patent privateering, litigation, and r&d incentives. Rand J. Econ. 48, 1004–1026. https://doi.org/10.1111/1756-2171.12211.

Loury, G.C., 1979. Market structure and innovation. Q. J. Econ. 93, 395–410.

Popofsky, M.S., Laufert, M.D., 2014. Antitrust attacks on patent assertion entities. Antitrust Law J. 79, 445–462.

Sipe, M., 2016. Patent privateers and antitrust fears. Mich. Telecommun. Technol. Law Rev. 22, 191–224.

Sokol, D.D., 2017. Patent privateering: the rise of hybrid patent assertion entities. In: Sokol, D.D. (Ed.), 1 ed.. Patent Assertion Entities and Competition Policy. Cambridge University Press, Cambridge, pp. 72–84.

Thumm, N., 2018. The good, the bad and the ugly - the future of patent assertion entities in Europe. Technol. Anal. Strateg. Manag. 30, 1046–1056. https://doi.org/10.1080/09537325.2018.1434875.