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Heterotic G_2-manifold compactifications with fluxes and fermionic condensates

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Abstract: We consider flux compactifications of heterotic string theory in the presence of fermionic condensates on $M_{1,2} \times X_7$ with both factors carrying a Killing spinor. In other words, $M_{1,2}$ is either de Sitter, anti-de Sitter or Minkowski, and X_7 possesses a nearly parallel G_2-structure or has G_2-holonomy. We solve the complete set of field equations and the Bianchi identity to order α'. The latter is satisfied via a non-standard embedding by choosing the gauge field to be a G_2-instanton. It is shown that none of the solutions to the field equations is supersymmetric.

Keywords: Flux compactifications, Superstring Vacua, Superstrings and Heterotic Strings, Solitons Monopoles and Instantons

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Contents

1 Introduction and summary

Many compactifications of string theory suffer from the severe problem of moduli stabilization, the existence of scalar fields whose vacuum expectation values are not fixed by a potential. A promising method for the heterotic string to stabilize these scalar fields is the introduction of fluxes and fermionic condensates, i.e. vacuum expectation values of some tensor fields and some fermionic bilinears along the internal manifold. Without fluxes and condensates, the Killing spinor equations demand the internal manifold to have reduced holonomy, e.g. SU(3) (G_2) for compactifications on a six-(seven-)dimensional manifold. Fluxes lead to a deformation of the internal manifold, resulting in an internal space with only reduced structure group but no reduced holonomy.

For compactifications on six-dimensional manifolds, deformations to non-Kähler SU(3)-manifolds have already been studied in detail [1, 2]. Also the effect of implementing gaugino and dilatino condensates has been analyzed [3–7]. Here, we investigate which aspects of these results carry over to compactifications on seven-dimensional manifolds X_7.

More specifically, we discuss compactifications on manifolds with G_2-holonomy as well as on their deformations to nearly parallel G_2-manifolds, in the presence of fermionic condensates. Assuming the space-time background to be a product of X_7 and a maximally symmetric Lorentzian space $M_{1,2}$, we solve the field equations to order α' and discuss the conditions under which the solutions preserve supersymmetry. The Bianchi identity is also satisfied to guarantee the absence of anomalies. The gauge field is taken to be a generalized instanton on the internal manifold X_7. This choice allows us to solve the Bianchi identity by a non-standard embedding and immediately takes care of the Yang-Mills equation. Furthermore, it also ensures the vanishing of the gaugino supersymmetry variation.

There are several aspects in which the considered compactifications to three dimensions differ from those to four dimensions. Most importantly, the fermionic condensates cannot be restricted to the internal manifold X_7 but must extend to $M_{1,2}$. Therefore, the field equations do not decouple into separate equations on $M_{1,2}$ and X_7. As a first consequence, the equations of motion allow not only for anti-de Sitter solutions but admit de Sitter and Minkowski space-times as well. Secondly, the radius of the de Sitter or anti-de Sitter space is not fixed but related to the amplitudes of the condensates and H-flux by the equations of motion. It turns out that none of these heterotic vacua is supersymmetric.

The paper is organized as follows. In section 2, we briefly review the action and the equations of motion of heterotic supergravity to first order in α'. We decompose the fields and their equations according to the space-time factorization $M_{1,2} \times X_7$, including the effect of gaugino and dilatino condensates. The geometric properties of manifolds with G_2-structure are the subject of section 3. In section 4 we solve the heterotic equations of motion for the six possible combinations of $M_{1,2}$ being either de Sitter, anti-de Sitter or Minkowski space-times X_7 carrying either G_2-holonomy or just a nearly parallel G_2-structure. Furthermore, we present the conditions for supersymmetric solutions and compute the fermion masses for all considered backgrounds.

2 Heterotic string with fermionic condensates

Action and equations of motion. The low-energy field theory limit of heterotic string theory is given by $d = 10$, $\mathcal{N} = 1$ supergravity coupled to a super-Yang-Mills multiplet, and it is defined on a ten-dimensional space-time M. The supergravity multiplet consists of the graviton g, which is a metric on M, the left-handed Rarita-Schwinger gravitino Ψ, the Kalb-Ramond two-form field B, the scalar dilaton ϕ and the right-handed Majorana-Weyl dilatino λ. Moreover, the vector supermultiplet consists of the gauge field one-form A and its superpartner, the left-handed Majorana-Weyl gaugino χ.

Rather than presenting the full action describing the propagation and interactions of the above fields [8, 9], we shall restrict ourselves to the part which is relevant for our purposes. In this paper we shall consider vacuum solutions where the fermionic expectation values are forced to vanish by requiring Lorentz invariance, but certain fermionic bilinears may acquire non-trivial vacuum expectation values. However, these vacuum expectation values will not involve the gravitino, whence we set the gravitino to zero from the very beginning, $\Psi = 0$. Then, in the string frame, the low-energy action up to and including terms of order α' reads as [10]

$$\mathcal{S} = \int_M \mathrm{d}^{10}x \sqrt{\det g}\; e^{-2\phi} \Big[\mathrm{Scal} + 4|\mathrm{d}\phi|^2 - \frac{1}{2}|H|^2 + \frac{1}{2}(H,\Sigma) - 2(H,\Delta) + \frac{1}{4}(\Sigma,\Delta) - \frac{1}{8}|\Sigma|^2 + \\ + \frac{1}{4}\alpha' \left(\mathrm{tr}|\tilde{R}|^2 - \mathrm{tr}\left(|F|^2 - 2\langle\chi,\mathcal{D}\chi\rangle - \frac{1}{3}\langle\lambda, \gamma^M\gamma^{AB}F_{AB}\gamma_M\lambda\rangle \right)\right) + 8\langle\lambda,\mathcal{D}\lambda\rangle \Big]. \quad (2.1)$$

Here, Scal is the scalar curvature of the Levi-Civita connection Γ^g on TM. Furthermore, $\tilde{R}$ is the curvature form of a connection $\tilde{\Gamma}$ on the TM. The choice of this connection is ambiguous and will be discussed in section 4. Here and in the following, traces are taken in the adjoint representation of SO(9,1) or of the gauge group, respectively, depending on the context.

The field strength H is defined by

$$H = \mathrm{d}B + \frac{1}{4}\alpha' \left(\omega_{\mathrm{CS}}(\tilde{\Gamma}) - \omega_{\mathrm{CS}}(A)\right) , \tag{2.2}$$

where the Chern-Simons forms of the connections $\tilde{\Gamma}$ and A are given by

$$\omega_{\mathrm{CS}}(\tilde{\Gamma}) = \mathrm{tr}\left(\tilde{R}\wedge\tilde{\Gamma} - \frac{2}{3}\tilde{\Gamma}\wedge\tilde{\Gamma}\wedge\tilde{\Gamma}\right) \quad \text{and} \quad \omega_{\mathrm{CS}}(A) = \mathrm{tr}\left(F\wedge A - \frac{2}{3}A\wedge A\wedge A\right). \tag{2.3}$$

For any two p-forms α, β we use the definitions

$$(\alpha,\beta) := \frac{1}{p!}\alpha_{M_1,\dots,M_p}\beta^{M_1,\dots,M_p} , \quad |\alpha|^2 := (\alpha,\alpha) . \tag{2.4}$$

$\mathcal{D} = \gamma^M\nabla_M$ denotes the Dirac operator coupled to Γ^g and A. Finally, we have defined the fermion bilinears

$$\Sigma = \frac{1}{24}\alpha' \mathrm{tr}\langle\chi,\gamma_{MNP}\chi\rangle\, e^{MNP} \qquad \text{and} \qquad \Delta = \frac{1}{6}\langle\lambda,\gamma_{MNP}\lambda\rangle e^{MNP} , \tag{2.5}$$

with $e^{MNP} \equiv e^M\wedge e^N\wedge e^P$ and $\{e^M\}$ being an othonormal frame on the space-time background M. By $\langle\cdot,\cdot\rangle$ we denote the inner product of spinors. For a suitable choice of the connection $\tilde{\Gamma}$, the action (2.1) is invariant under $\mathcal{N}=1$ supersymmetry transformations, which act on the fermions as

$$\delta\Psi_M = \nabla_M\epsilon - \frac{1}{8}H_{MNP}\gamma^{NP}\epsilon + \frac{1}{96}\Sigma\cdot\gamma_M\epsilon , \tag{2.6a}$$

$$\delta\lambda = -\frac{\sqrt{2}}{4}\left(d\phi - \frac{1}{12}H - \frac{1}{48}\Sigma + \frac{1}{48}\Delta\right)\cdot\epsilon , \tag{2.6b}$$

$$\delta\chi = -\frac{1}{4}F\cdot\epsilon + \langle\chi,\lambda\rangle\,\epsilon - \langle\epsilon,\lambda\rangle\,\chi + \langle\chi,\gamma_M\epsilon\rangle\,\gamma^M\lambda , \tag{2.6c}$$

where ϵ is the supersymmetry parameter, a left-handed Majorana-Weyl spinor.

The equations of motion may be obtained by varying the action (2.1) and take the form

$$\begin{aligned} 0 = {} & \mathrm{Ric}_{MN} + 2(\nabla\mathrm{d}\phi)_{MN} - \frac{1}{8}\left(H - \frac{1}{2}\Sigma + 2\Delta\right)_{PQ(M}H_{N)}{}^{PQ} + \\ & \frac{1}{4}\alpha'\left[\tilde{R}_{MPQR}\tilde{R}_N{}^{PQR} - \mathrm{tr}\left(F_{MP}F_N{}^P + \frac{1}{2}\langle\chi,\gamma_{(M}\nabla_{N)}\chi\rangle\right)\right] + 2\langle\lambda,\gamma_{(M}\nabla_{N)}\lambda\rangle , \end{aligned} \tag{2.7a}$$

$$\begin{aligned} 0 = {} & \mathrm{Scal} - 4\Delta\phi + 4|\mathrm{d}\phi|^2 - \frac{1}{2}|H|^2 + \frac{1}{2}(H,\Sigma) - 2(H,\Delta) + \frac{1}{4}(\Sigma,\Delta) - \frac{1}{8}|\Sigma|^2 \\ & + \frac{1}{4}\alpha'\mathrm{tr}\left[|\tilde{R}|^2 - |F|^2 - 2\langle\chi,\mathcal{D}\chi\rangle\right] + 8\langle\lambda,\mathcal{D}\lambda\rangle , \end{aligned} \tag{2.7b}$$

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$$0 = e^{2\phi}\mathrm{d} * (e^{-2\phi}F) + A \wedge *F - *F \wedge A + * \left(H - \frac{1}{2}\Sigma + 2\Delta\right) \wedge F\,, \tag{2.7c}$$

$$0 = \mathrm{d} * e^{-2\phi}\left(H - \frac{1}{2}\Sigma + 2\Delta\right)\,, \tag{2.7d}$$

$$0 = \left(\mathcal{D} - \frac{1}{24}\left(H - \frac{1}{2}\Sigma + \frac{1}{2}\Delta\right)\cdot\right) e^{-2\phi}\chi\,, \tag{2.7e}$$

$$0 = \left(\mathcal{D} - \frac{1}{24}\left(H - \frac{1}{8}\Sigma\right)\cdot\right) e^{-2\phi}\lambda\,. \tag{2.7f}$$

They are complemented by the Bianchi identity

$$\mathrm{d}H = \frac{1}{4}\alpha' \left(\mathrm{tr}(\tilde{R} \wedge \tilde{R}) - \mathrm{tr}(F \wedge F)\right)\,, \tag{2.8}$$

which follows from the definition (2.2). In order to simplify the equations of motion, we will assume for the remainder of the paper that the dilation vanishes, i.e.

$$\phi = 0\,. \tag{2.9}$$

Space-time and spinor factorization. We will consider space-time backgrounds M of the form

$$M = M_{1,2} \times X_7\,, \tag{2.10}$$

with $M_{1,2}$ being a maximally symmetric Lorentzian manifold and X_7 a being seven-dimensional compact Riemannian manifold. Furthermore, we assume that X_7 possesses a nowhere vanishing real Killing spinor,[1] i.e. a spinor satisfying

$$\nabla^g_X \eta = i\,\mu_2\, X \cdot \eta \tag{2.11}$$

for some real constant μ_2. If $\mu_2 = 0$, then X_7 admits G_2-holonomy. Manifolds with non-vanishing μ_2 are known as nearly parallel G_2-manifolds (see section 3). We denote the metric on M by g, whereas the metrics on $M_{1,2}$ and X_7 will be labeled by g_3 and g_7, respectively.

The factorization of the space-time background M induces a splitting of the SO(9,1) Clifford algebra. We employ a standard representation of the SO(9,1) Clifford algebra

$$\{\gamma^A, \gamma^B\} = 2\eta^{AB} = 2\,\mathrm{diag}(-1, +1, \ldots, +1)^{AB} \tag{2.12}$$

via

$$\gamma^\mu = \gamma^\mu_{(3)} \otimes \mathbb{1}_8 \otimes \sigma_2 \qquad \text{for} \quad \mu = 0, 1, 2\,, \tag{2.13a}$$

$$\gamma^{a+2} = \mathbb{1}_2 \otimes \gamma^a_{(7)} \otimes \sigma_1 \qquad \text{for} \quad a = 1, \ldots, 7\,. \tag{2.13b}$$

[1] Note that the notion of real an imaginary Killing spinors differs from parts of the mathematical literature, due to a different sign in the definition of the Clifford algebra (see (2.12)).

Here, $\mathbb{1}_n$ are $n \times n$-unit matrices and $\gamma^a_{(7)}$ are SO(7) gamma matrices. Furthermore, $\gamma^\mu_{(3)}$ and σ_i denote the SO(2,1) gamma matrices and Pauli matrices, respectively:

$$\gamma^0_{(3)} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} , \qquad \gamma^1_{(3)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \qquad \gamma^2_{(3)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \tag{2.14}$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{2.15}$$

The chirality and the charge conjugation operator in this representation read

$$\Gamma^{11} = \mathbb{1}_2 \otimes \mathbb{1}_8 \otimes \sigma^3 \qquad \text{and} \qquad \mathcal{C} = C_{(3)} \otimes \mathcal{C}_{(7)} \otimes \mathbb{1}_2 , \tag{2.16}$$

with

$$\mathcal{C}_{(7)} = i\gamma^2_{(7)}\gamma^4_{(7)}\gamma^6_{(7)} \qquad \text{and} \qquad \mathcal{C}_{(3)} = \gamma^0_{(3)} . \tag{2.17}$$

We assume that the dilatino and gaugino decompose as

$$\chi = \widehat{\chi} \otimes \eta \otimes (1,0)^t , \tag{2.18a}$$

$$\lambda = \widehat{\lambda} \otimes \eta \otimes (0,1)^t , \tag{2.18b}$$

with $\widehat{\chi}$ and $\widehat{\lambda}$ being Grassmann-valued SO(2,1) Majorana spinors and η being the real Majorana Killing spinor on X_7. The last factor in the products accounts for the opposite chirality of the gaugino and the dilatino. The only non-vanishing spinor bilinears constructed with a single spinor η are $\langle \eta, \eta \rangle$ and $\langle \eta, \gamma_{abc} \eta \rangle$. Hence, Σ and Δ simplify to

$$\Sigma = m \left(-\mathrm{vol}^{(3)} + Q \right) , \tag{2.19}$$

$$\Delta = n \left(-\mathrm{vol}^{(3)} + Q \right) , \tag{2.20}$$

with

$$m = \frac{1}{24} \alpha' \mathrm{tr} \langle \widehat{\chi}, \widehat{\chi} \rangle \qquad \text{and} \qquad n = \frac{1}{6} \langle \widehat{\lambda}, \widehat{\lambda} \rangle . \tag{2.21}$$

Here, $\mathrm{vol}^{(3)}$ is the volume form on $M_{1,2}$, and

$$Q = -\frac{i}{3!} \langle \eta, (\gamma_{(7)})_{mnp}\, \eta \rangle \, e^{mnp} . \tag{2.22}$$

The properties of the three-form Q will be discussed in section 3.

From now on, we will replace all terms depending on fermion bilinears by their quantum expectation values. Furthermore, we assume that the only non-vanishing expectation values are Σ and Δ. Note that the form of the condensates (2.19) and (2.20) differs crucially from condensates considered previously in compactifications to four-dimensional space-times. In the latter case one may consistently confine the condensate to the compactification space. For a compactification to a three-dimensional space-time, however, a non-vanishing condensate must always have a space-time component, due to the fact that

$$\Gamma^0 \Gamma^1 \Gamma^2 = -\mathbb{1} . \tag{2.23}$$

Geometric data of maximally symmetric Lorentzian manifolds. For future reference we review some aspects of the geometry of maximally symmetric Lorentzian manifolds, i.e. de Sitter, anti-de Sitter and Minkowski spaces. These spaces possess a Killing spinor with Killing number μ_1, meaning a spinor ζ satisfying

$$\nabla^g_X \zeta = \mu_1 X \cdot \zeta \,. \tag{2.24}$$

For de Sitter space, μ_1 is real, whereas for anti-de Sitter space, μ_1 is purely imaginary. It vanishes on Minkowski space. We define the (anti-)de Sitter radius $|\rho_1|$ by setting

$$\mu_1 = \begin{cases} \rho_1^{-1} & \text{for de Sitter space} \\ i\,\rho_1^{-1} & \text{for anti-de Sitter space} \end{cases} . \tag{2.25}$$

The Ricci tensor and scalar curvature of (anti-)de Sitter space are given by

$$\mathrm{Scal}_3 = \pm \frac{24}{\rho_1^2} \qquad \text{and} \qquad \mathrm{Ric}_3 = \pm \frac{8}{\rho_1^2}\, g_3 \,. \tag{2.26}$$

Moreover, the curvature dependent quantities entering the Einstein and dilaton equations (2.7a) and (2.7b) are given by

$$(R_3)_{\mu\alpha\beta\gamma}(R_3)_\nu{}^{\alpha\beta\gamma} = \frac{64}{\rho_1^4}\, g_3 \qquad \text{and} \qquad |R|^2 = \frac{192}{\rho_1^4} \,. \tag{2.27}$$

Note that the curvature only depends on even powers of ρ_1 and, hence, the sign of ρ_1 will only enter the supersymmetry variations but not the equations of motion.

3 The geometry of manifolds with G_2-structure

G_2-manifolds. Manifolds with a G_2-structure are by definition seven-dimensional Riemannian manifolds possessing a nowhere vanishing G_2-invariant three-form Q. Equivalently, they can be defined by the existence of a nowhere vanishing spinor η. One can always find an orthonormal frame $\{e^a\}$ such that the three-form Q can be written as

$$Q = e^{123} - e^{156} + e^{246} - e^{345} + e^{147} + e^{257} + e^{367} \,. \tag{3.1}$$

Here and in the following we use the abbreviation $e^{abc} \equiv e^a \wedge e^b \wedge e^c$. The G_2-structure defined by Q is compatible with the metric g_7 in the sense that $*((X \lrcorner Q) \wedge (Y \lrcorner Q) \wedge Q) = 6\, g_7(X,Y)$ for all vector fields X and Y. The three-form Q is related to the spinor η by

$$Q = -\frac{i}{3!} \langle \eta, \gamma_{mnp} \eta \rangle\, e^{mnp} \,. \tag{3.2}$$

From this, one can deduce the action of Q on η under Clifford multiplication:

$$Q \cdot \eta = 7i\, \eta \,, \tag{3.3}$$

$$Q \cdot (X \cdot \eta) = -i\, X \cdot \eta \,, \tag{3.4}$$

$$(X \lrcorner Q) \cdot \eta = 3i\, X \cdot \eta \,. \tag{3.5}$$

Form decomposition. Under the action of G_2 the spaces of p-forms on X_7, Λ^p, split into irreducible representations. For the subsequent discussion we need the decompositions [11]

$$\Lambda^2 = \Lambda^2_{(7)} \oplus \Lambda^2_{(14)} \,, \tag{3.6}$$

$$\Lambda^4 = \Lambda^4_{(1)} \oplus \Lambda^4_{(7)} \oplus \Lambda^4_{(27)} \,, \tag{3.7}$$

with

$$\Lambda^2_{(7)} = \{v \lrcorner Q | v \in T_m X_7\} \,, \tag{3.8a}$$

$$\Lambda^2_{(14)} = \{\beta \in \Lambda^2 | (*Q) \wedge \beta = 0\} \tag{3.8b}$$

and

$$\Lambda^4_{(1)} = \{\mu * Q | \mu \in \mathbb{R}\} \,, \tag{3.9a}$$

$$\Lambda^4_{(7)} = \{\alpha \wedge Q | \alpha \in T^*_m X_7\} \,, \tag{3.9b}$$

$$\Lambda^4_{(27)} = \{\gamma \in \Lambda^4 | \gamma \wedge Q = 0 \text{ and } * \gamma \wedge Q = 0\} \cong S^2_0 \,. \tag{3.9c}$$

The subscripts of Λ^p label the dimension of the G_2 representation, and S^2_0 is the space of traceless symmetric two-tensors.

Manifolds with G_2-holonomy. G_2-structure manifolds are classified by the derivative of the three-form Q or equivalently of the spinor η. On manifolds with G_2-holonomy, the three-form Q is closed and coclosed, and the spinor η is parallel everywhere with respect to the Levi-Civita connection,

$$\mathrm{d}Q = \mathrm{d} * Q = 0 \qquad \text{and} \qquad \nabla^g \eta = 0 \,. \tag{3.10}$$

On the other hand, either the closedness and coclosedness of Q or the vanishing of $\nabla^g \eta$ imply that the holonomy is contained in G_2. As a result of (3.10), manifolds with G_2-holonomy are Ricci flat.

Nearly parallel G_2-manifolds. By definition, the exterior derivative of the three-form Q on a nearly parallel G_2-manifold [12] is proportional to its Hodge dual,

$$\mathrm{d}Q = -8\mu_2 * Q \tag{3.11}$$

for some $\mu_2 \in \mathbb{R}\backslash\{0\}$. This is equivalent to demanding the spinor η to be a real Killing spinor with nonzero Killing number μ_2,

$$\nabla^g_X \eta = i\,\mu_2\, X \cdot \eta \,. \tag{3.12}$$

Since the Killing number transforms under a conformal transformation $g \to \lambda^2 g$ as $\mu_2 \to \lambda^{-1}\mu_2$, its inverse modulus $|\mu_2^{-1}|$ measures the size of the nearly parallel G_2-manifold. Thus, analogously to the parameter ρ_1 for (A)dS$_3$-spaces, on nearly parallel G_2-manifolds we set

$$\mu_2 = \rho_2^{-1} \,. \tag{3.13}$$

On every nearly parallel G_2-manifold we have two prominent connections: the Levi-Civita connection ∇^g and the canonical connection ∇^C. The latter is defined as the unique metric connection with respect to which Q and equivalently η are constant [12]. It differs from the Levi-Civita connection by a totally skew-symmetric torsion T,

$$g_7(\nabla^C_X Y, Z) = g_7(\nabla^g_X Y, Z) + T(X,Y,Z) \quad \text{with} \quad T = -\frac{4}{3}\,\mu_2\,Q\,. \tag{3.14}$$

For later reference, we additionally define an interpolating connection

$$\nabla^\kappa = \kappa\nabla^g + (1-\kappa)\nabla^C \qquad \text{with} \quad \kappa\in\mathbb{R}\,. \tag{3.15}$$

Curvature of nearly parallel G_2-manifolds. With respect to the Levi-Civita connection, nearly parallel G_2-manifolds are Einstein [12] with Einstein constant $24\mu_2^2$,

$$\mathrm{Ric}^g = 24\mu_2^2\,g_7\,. \tag{3.16}$$

Note that, analogously to de Sitter and anti-de Sitter spaces, the curvature only depends on the square of μ_2 and, hence, the sign of μ_2 or equivalently ρ_2 will enter only the supersymmetry variations but not the equations of motion.

The canonical connection ∇^C is a G_2-instanton connection, meaning that its curvature form R^C satisfies

$$*\,R^C = -Q\wedge R^C\,. \tag{3.17}$$

This is equivalent to R^C annihilating the Killing spinor η by its Clifford action,

$$R^C\cdot\eta = 0\,. \tag{3.18}$$

Beyond that, we also need the quantities $R_{abcd}R_e{}^{bcd}$, $|R|^2$ and $\mathrm{tr}(R\wedge R)$ to discuss the field equations (2.7) of the action (2.1). We do not calculate these quantities explicitly, but relate them for the connection ∇^κ to their value for the canonical connection. In this way, the curvature tensor R^κ of the connection ∇^κ reads

$$(R^\kappa)_{abcd} = (R^C)_{abcd} - \frac{16}{9}\,\mu_2^2\,\kappa^2\,(*Q)_{abcd} + \frac{4}{9}\,\mu_2^2\,\kappa^2\left((g_7)_{ac}(g_7)_{bd} - (g_7)_{ad}(g_7)_{bc}\right). \tag{3.19}$$

For $\kappa=1$ we obtain the curvature of the Levi-Civita connection. Using the identities for Q in (A.2) we obtain

$$(R^\kappa)_{acde}(R^\kappa)_b{}^{cde} = (R^C)_{acde}(R^C)_b{}^{cde} - \frac{64}{9}\,\mu_2^4\,\kappa^2\,(16-11\kappa^2)\,(g_7)_{ab}\,, \tag{3.20}$$

$$|R^\kappa|^2 = |R^C|^2 - \frac{448}{9}\,\mu_2^4\,\kappa^2\,(16-11\kappa^2)\,, \tag{3.21}$$

$$\mathrm{tr}(R^\kappa\wedge R^\kappa) = \mathrm{tr}(R^C\wedge R^C) - \frac{256}{27}\,\mu_2^4\,\kappa^2\,(1+\kappa^2)\,(*Q)\,. \tag{3.22}$$

At this point we also note the following fact [13]: for a generic two-form $\beta\in\Lambda^2_{(14)}$, the wedge product $\beta\wedge\beta$ lies in $\Lambda^4_{(1)}\oplus\Lambda^4_{(27)}$. Furthermore, the components of $\beta\wedge\beta$ in $\Lambda^4_{(1)}$ and $\Lambda^4_{(27)}$ cannot vanish separately. As the curvature form of the canonical connection is in $\Lambda^2_{(14)}\otimes\mathfrak{g}_2$, this also applies to $\mathrm{tr}(R^C\wedge R^C)$. Moreover, as $\mathrm{tr}(R^\kappa\wedge R^\kappa)$ differs from $\mathrm{tr}(R^C\wedge R^C)$ by a term in $\Lambda^4_{(1)}$, the $\Lambda^4_{(27)}$ component of $\mathrm{tr}(R^\kappa\wedge R^\kappa)$ cannot vanish for any choice of κ. This fact will pose severe constraints on the possible ansätze for the gauge field.

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4 The heterotic string on nearly parallel G_2-manifolds

In this section we discuss the solutions of the field equations and Bianchi identity of the heterotic string on

$$M = M_{1,2} \times X_7 \,, \tag{4.1}$$

with $M_{1,2}$ being three-dimensional de Sitter, anti-de Sitter or Minkowski space, and X_7 being a nearly parallel G_2-manifold or a manifold with G_2-holonomy. Moreover, we calculate the supersymmetry variations and the masses of the fermions for any of these solutions.

As the gaugino and dilatino condensates possess components on both $M_{1,2}$ and X_7, the vanishing of the dilatino supersymmetry variation (2.6b) demands that the same holds true for the three-form flux H. Hence, it is natural to choose

$$H = -h_1 \,\mathrm{vol}^{(3)} + \, h_2 \, Q \tag{4.2}$$

with h_1, $h_2 \in \mathbb{R}$, as an ansatz also for not necessarily supersymmetric solutions.

The possible ansätze for the gauge field F and the connection $\tilde{\Gamma}$ are highly restricted by the Bianchi identity. Here, we choose F to be the curvature of the canonical connection ∇^C on X_7. Although it is in principle possible to find non-instanton solutions, instanton solutions are distinguished by allowing for a non-standard embedding in the Bianchi identity and by immediately solving the Yang-Mills equation. Furthermore, for an instantonic gauge field, supersymmetry variation of the gaugino vanishes. We identify the curvature $\tilde{R}$ of the connection $\tilde{\Gamma}$ with the curvature of the interpolating connection ∇^κ. As discussed in section 3, this choice ensures the vanishing of the $\Lambda^4_{(27)}$ component of the right hand side of the Bianchi identity (2.8), which is required as its left hand side, $\mathrm{d}H$, is proportional to $*Q \in \Lambda^4_{(1)}$. Summarizing, our ansatz is

$$F = R^C \qquad \text{and} \qquad \tilde{R} = R^\kappa \,. \tag{4.3}$$

4.1 Equations of motion

For our ansatz, the equations of motion (2.7) reduce to a set of algebraic equations for the parameters $\mu_1, \mu_2, h_1, h_2, m, n$ and κ. It is convenient to replace the parameters m and n by $\hat{m}$ and $\hat{n}$, defined as

$$2\,\hat{m} = 4n - m \qquad \text{and} \qquad 2\,\hat{n} = 3m - 4n \,. \tag{4.4}$$

The Einstein equation splits into one equation on $M_{1,2}$ and one on X_7:

$$64\mu_1^2(1 + 2\alpha'\mu_1^2) - 2h_1(h_1 + \hat{m}) = 0 \,, \tag{4.5a}$$

$$64\,\mu_2^2 \left(1 - \frac{2}{27}\alpha'\mu_2^2 a(\kappa)\right) - 2h_2(h_2 + \hat{m}) = 0 \,, \tag{4.5b}$$

with

$$a(\kappa) = \kappa^2(16 - 11\kappa^2) \,. \tag{4.5c}$$

Furthermore, the dilaton equation and the Bianchi identity read

$$-48\mu_1^2(1 - \alpha'\mu_1^2) + 336\mu_2^2 - \frac{112}{9}\alpha'\mu_2^4\, a(\kappa) - h_1^2 - 7h_2^2 - 2(h_1 + 7h_2)\hat{m} + \hat{m}^2 - \hat{n}^2 = 0 \tag{4.5d}$$

and

$$\mu_2 h_2 = \frac{8}{27}\mu_2^4\,\alpha'\,b(\kappa) \tag{4.6}$$

with

$$b(\kappa) = \kappa^2(1+\kappa^2)\,, \tag{4.7}$$

respectively. Note that $\hat{n}$ enters the equations of motions only quadratically. Hence, solely $\hat{n}^2$ will be determined by the field equations, but the sign of $\hat{n}$ will not be fixed.

4.2 Solution to the equations of motion

4.2.1 De Sitter and anti-de Sitter solutions

Nearly parallel G_2 compactifications. We begin with the more general situation of compactifications on nearly parallel G_2-manifolds, i.e. we assume $\mu_2 \neq 0$. As discussed in section 3, we set $\mu_2 = \rho_2^{-1}$ in this case, where $|\rho_2|$ measures the size of X_7. As the solutions for de Sitter and anti-de Sitter backgrounds are very similar, we treat them in parallel. We remind the reader that the radius of the (anti-) de Sitter space is given by $|\rho_1| = |\mu_1|^{-1}$, with μ_2 being either real or purely imaginary. The equations of motion (4.5) and the Bianchi identity (4.6) form a system of four algebraic equations for the seven parameters of our ansatz,

$$\rho_1, \rho_2, \kappa, h_1, h_2, \hat{m} \text{ and } \hat{n}\,. \tag{4.8}$$

It can be solved for the parameters of the H-flux and the fermionic condensates,

$$h_1(\rho_1,\rho_2,\kappa)\,, \quad h_2(\rho_1,\rho_2,\kappa)\,, \quad \hat{m}(\rho_1,\rho_2,\kappa)\,, \quad \hat{n}(\rho_1,\rho_2,\kappa)\,. \tag{4.9}$$

The explicit expressions are as follows,

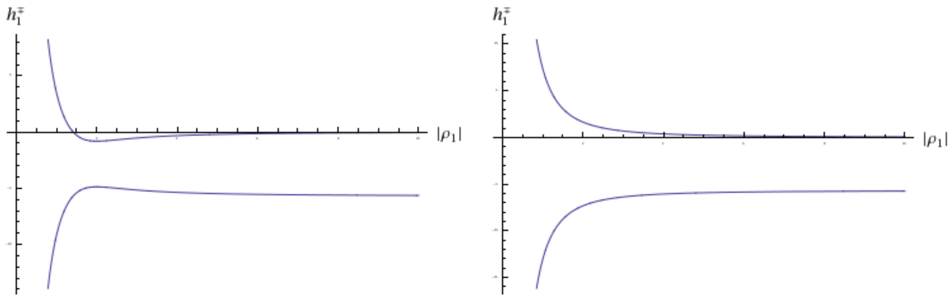
$$\begin{aligned} h_1^{\pm} &= -\rho_2\left[\frac{54}{\alpha' b(\kappa)} - \frac{4a(\kappa)}{\rho_2^2 b(\kappa)} - \frac{4\alpha' b(\kappa)}{27\rho_2^4}\right] \\ &\quad \pm \rho_2\sqrt{\left[\frac{54}{\alpha' b(\kappa)} - \frac{4a(\kappa)}{\rho_2^2 b(\kappa)} - \frac{4\alpha' b(\kappa)}{27\rho_2^4}\right]^2 + \frac{32\left(2\alpha' + \delta\rho_1^2\right)}{\rho_1^4\rho_2^2}}\,, \end{aligned} \tag{4.10a}$$

$$h_2 = \frac{8\alpha'}{27\rho_2^3}b(\kappa) \tag{4.10b}$$

and

$$\hat{m} = 2\rho_2\left[\frac{54}{\alpha' b(\kappa)} - \frac{4a(\kappa)}{\rho_2^2 b(\kappa)} - \frac{4\alpha' b(\kappa)}{27\rho_2^4}\right]\,, \tag{4.10c}$$

$$\begin{aligned} (\hat{n}^{\pm})^2 &= -16\frac{\alpha' - \delta\rho_1^2}{\rho_1^4} + 16\left[\frac{2187\rho_2^2}{2\alpha'^2 b(\kappa)^2} - \frac{162a(\kappa)}{\alpha' b(\kappa)^2} - \frac{13}{\rho_2^2} + \frac{6a(\kappa)^2}{\rho_2^2 b(\kappa)^2} + \frac{47\alpha' a(\kappa)}{27\rho_2^4} + \frac{34\alpha'^2 b(\kappa)^2}{729\rho_2^6}\right] \\ &\quad \pm 2\rho_2^2\left[\frac{54}{\alpha' b(\kappa)} - \frac{4a(\kappa)}{\rho_2^2 b(\kappa)} - \frac{4\alpha' b(\kappa)}{27\rho_2^4}\right]\sqrt{\left[\frac{54}{\alpha' b(\kappa)} - \frac{4a(\kappa)}{\rho_2^2 b(\kappa)} - \frac{4\alpha' b(\kappa)}{27\rho_2^4}\right]^2 + \frac{32(2\alpha' + \delta\rho_1^2)}{\rho_1^4\rho_2^2}}\,, \end{aligned} \tag{4.10d}$$



(a) Anti-de Sitter backgrounds (b) De Sitter backgrounds

Figure 1. Plots of $h_1^\pm$ for a fixed value of ρ_2 and κ.

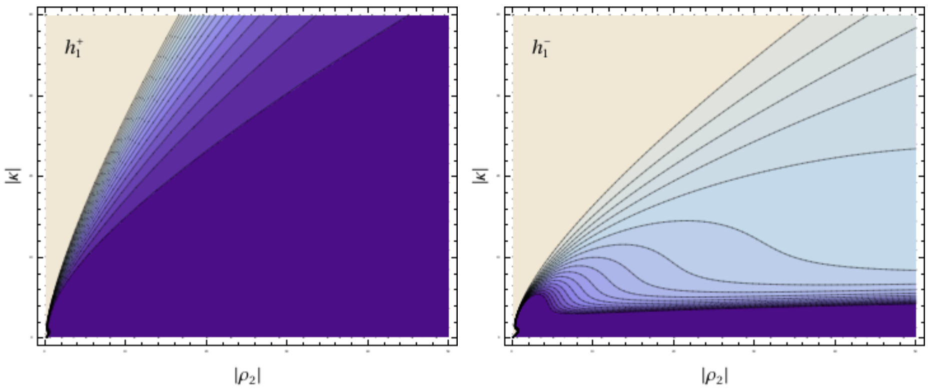


Figure 2. Contour plots of $h_1^\pm$ for a fixed value of ρ_1.

respectively, with the plus/minus signs in (4.10a) and (4.10d) being correlated. We have also defined

$$\delta = \begin{cases} -1 & \text{for anti-de Sitter backgrounds} \\ 1 & \text{for de Sitter backgrounds} \end{cases} . \tag{4.11}$$

For fixed values of ρ_1, ρ_2 and κ, the field equations possess up to four solutions for the parameters h_1, h_2, $\hat{m}$ and $\hat{n}$, distinguished by the choice of the plus/minus-signs in (4.10a) and (4.10d) and the sign of $\hat{n}$. However, the equations of motion are not solvable for all values of (ρ_1, ρ_2, κ). The excluded regions in the parameter space (ρ_1, ρ_2, κ) are discussed in appendix B.

The dependence of h_1, $\hat{m}$ and $\hat{n}$ on the parameters ρ_1, ρ_2 and κ cannot be read off easily from (4.10a), (4.10b) and (4.10d). It is depicted qualitatively in the figures 1, 2, 3 and 4.

Solutions with $h_1 = 0$ or $h_2 = 0$. In general, the tensor field H has components proportional to Q as well as to $\mathrm{vol}^{(3)}$. We remark that there is no solution with $h_2 = 0$: in this case, the Bianchi identity (4.6) implies $\kappa = 0$, but obviously the Einstein equation (4.5b) possesses no solution with $h_2 = \kappa = 0$. On the other hand, $h_1 = 0$ is possible: from (4.5a) we can read off that, in anti-de Sitter backgrounds, $h_1 = 0$ only implies $\rho_1^2 = 2\alpha'$. Since h_2

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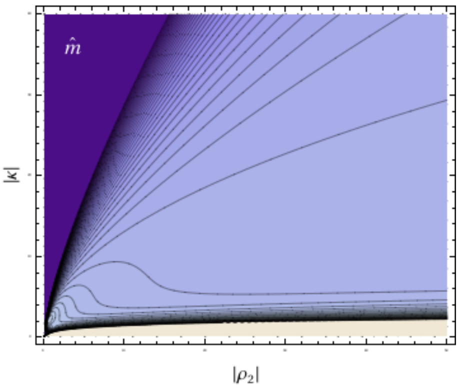
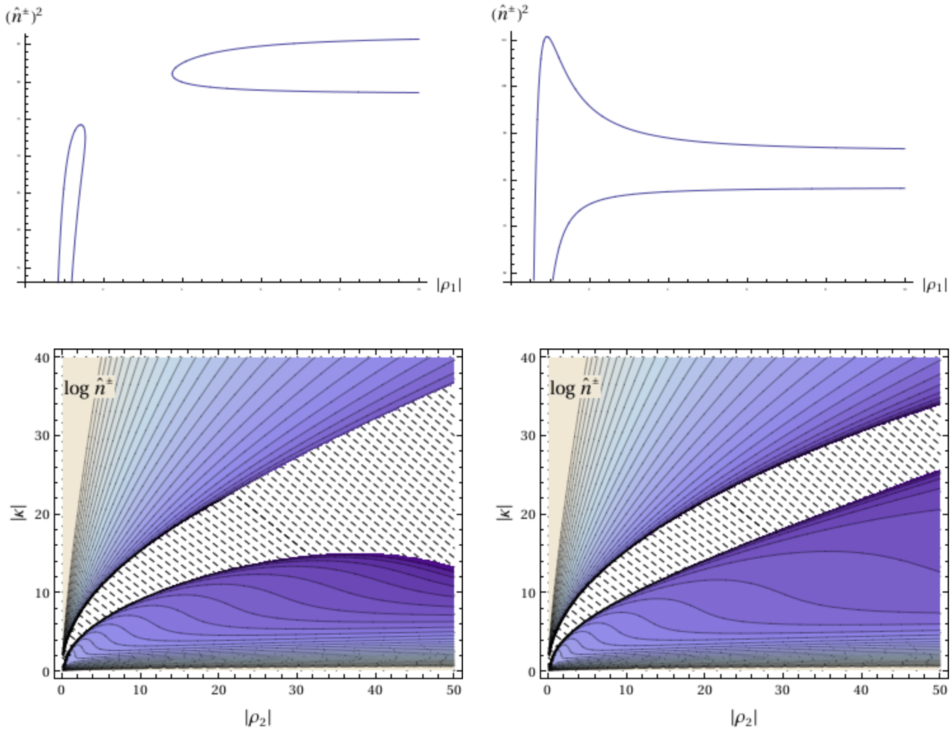


Figure 3. Contour plot of $\hat{m}$.



(a) Anti-de Sitter backgrounds

(b) De Sitter backgrounds

Figure 4. Plots of $\hat{n}^{\pm}$ for a fixed value of ρ_2 and κ (top) and contour plots of $\hat{n}^{-}$ (bottom) for a fixed value of ρ_1. In the striped area, no solution to the equations of motion exist.

and $\hat{m}$ do not depend on ρ_1, their expressions are not simplified for this special solution. However, (4.10d) reduces to

$$(\hat{n})^2 = -\frac{12}{\alpha'} - \frac{112}{\rho_2^2} + \frac{560\alpha' k(\kappa)}{27\rho_2^4} + \frac{448\alpha'^2 b(\kappa)^2}{729\rho_2^6} + 4\rho_2^2 \left[\frac{54}{\alpha' b(\kappa)} - \frac{4k(\kappa)}{\rho_2^2 b(\kappa)} - \frac{4\alpha' b(\kappa)}{27\rho_2^4}\right]^2 . \quad (4.12)$$

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G_2-holonomy compactifications. If we choose, more specially, X_7 to have G_2-holonomy, the Bianchi identity can only be solved by the standard embedding,

$$F = \tilde{R} \qquad \Rightarrow \qquad \kappa = 0\,. \tag{4.13}$$

The reason is that, on G_2-holonomy manifolds, the three form Q, and thus H, is closed, and thereby the left-hand side of the Bianchi identity (2.8) vanishes. Furthermore, since $\mu_2 = 0$, we send $\rho_2 \to \infty$. This leaves us with 3 equations for 5 parameters, which we can solve for

$$h_1(\rho_1, \hat{m})\,, \quad h_2(\rho_1, \hat{m}) \qquad \text{and} \qquad \hat{n}(\rho_1, \hat{m})\,. \tag{4.14}$$

The Einstein equation on (A)dS$_3$ (4.5a) is solved by

$$h_1^{\pm} = -\frac{\hat{m}}{2} \pm \frac{1}{\rho_1^2}\sqrt{32(2\alpha' + \delta\rho_1^2) + \frac{1}{4}\hat{m}\rho_1^4}\,. \tag{4.15a}$$

The Einstein equation on X_7 (4.5b) admits only two solutions, which we distinguish by introducing an auxiliary parameter θ:

$$h_2 = 0 \qquad \Leftrightarrow \qquad \theta = 3 \qquad \text{or} \tag{4.15b}$$

$$h_2 = -\hat{m} \qquad \Leftrightarrow \qquad \theta = 17\,. \tag{4.15c}$$

Finally, the dilaton equation (4.5d) is solved by

$$(\hat{n}^{\pm})^2 = 16\frac{\delta\rho_1^2 + \alpha'}{\rho_1^4} + \frac{\theta}{2}\hat{m}^2 \pm \frac{\hat{m}}{\rho_1^2}\sqrt{32(2\alpha' + \delta\rho_1^2) + \frac{1}{4}\hat{m}^2\rho_1^4}\,, \tag{4.15d}$$

with the plus/minus signs in (4.15a) and (4.15d) being correlated. Thus, in contrast to nearly parallel G_2 compactifications, there are up to eight solutions to the field equations, parametrized by ρ_1 and $\hat{m}$ and distinguished by the choice of the plus/minus-sign in (4.15a) and (4.15d), the parameter θ and the sign of $\hat{n}$. The qualitative dependence of h_1 and $\hat{n}$ on ρ_1 and $\hat{m}$ are depicted in figures 5 and 6. Unlike in the case of nearly parallel G_2 compactifications, there now exist solutions with H-flux confined to $M_{1,2}$, i.e. $h_2 = 0$ and $h_1 \neq 0$.

As for nearly parallel G_2 compactifications, the solutions on anti-de Sitter backgrounds simplify for $\rho_1^2 = 2\alpha'$:

$$h_1^{\pm} = \frac{-\hat{m} \pm |\hat{m}|}{2}\,, \tag{4.16}$$

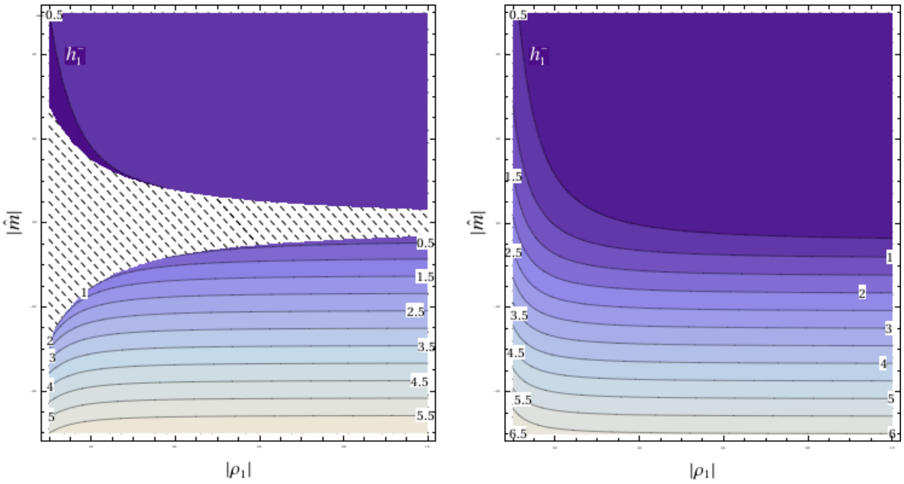
$$(\hat{n}^{\pm})^2 = \frac{12}{\alpha'} + \frac{\hat{m}(\theta\,\hat{m} \pm |\hat{m}|)}{2}\,. \tag{4.17}$$

Obviously, this yields also solutions with vanishing flux on $M_{1,2}$,

$$h_1 = 0 \qquad \text{and} \qquad \hat{n}^2 = \frac{12}{\alpha'} + \frac{(1+\theta)\hat{m}^2}{2}\,. \tag{4.18}$$

Moreover, there is also a solution with completely vanishing H-flux and condensates only,

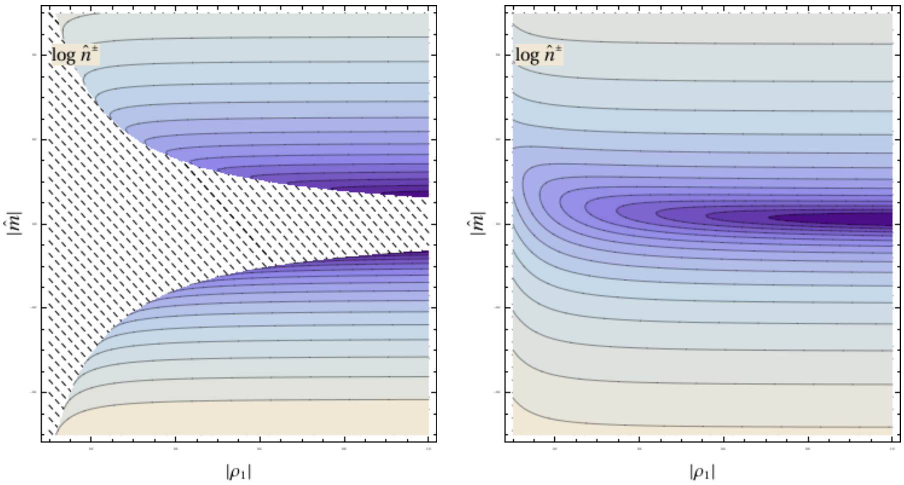
$$h_1 = h_2 = 0 \qquad \text{and} \qquad \hat{n}^2 = \frac{12}{\alpha'} + 2\hat{m}^2\,. \tag{4.19}$$



(a) Anti-de Sitter background

(b) De Sitter background

Figure 5. Contour plots of h_1^- for compactifications on manifolds with G_2-holonomy.



(a) Anti-de Sitter backgrounds

(b) De Sitter backgrounds

Figure 6. Plots of $\hat{n}^{\pm}$ for compactifications on manifolds with G_2-holonomy. In the striped area, no solution to the equations of motion exist.

4.2.2 Minkowski solutions

Nearly parallel G_2 compactifications. For compactifications to three-dimensional Minkowski space, we have to set $\mu_1 = 0$. The solutions to the Bianchi identity (4.6) and the Einstein equation on X_7 (4.5b) are the same as in the (A)dS-case,

$$h_2 = \frac{8\alpha'}{27\rho_2^3} b(\kappa) \,. \tag{4.20a}$$

$$\hat{m} = 2\rho_2 \left[\frac{54}{\alpha' b(\kappa)} - \frac{4a(\kappa)}{\rho_2^2 b(\kappa)} - \frac{4\alpha' b(\kappa)}{27\rho_2^4} \right] \,. \tag{4.20b}$$

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The Einstein equation on $M_{1,2}$ (4.5a) on the other hand is solved by either

$$h_1^- = 0 \qquad \text{or} \tag{4.20c}$$

$$h_1^+ = -2\rho_2 \left[\frac{54}{\alpha' b(\kappa)} - \frac{4a(\kappa)}{\rho_2^2 b(\kappa)} - \frac{4\alpha' b(\kappa)}{27\rho_2^4} \right] . \tag{4.20d}$$

Finally, the solution to the equation of motion for the dilaton (4.5d) yields

$$(\hat{n}^\pm)^2 = 112 \left(-\frac{1}{\rho_2^2} + \frac{5\alpha' a(\kappa)}{27\rho_2^4} + \frac{4\alpha'^2 b(\kappa)^2}{729\rho_2^6} \right) + (6 \pm 2)\rho_2^2 \left(\frac{54}{\alpha' b(\kappa)} - \frac{4a(\kappa)}{\rho_2^2 b(\kappa)} - \frac{4\alpha' b(\kappa)}{27\rho_2^4} \right)^2 . \tag{4.20e}$$

The superscripts $+$ and $-$ refer to the solutions (4.20c) and (4.20d) for H_1. As for the solutions on de Sitter and anti-de Sitter backgrounds, the dependence of $\hat{m}$ and $\hat{n}$ on ρ_2 and κ cannot be easily seen from (4.20a) and (4.20e). The solution for $\hat{m}$, however, is identical to the (anti-)de Sitter case (see (4.10b) and figure 3). Furthermore, the qualitative dependence of the solution for $\hat{n}$ on ρ_2 and κ is the same as for (anti-)de Sitter backgrounds (see figure 4), as (4.20e) is the $|\rho_1| \to \infty$ limit of (4.10d).

G_2-holonomy compactifications. Finally, we discuss the case of compactifications to Minkowski space on manifolds with G_2-holonomy. As in the previous cases of G_2-holonomy compactifications, the Bianchi identity requires

$$\tilde{R} = F \qquad \Rightarrow \qquad \kappa = 0 . \tag{4.21}$$

After sending $\rho_2 \to \infty$, the field equations are solved by

$$h_2 = -\hat{m} \tag{4.22a}$$

and either

$$h_1 = 0 , \qquad \hat{n}^2 = 8\hat{m}^2 \qquad \text{or} \tag{4.22b}$$

$$h_1 = -\hat{m} , \quad \hat{n}^2 = 9\hat{m}^2 . \tag{4.22c}$$

Hence, there are four independent solutions to the field equations. In contrast to the previous cases, the equations of motion possess solutions for all values of the parameter $\hat{m}$.

4.3 Supersymmetry conditions and supersymmetric solutions

It is of interest to find out which subset of our heterotic backgrounds are supersymmetric. To this end, we investigate the supersymmetry conditions (2.6).[2] We begin with the gravitino variation (2.6a). Recall that $M_{1,2}$ carries either a real or imaginary Killing spinor ζ

[2]The literature is not consistent on the relative choices of connections in the Bianchi identity and field equations on the one hand and in the supersymmetry equations on the other hand. For example, [14] employ $\Gamma^+ \equiv \Gamma^{\kappa=2}$ in the Bianchi identity and argue that the corresponding action is invariant under supersymmetry variations with $\Gamma^- \equiv \Gamma^{\kappa=0}$. Likewise, [15] proved that the heterotic supersymmetry equations with Γ^- imply the field equations with Γ^+. Since we also consider supersymmetry equations with Γ^-, adhering to [15] would imply setting κ=2 in the equations of motions and Bianchi identity. Our conclusions however will not depend on such a choice.

with Killing number μ_1 and that X_7 possesses a real Killing spinor with Killing number μ_2. Furthermore, using (3.3) and (3.5) it is straightforward to compute

$$(X \lrcorner H) \cdot \epsilon = \begin{cases} -2\, h_1\, (X \cdot \widehat{\epsilon}) \otimes \eta \otimes (1,0)^t & \text{for } X \in TM_{1,2} \\ 6i\, h_2\, \epsilon \otimes (X \cdot \eta) \otimes (1,0)^t & \text{for } X \in TX_7 \end{cases} \tag{4.23}$$

and

$$\Sigma \cdot X \cdot \epsilon = \begin{cases} -m\ \ (X \cdot \widehat{\epsilon}) \otimes \eta \otimes (1,0)^t & \text{for } X \in TM_{1,2} \\ -7i\, m\ \ \widehat{\epsilon} \otimes (X \cdot \eta) \otimes (1,0)^t & \text{for } X \in TX_7 \end{cases} . \tag{4.24}$$

Inserting these relations in the gravitino variation yields

$$96\mu_1 = 12h_1 + m \ \ = 12h_1 + \hat{m} + \hat{n}\,, \tag{4.25a}$$

$$96\mu_2 = 12h_2 + 7m = 12h_2 + 7(\hat{m} + \hat{n})\,. \tag{4.25b}$$

As we set the dilaton to zero, the dilatino variation (2.6b) reads

$$4H + \Sigma - \Delta = 0 \tag{4.26}$$

and therefore we obtain

$$16h_1 = 16h_2 = 4(n - m) = -\hat{m} - 3\hat{n}\,. \tag{4.27}$$

As already mentioned at the beginning of this section, the gaugino variation (2.6c) vanishes since F is an instanton.

Together with the equations of motion, the conditions for the vanishing of the gravitino and the dilatino variation, (4.25) and (4.27), form a set of eight equations for the seven parameters μ_1, μ_2, h_1, h_2, $\hat{m}$, $\hat{n}$ and κ. It can be checked straightforwardly that this system is not solvable. Hence, our compactifications on nearly parallel G_2-manifolds do not yield any supersymmetric solutions for either de Sitter, anti-de Sitter or Minkowski space-times, not even in the G_2-holonomy limit.

4.4 Fermion masses

Employing the decomposition of the fermions (2.18) and the Killing property of η as well as the knowledge of the Clifford action of $\mathrm{vol}^{(3)}$ (2.23) and Q (3.3), it is straightforward to rewrite the Dirac equations for the gaugino (2.7e) and dilatino (2.7f) as

$$0 = \left(\tilde{\mathcal{D}} + \mathcal{D}^- - \frac{1}{24}\left(H - \frac{1}{2}\Sigma + \frac{1}{2}\Delta\right)\cdot\right)\chi = (\tilde{\mathcal{D}} - M_\chi)\,\chi\,, \tag{4.28}$$

$$0 = \left(\tilde{\mathcal{D}} + \mathcal{D}^- - \frac{1}{24}\left(H - \frac{1}{8}\Sigma\right)\cdot\right)\lambda = (\tilde{\mathcal{D}} - M_\lambda)\,\lambda\,, \tag{4.29}$$

where $\tilde{\mathcal{D}}$ is the Dirac operator on $M_{1,2}$. The masses of the gaugino and dilatino are readily computed to be

$$M = \frac{1}{24}\,(h_1 + 7h_2 - \hat{m} - c\hat{n}) \qquad \text{with} \qquad c_\chi = 3 \qquad \text{and} \qquad c_\lambda = 1\,. \tag{4.30}$$

In the following, we give the expressions of the masses in terms of the parameters (ρ_1, ρ_2, κ) or $(\rho_1, \hat{m})$, respectively, for the compactifications considered previously.

De Sitter and anti-de Sitter backgrounds. Recall that for given values of (ρ_1, ρ_2, κ) the field equations possess up to four solutions in the case of compactifications on nearly parallel G_2-manifolds and up to eight solutions for compactifications on manifolds with G_2-holonomy. There are up to two solutions for h_1 labeled by the superscript $\pm$. Then, for a given solution for h_1, the field equations fix the value of $\hat{n}^2$, but not the sign of $\hat{n}$ (see (4.10d)). Hence, in the following we set

$$\hat{n} = \nu\,|\hat{n}| \qquad \text{with} \qquad \nu \in \{-1, +1\}\,. \tag{4.31}$$

For compactifications on nearly parallel G_2-manifolds the fermion masses are given by

$$\begin{aligned} M^{\pm} = &-\frac{27\rho_2}{4\alpha' b(\kappa)} + \frac{a(\kappa)}{2\rho_2 b(\kappa)} + \frac{17\alpha' b(\kappa)}{162\rho_2^3} \\ &\mp \frac{\rho_2}{12}\sqrt{\frac{8\left(2\alpha' + \delta\rho_1^2\right)}{\rho_1^4\rho_2^2} + \left(\frac{27}{\alpha' b(\kappa)} - \frac{2a(\kappa)}{\rho_2^2 b(\kappa)} - \frac{2\alpha' b(\kappa)}{27\rho_2^4}\right)^2} \\ &- \frac{\nu c|\rho_2|}{6\sqrt{2}}\left[\pm\left(\frac{27}{\alpha' b(\kappa)} - \frac{2a(\kappa)}{\rho_2^2 b(\kappa)} - \frac{2\alpha' b(\kappa)}{27\rho_2^4}\right)\right. \\ &\sqrt{\frac{8\left(2\alpha' + \delta\rho_1^2\right)}{\rho_1^4\rho_2^2} + \left(\frac{27}{\alpha' b(\kappa)} - \frac{2a(\kappa)}{\rho_2^2 b(\kappa)} - \frac{2\alpha' b(\kappa)}{27\rho_2^4}\right)^2} \\ &\left.-\frac{2\left(\delta\rho_1^2 + \alpha'\right)}{\rho_1^4\rho_2^2} + \frac{3\left(27\rho_2^2 - 2\alpha' a(\kappa)\right)^2}{\alpha'^2 b(\kappa)^2\rho_2^4} - \frac{26}{\rho_2^4} + \frac{94\alpha' a(\kappa)}{27\rho_2^6} + \frac{68\alpha'^2 b(\kappa)^2}{729\rho_2^8}\right]^{1/2}, \end{aligned} \tag{4.32}$$

whereas for compactifications on manifolds with G_2-holonomy they read[3]

$$\begin{aligned} M^{\pm} = &-\theta\frac{\hat{m}}{48} \pm \frac{1}{48\rho_1^4}\sqrt{128\rho_1^4(\delta\rho_1^2 + 2\alpha') + \rho_1^8\hat{m}^2} \\ &- \frac{c\nu}{24\sqrt{2}\rho_1^2}\sqrt{32\left(\delta\rho_1^2 - \alpha'\right) + \theta\rho_1^4\hat{m}^2 \pm \rho_1^2\hat{m}\sqrt{128(\delta\rho_1^2 + 2\alpha') + \rho_1^4\hat{m}^2}}\,. \end{aligned} \tag{4.33}$$

Minkowski backgrounds. On Minkowski backgrounds, the masses of the gaugino and dilatino are given by

$$\begin{aligned} M^{\pm} = &\frac{(6\pm 2)k(\kappa)}{3\rho_2 b(\kappa)} - \frac{9(6\pm 2)\rho_2}{2\alpha' b(\kappa)} + \frac{(13\pm 2)\alpha' b(\kappa)}{81\rho_2^3} \\ &- \frac{c\nu}{8}\sqrt{112\left(-\frac{1}{\rho_2^2} + \frac{5\alpha' a(\kappa)}{27\rho_2^4} + \frac{4\alpha'^2 b(\kappa)^2}{729\rho_2^6}\right) + (6\pm 2)\rho_2^2\left(\frac{54}{\alpha' b(\kappa)} - \frac{4a(\kappa)}{\rho_2^2 b(\kappa)} - \frac{4\alpha' b(\kappa)}{27\rho_2^4}\right)^2} \end{aligned} \tag{4.34}$$

for compactifications on nearly parallel G_2-manifolds.

Finally, for compactifications on manifolds with G_2-holonomy the fermion masses read

$$M^{\pm} = -\frac{1}{24}(a\,\hat{m} + c\,b\,\nu|\hat{m}|)\,. \tag{4.35}$$

[3]Recall, that $\theta = 3$ for solutions with $h_2 = 0$ and $\theta = 17$ for $h_2 = -\hat{m}$ (see (4.15)).

The values of (a, b) are given in the following table for all solutions to the field equations:

(a,b)	$h_1 = 0$	$h_1 = -\hat{m}$
$h_2 = 0$	$(1,\, 1)$	$(2,\, \sqrt{2})$
$h_2 = \hat{m}$	$(4,\, 2\sqrt{2})$	$(9,\, 3)$

(4.36)

Acknowledgments

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A Useful identities for nearly parallel G_2-manifolds

In this appendix we will list some useful identities for the G_2-invariant three-form Q on manifolds with G_2-structure. In the following we will denote the Hodge-dual of Q by $\widehat{Q}$,

$$\widehat{Q} \equiv *Q = e^{4567} - e^{2347} + e^{1357} - e^{1267} + e^{2356} + e^{1245} + e^{1346} \,. \tag{A.1}$$

It is straightforward to derive the following identities of contractions of Q and $\widehat{Q}$ [16]:

$$Q_{abe}Q_{cde} = -\widehat{Q}_{abcd} + \delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc} \,, \tag{A.2a}$$

$$Q_{acd}Q_{bcd} = 6\,\delta_{ab} \,, \tag{A.2b}$$

$$Q_{abp}\widehat{Q}_{pcde} = 3Q_{a[cd}\delta_{e]b} - 3Q_{b[cd}\delta_{e]a} \,, \tag{A.2c}$$

$$\widehat{Q}_{abcp}\widehat{Q}_{defp} = -3\widehat{Q}_{ab[de}\delta_{f]c} - 2\widehat{Q}_{def[a}\delta_{b]c} - 3Q_{ab[d}Q_{ef]c} + 6\delta_a^{[d}\delta_b^{e}\delta_c^{f]} \,, \tag{A.2d}$$

$$\widehat{Q}_{abpq}Q_{pqc} = -4Q_{abc} \,, \tag{A.2e}$$

$$\widehat{Q}_{abpq}\widehat{Q}_{pqcd} = -2\widehat{Q}_{abcd} + 4(\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}) \,, \tag{A.2f}$$

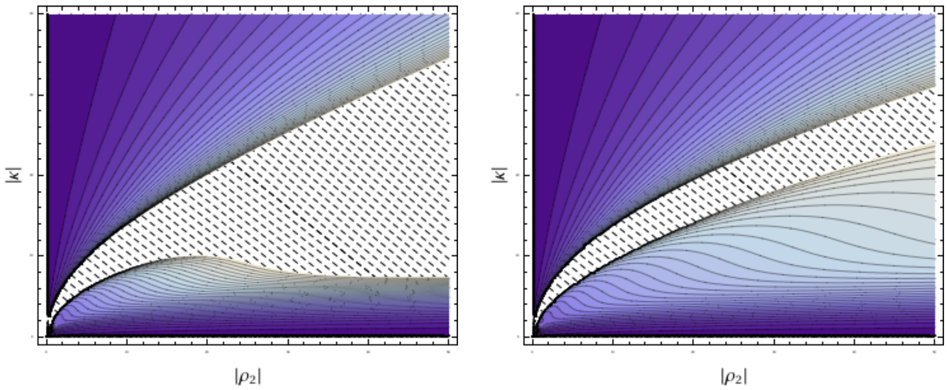
$$\widehat{Q}_{apqr}\widehat{Q}_{bpqr} = 24\delta_{ab} \,, \tag{A.2g}$$

$$\begin{aligned} Q_{abp}Q_{pcq}Q_{qde} = {} & Q_{abd}\delta_{ce} - Q_{abe}\delta_{cd} - Q_{ade}\delta_{bc} + Q_{bde}\delta_{ac} \\ & - Q_{acd}\delta^{be} + Q_{ace}\delta_{bd} + Q_{bcd}\delta_{ae} - Q_{bce}\delta_{ad} \,, \end{aligned} \tag{A.2h}$$

$$Q_{paq}Q_{qbs}Q_{scp} = 3Q_{abc} \,. \tag{A.2i}$$

B Restrictions on the parameter space of solutions

In general, the equations of motion do not possess solutions for all values of the parameters $\mu_1, \mu_2, h_1, h_2, \hat{m}, \hat{n}$ and κ of the ansatz considered in section 4. In this appendix we discuss for which values of the parameters the field equations are not solvable.



(a) Anti-de Sitter backgrounds (b) De Sitter backgrounds

Figure 7. Contour plot of the lower bound for $|\rho_1|$ for which solutions to the equations of motion for (anti-)de Sitter backgrounds exists. For $\rho_2 \to 0$ or $\kappa \to 0$ the lower bound approaches zero. In the striped area, the equations of motion cannot be solved for any value of ρ_1.

B.1 Nearly parallel G_2 compactifications

For both, anti-de Sitter and de Sitter backgrounds, the equations of motion were solved for $h_1, h_2, \hat{m}$ and $\hat{n}$ and are parametrized by $\rho_1 = \mu_1^{-1}$, $\rho_2 = \mu_2^{-1}$ and κ. In both cases it is obvious that there exists a lower bound for $|\rho_1|$ imposed by demanding the solution for $\hat{n}^2$, (4.10d) or (4.10d), respectively, to be positive. The same conditions exclude for fixed values of ρ_1 a region in the parameter space spanned by ρ_2 and κ in which no solution to the field equations exists. Additionally, in the case of anti-de Sitter backgrounds, the argument in the square root in (4.10a) and (4.10d) becomes negative for certain values of ρ_1, ρ_2 and κ. Contour plots of the resulting lower bound on $|\rho_1|$ and the excluded region in the parameter space spanned by ρ_2 and κ are depicted in figure 7.

For compactifications on Minkowski backgrounds, the parameter μ_1 vanishes. The parameter spaces spanned by ρ_2 and κ on the other hand is restricted by the same conditions as in the de Sitter and anti-de Sitter cases. The area in the parameter spaces in which no solutions to the field equations with Minkowski background can be given is depicted in figure 8.

B.2 G_2-holonomy compactifications

The solutions to the equations for compactifications on G_2-holonomy manifolds as discussed in section 4 depend on the parameters ρ_1 and $\hat{m}$. For anti-de Sitter backgrounds, as in the nearly parallel G_2 case, the field equations only possess solutions if the value $|\rho_1|$ exceeds a lower bound. For de Sitter backgrounds on the other hand, there is no lower bound on $|\rho_1|$. Additionally, on both backgrounds, there is a lower bound on $|\hat{m}|$. Plot of the areas in the parameter space excluded by these lower bounds are shown in figure 9. Finally, for compactifications to Minkowski space on G_2-holonomy manifolds, the equations of motion are solvable for all values of the parameters.

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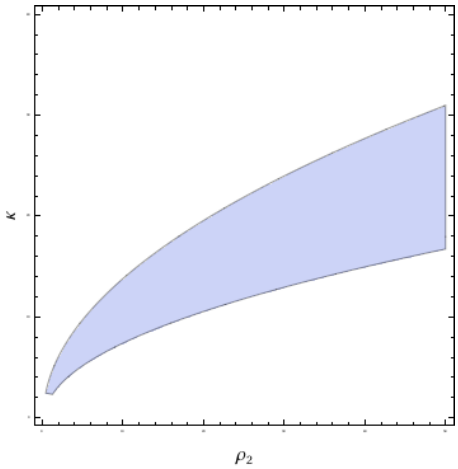
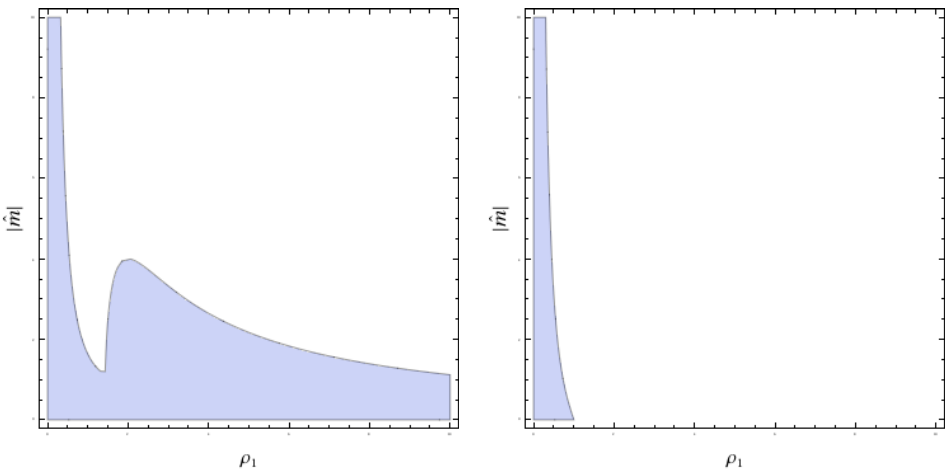


Figure 8. Plot of the area in the parameter space spanned by ρ_2 and κ for which no solutions to the field equations exist.



(a) Anti-de Sitter backgrounds (b) De Sitter backgrounds

Figure 9. Plot of the area in the parameter space spanned by ρ_1 and $\hat{m}$ for which no solutions to the field equations for G_2-holonomy compactifications exist.

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