PAPER • OPEN ACCESS

An open-source framework for the uncertainty quantification of aeroelastic wind turbine simulation tools

To cite this article: H Verdonck *et al* 2022 *J. Phys.: Conf. Ser.* **2265** 042039

View the article online for updates and enhancements.

Journal of Physics: Conference Series **2265** (2022) 042039 doi:10.1088/1742-6596/2265/4/042039

An open-source framework for the uncertainty quantification of aeroelastic wind turbine simulation tools

H Verdonck[1], O Hach[1], J D Polman[2], O Braun[2], C Balzani[2], S Müller[3], J Rieke[3]

[1]German Aerospace Center, Institute of Aeroelasticity, Bunsenstr. 10, 37073 Göttingen, Germany

[2]Leibniz University Hannover, Institute for Wind Energy Systems, Appelstr. 9A, 30167 Hannover, Germany

[3]Nordex Energy GmbH, Langenhorner Chaussee 600, 22419 Hamburg, Germany

E-mail: `hendrik.verdonck@dlr.de`

Abstract. The uncertainty quantification of aeroelastic wind turbine simulations is an active research topic. This paper presents a dedicated, open-source framework for this purpose. The framework is built around the *uncertainpy* package, likewise available as open source. Uncertainty quantification is done with a non-intrusive, global and variance-based surrogate model, using PCE (i.e., polynomial chaos expansion). Two methods to handle the uncertain parameter distribution along the blades are presented. The framework is demonstrated on the basis of an aeroelastic stability analysis. A sensitivity analysis is performed on the influence of the flapwise, edgewise and torsional stiffness of the blades on the damping of the most critical mode for both a Bladed linearization and a Bladed time domain simulation. The sensitivities of both models are in excellent agreement and the PCE surrogate models are shown to be accurate approximations of the true models.

1. Introduction

Aeroelastic simulations form a cornerstone in the design and certification process of wind turbines. Comprehensive models covering the interaction of aerodynamics, structural dynamics and controllers are needed to understand and optimize the characteristics of new turbine designs. Due to their interdisciplinary nature, these models tend to cover a large number of parameters, whose interaction makes it often difficult to identify driving factors for a specific behavior. Yet, understanding exactly this interaction and the influence each parameter has on investigated wind turbine characteristics, can have significant importance in the design process. The analysis of uncertain parameter interaction and the influence of these parameters on model outcomes is covered by Uncertainty Quantification (UQ). Uncertainty quantification on wind turbine aeroelastic simulations is an active research topic. A literature overview is given by van den Bos et al. [1]. Example research topics are the influence of parametric uncertainties on wind turbine aerodynamics and loads [2, 3, 4, 5, 6] and uncertainties in blade flutter [7, 8, 9].

One of the main hurdles which has to be overcome, is the inherent significant computational cost of aeroelastic wind turbine simulations. This requires dedicated methods to allow for the investigation of multiple uncertain parameters simultaneously, while also accounting for their interaction. Promising on this aspect are meta model approaches, which approximate the true model with a cost efficient surrogate, retaining the critical information on parameter interaction and sensitivity. These methods are particularly efficient for systems with a small amount of uncertain parameters. In this case, the computational effort compared to standard UQ methods can be reduced by multiple orders of magnitude [10]. Extensive work has been done on the development of these methods by Sudret et al. [10, 11]. These meta model based uncertainty quantification methods have been applied to different research questions on wind turbine loads by multiple authors [12, 13, 14, 15, 16, 17, 18] and recently also on the topic of blade flutter by Li et al. [19].

Most of these methods are non-intrusive, meaning that the UQ methodology can be implemented without modification of the investigated model. This has the primary benefit that the considered models can usually be exchanged with ease. This independence of the model explains the usefulness of open-source development of these methodologies to enhance the cross-disciplinary knowledge exchange. Multiple open-source libraries have been developed over the last years [20, 21, 22]. However, some wind turbine specific functionalities were not open-source available before. Uncertain parameters are often distributed along the blades, which requires a parameterization of the uncertainty in this dimension. Furthermore, the interfaces towards different tools can be generalized to ease the implementation of further tools.

The goal of this research is to develop a framework for uncertainty quantification on aeroelastic wind turbine simulations taking into account limitations due to the high computational cost and additional requirements due to the radius resolved uncertain parameters. A further goal is to establish the framework in a modular and generic way which facilitates its usage for other simulation tools and for other uncertain parameters or quantities of interest. To this end, the full python code of the *wtuq* framework has been made open-source accessible [23]. In this paper, and as part of the open-source framework, the influence of the edgewise, flapwise and torsional stiffness of the blades on the aeroelastic stability of the turbine is used exemplarily. Stability analysis of wind turbines is a prime example of a time-consuming simulation with complex, multi-parameter models.

This paper has two main parts. Part one will introduce the uncertainty quantification framework in all its theoretical and practical aspects. Part two will exemplify the usage of the framework for the investigation of aeroelastic stability of wind turbines.

2. Uncertainty Quantification Framework

The uncertainty quantification framework is built around the open-source *uncertainpy* package [20]. This package provides methods for global, variance based uncertainty quantification based on surrogate models. How and why these methods and this package have been selected will be briefly explained in this section, followed by an explanation of the radially resolved input parameterization and an overview of the practical implementation of the open-source uncertainty quantification framework *wtuq* [23].

2.1. Variance based uncertainty quantification

The implemented framework bases on non-intrusive, global, variance-based uncertainty quantification. The meaning of this classification is briefly clarified in the following.

Intrusive UQ methods are introduced on the equation level in the models themselves, while non-intrusive methods can be seen as a wrapper around the actual model. Intrusive methods are often more efficient, because they do not require an expensive sampling of the investigated model. However, the required implementation modifications are a significant overhead, which hinders widespread application.

Another distinction are local vs. global approaches. Local methods, commonly analyzed by means of partial derivatives, describe the sensitivity around a fixed reference point. Global approaches quantify the uncertainty over the full input uncertainty distributions. On top of that, certain efficient global methods, such as Elementary Effects (EE), capture the full input distribution, but vary input parameters independently, which neglects possible interactions between parameters. Variance-based UQ methods capture both the full input distribution domain and possible interactions between parameters. They attempt to decompose the variance of the output Quantity of Interest (QoI) in contributions by each of the input parameters individually and contributions due to interactions between these parameters. The full mathematical derivation is not included here and can be found in section 2 in [10] or in section 4.1 in [13]. To understand the outcomes of the uncertainty quantification, following mathematical concepts are introduced. Assuming that the variance of the true model Y can be decomposed in

$$\mathrm{Var}(Y) = \sum_{i=1}^{d} V_i + \sum_{1\leq i<j\leq d} V_{ij} + ... \tag{1}$$

where d is the number of uncertain parameters. This relation shows the decomposition of the total variance $\mathrm{Var}(Y)$ in first order terms V_i, attributable to each input, and higher order terms V_{ij}, representing the interactions. Note that this series will be further expanded depending on the total amount of uncertain parameters. Derived variance-based sensitivity measures are the Sobol indices. The First Order Sobol index represents the isolated contribution of each uncertain parameter on the total variance. It is defined for each uncertain parameter as

$$S_i = \frac{V_i}{\mathrm{Var}(Y)}, \text{ where } i \in [1, ..., d]. \tag{2}$$

The Total Sobol index represents the contribution of each uncertain parameter to the total variance, including contributions due to parameter interactions. It is defined as

$$S_{\mathrm{T}i} = \frac{V_i + \sum_{j=1}^{d} V_{ij} + ...}{\mathrm{Var}(Y)}, \text{ where } i \in [1, ..., d]. \tag{3}$$

2.2. Surrogate model

The most well-known non-intrusive, global UQ methodology is the Monte-Carlo method. Its methodology bases on a dense sampling of the input uncertainty distributions to get a statistically meaningful description of the model output. The required sampling density is one of the main obstacles in non-intrusive UQ. Monte-Carlo simulation is computationally inefficient, limiting its direct usage to fast models. The essential aspect of *uncertainpy*, which makes it suitable for cost-intensive models, is the introduction of a surrogate model. The stochastic uncertainty quantification is done on a computationally efficient Polynomial Chaos Expansion (PCE) surrogate model. A detailed description of this approach can be found in [20] and [10]. A short summary is given here. PCE approximates the true model by a sum of orthogonal polynomials with unknown coefficients. Samples of the true model are used as training data for the surrogate model. An overdetermined system of equations is established, which is solved for the optimal polynomial coefficients in a least-squares sense. The orthogonality of the basis polynomials with respect to the input uncertainty distributions assures an optimal convergence of the approximation error with the order of the polynomials. The shape of the polynomial basis is fully defined by the orthogonality requirement with respect to the input uncertainty distributions. A higher polynomial order will reduce the approximation error, but will also increase the number of unknown coefficients and the corresponding required number of training data samples. The truncation order and other aspects of the experimental design, such as the sampling method and the number of samples, rely on user-defined settings. Adaptive methodologies which update the experimental design iteratively based on the approximation error or other performance metrics of the surrogate model could be a possible future improvement [24].

A verification of the surrogate model is needed to guarantee that it is an accurate representation of the true model. A leave-one-out test is applied to assure cross-validation. This test is done by computing a new surrogate model at each of the training data coordinates, with exclusion of said training data node. These leave-one-out surrogate models are evaluated at the same coordinates and compared with the excluded training data samples. This error measure has two main benefits. No additional model evaluations are needed for the verification and oversampling effects are taken into account, which would not be the case if the surrogate model is directly compared with its training data samples.

2.3. Input parameterization

Two approaches to handle the uncertainty distribution in radial direction along the blades are implemented in the framework. Figure 1 visualizes both approaches. Methodology one is shown on the left. This method is similar to the work by Kumar et al. [13]. Multiple control points are distributed in radial direction. Each of these points can be an uncertain parameter or can have a fixed value. A Non-Uniform Rational Basis Splines (NURBS) interpolation is used to get a smooth function in radial direction. Three exemplary random samples are shown and the influence they would have on a structural parameter is shown in the bottom left plot. The second method serves two purposes. It aids the generation of individually defined distributions and it reduces the total number of uncertain parameters. A fixed distribution can be given as input, and the uncertain parameter only describes the amplitude of the uncertainty. The right-hand side plots in figure 1 visualize this. The upper and lower bounds of the user-defined distributions are marked in black, three random samples are picked between these bounds. The lower right-hand side plot shows their influence on an exemplary structural parameter.

Each parameterization could be used for a different research question. The NURBS curves approach has multiple uncertain parameters along the span, which could be used for studies where the sensitivity of a specific parameter along the span is in question. The user-defined distributions are useful if the uncertainty distribution of a specific parameter along the span is known, for example through experiments or by prior knowledge.

Each uncertain parameter individually can be represented by means of a uniform distribution, described by its minimum and maximum value, or a normal distribution, described by its mean and standard deviation. The implementation of other distributions only requires minor modifications and will be undertaken in the future.

2.4. Framework overview

A generalized overview of the uncertainty quantification framework is shown in figure 2. The box on the left is governed by the open-source *uncertainpy* and *chaospy* python modules [20, 21]. The box on the right-hand side is specific with respect to the particular model and research question. A preprocessor is needed which understands the definition of the uncertain parameters and can translate them into specific inputs for the employed simulation tool. The postprocessor takes simulation results as input and extracts the predefined Quantity of Interest (QoI). An adequate number of model samples covering the full uncertainty range is used to establish the surrogate model. This computationally efficient model can then be used to determine all statistical characteristics through Monte-Carlo sampling. Note that this methodology can also be applied in case of multiple Quantities of Interest (QoIs). This will require the (automatic) setup of multiple surrogate models, but does not require more model evaluations if all QoIs can

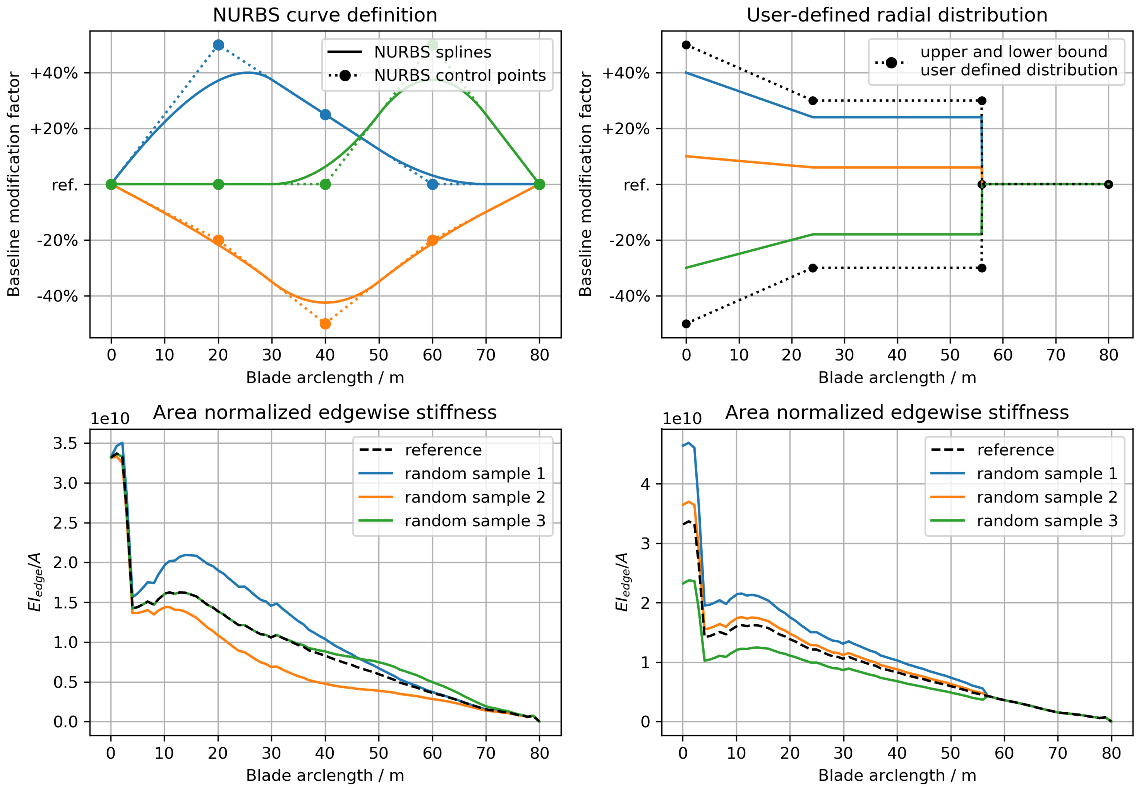


Figure 1: Example of NURBS curves parameterization (left) and user-defined distribution (right)

be extracted from the same simulations. The level of detail in this flowchart is intentionally low, illustrating the suitability of the framework to a range of different models and research questions. Uncertain parameters which could be considered in future investigations are further beam properties, e.g. shear center and elastic center positions, aerodynamic model properties, e.g. polar lift slope and maximum lift coefficient. Additional to these numerical model properties, physical quantities such as spar positioning, laminate angles, surface roughness or blade erosion due to uncertainty in manufacturing defects or lifetime depreciation could be highly interesting. Uncertainties in the wind field or other non-blade components will not be distributed along the blade, but can also be covered by the framework. Among others, interesting QoIs can be the modal quantities of critical modes, flutter speed, ultimate or damage equivalent loads, tower clearance or the annual energy production.

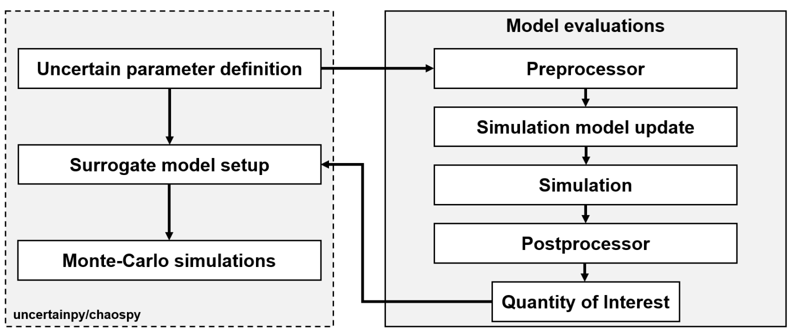


Figure 2: Generalized framework overview

3. Case Study: Aeroelastic Wind Turbine Stability

The capabilities of the framework are demonstrated by an exemplary case study. The reference model is based on the open-source IWT-7.5-164 turbine [25]. The blade stiffness was reduced significantly to establish an instability under nominal operating conditions [26]. The analysis is limited to a single, critical operating condition of the turbine at a steady, uniform wind speed of 12 m/s, a pitch angle of 0° and a rotational speed of 10 rpm. Symmetrical conditions are enforced by neglecting gravitational loads, tilt, wind shear or veer and tower effects on the wind field. An edgewise instability formed with negative damping for both the 1st and 2nd backward whirling modes. A detailed description of this reference instability condition is given in [26]. The damping ratio of the most negatively damped mode is used as single QoI. The flapwise, edgewise and torsional stiffness are used as uncertain parameters, this will be detailed in section 3.2. The models for this case study are available as use case in the open-source framework.

3.1. Wind turbine stability analysis

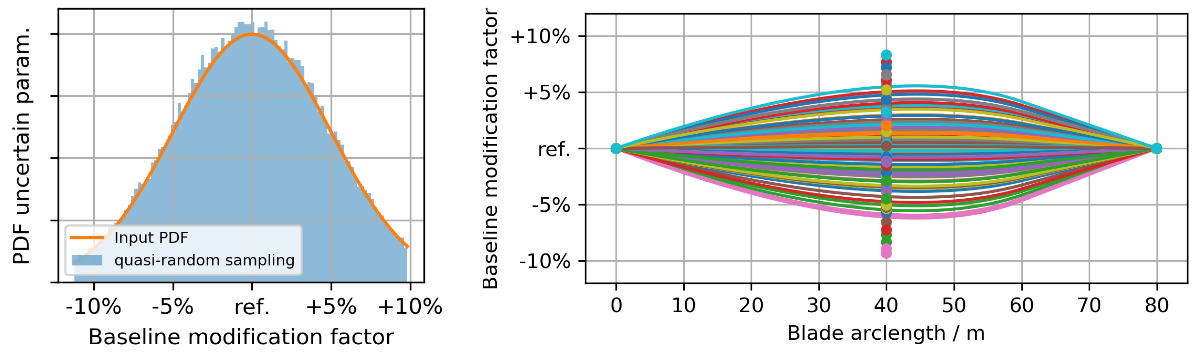
The aeroelastic analysis is done with the commercial simulation code Bladed, version 4.9. The main model components to point out are the blades, tower and aerodynamics. The tower is modeled as a single, modal reduced body with 12 enabled modes. Each blade is modeled as a multi-segmented structure with 6 individual sub-bodies, each sub-body contains 12 modes. The Beddoes-Leishmann dynamic stall model and the Øye dynamic wake model are enabled. The fixed operating point eliminates the need for an active control system.

Two methods are applied for the stability analysis. The standard Bladed linearization functionality linearizes the model around the non-linear reference condition. The eigenvalue analysis of the linearization matrix gives a modal decomposition of the system with a frequency and damping for each aeroelastic mode. The QoI is therefore a direct simulation output.

The second method uses the standard Bladed time domain simulation functionality. Postprocessing of the time signals is required to determine the damping of the most critical mode. Multiple simplified methods were tested unsuccessfully. Determining the logarithmic damping between subsequent oscillation peaks of the signals or exponential curve fitting to original data both assume the system to only have a single degree of freedom. These methods could therefore only produce accurate results if the full system was completely dominated by the critical mode, which was not always the case. Instead, Dynamic Mode Decomposition (DMD) was used to overcome this problem. The main idea of this methodology is to fit a linear operator to the actual non-linear system. The eigenvalues of this linear operator can be interpreted analogously to those of an actual linearization of the system. A detailed explanation of the theoretical basis of DMD and a detailed analysis of the DMD results on the non-linear aeroelastic time signals is out of the scope of this study. The framework uses the Higher-Order DMD implementation in the open-source *pyDMD* package [27]. Le Clainche et al. [28] serve as reference for the theoretical background.

3.2. Uncertainty Quantification

Three structural beam properties are defined as uncertain parameters, the flapwise, edgewise and torsional stiffness. Each of them is described by the same normal distribution around their respective reference values with a standard deviation of 5%. The probability density function (PDF) of this input distribution is given by the orange curve in figure 3a. The parameterization along the blade is made with the NURBS curve methodology presented in section 2.3. The applied NURBS curve has two fixed control points at the root and tip of the blade and a variable control point in the middle. Each physical parameter has therefore only one uncertain parameter. A snapshot of the random samples is shown in figure 3b. These input distributions are chosen as academic example and are not meant to represent any real physical deviations that could occur in a wind turbine blade.



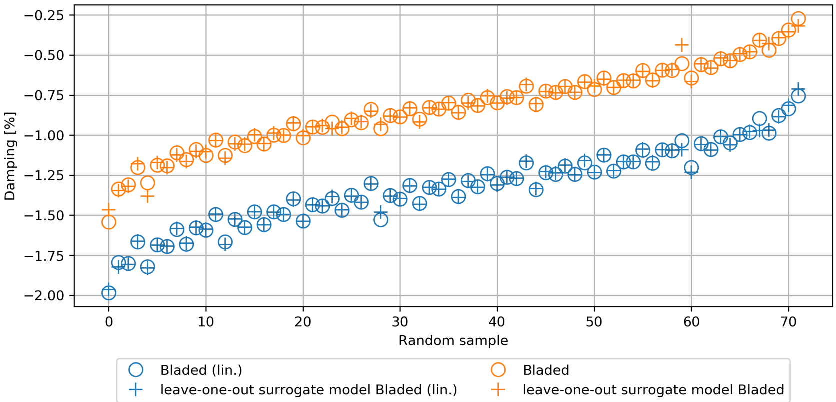
(a) Input uncertain param. distribution (b) Radial parameterization of the uncertain parameters

Figure 3: Definition of the input uncertain parameters

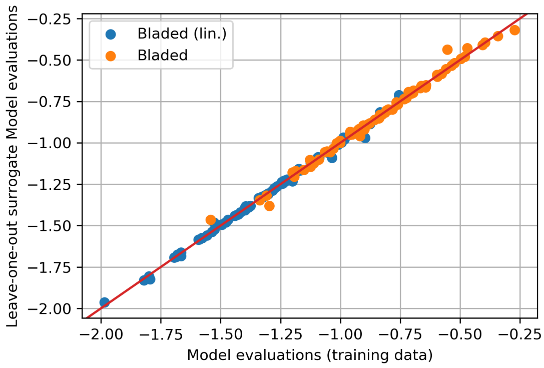
The design of experiments of the surrogate model was identical for both tools and fixed from the outset. A fourth order polynomial is used, applying the quasi-random, minimal discrepancy Hammersley sampling method. According to the best-practice study by Hosder et al. [29], the input distribution space was oversampled by a factor of 2, resulting in a total of 72 random samples. These settings were verified by trial-and-error, the accuracy of the surrogate model could not be improved significantly by increasing its order or the number of samples. As mentioned before, an adaptive design of experiments, as e. g. presented by Lüthen et al., will be a meaningful extension of the UQ framework in the future [24].

The damping ratio training samples for both tools are shown by the circular markers in figure 4a. For clarity, some important remarks have to be made. The Hammersley sampling method is quasi-random. Training data coordinates are chosen to minimize the discrepancy. In this implementation, the torsional stiffness uncertain parameter varies linearly from its minimum value at random sample node 0, up to its maximum value at random sample node 71. This results in a clear 'trend' in the damping values, rather than completely random results. The samples are identical between the two tools, e.g., the simulation at node 0 is in both tools done with the exact same set of input parameters.

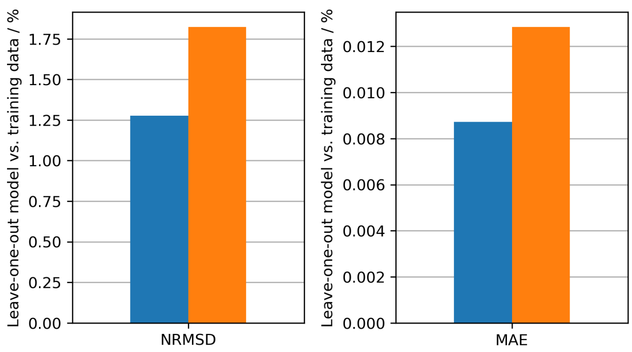
There is a constant offset of approximately 0.5 % between the Bladed linearization damping and the Bladed time domain damping, independent of the uncertain parameters. This discrepancy is also present for simulations under nominal uncertain parameter conditions. The reason for this offset is not yet fully understood and considered beyond the scope of the article and subject of future investigations. The spread of the damping values for the random samples around the nominal values shows an almost identical trend for both tools. This is a first indication that the three investigated uncertain parameters have an identical influence on the QoI for both methodologies. In the same plot, the evaluations of the leave-one-out surrogate models are visualized by the + markers. An excellent agreement can be seen for most nodes. This verifies the approximation of the true model by the PCE surrogate model. Another visualization of the same training data samples and leave-one-out model verification data is shown in figure 4b. The markers would be on the diagonal red line for an exact approximation of the training data by the leave-one-out model. This figure highlights again the excellent agreement. One further finding is that especially for the Bladed time domain model, the approximation quality seems to deteriorate at the edges of its domain, i. e., for the lowest and highest damping values. At these values, the input parameters are likely at their respective bounds too. As discussed in section 2.2, the leave-one-out test is done by evaluating the surrogate model at the training data point excluded from the training dataset. The Hammersley sampling method will therefore



(a) 72 training data samples and leave-one-out model evaluations



(b) Leave-one-out surrogate model evaluations vs. training data



(c) NRMSD and MAE error metrics for the leave-one-out surrogate model

Figure 4: Verification of the PCE surrogate model by a comparison of the damping training data samples and the leave-one-out evaluations

locally lose its favorable low discrepancy characteristic. This will result in an exaggerated error in the leave-one-out test compared to the actual quality of the surrogate model. It is therefore likely that these small errors at the edges of the domain are rather an effect of the leave-one-out verification methodology. Finally, the total approximation error of the leave-one-out test can also be condensed into an error metric value. Following two metrics are defined, namely the normalized root-mean-square deviation (NRMSD), and the Mean Absolute Error (MAE), which are given by

$$\mathrm{NRMSD} = \frac{\sqrt{\frac{\sum \left(\widehat{Z} - Z\right)^2}{n}}}{\max Z - \min Z}, \tag{4}$$

$$\mathrm{MAE} = \frac{\sum \left|\widehat{Z} - Z\right|}{n}, \tag{5}$$

where Z is the set of damping values of the true model, $\widehat{Z}$ is the set of approximated damping values of the leave-one-out surrogate model, and n is the number of samples. The Mean Absolute Error (MAE) has the benefit that it describes the error in the same unit as the samples. This makes it easier to interpret the severity of a specific error. The Normalised Root-Mean-Square

Deviation (NRMSD) uses a normalization of the error by the spread of the samples. For this case study, both error metrics show the accuracy of the PCE model with a relative NRMSD below 2% and an MAE close to 0.01% damping, as visualized in figure 4c. The approximation error of the time domain model is slightly larger compared to the linearized model.

The main result of the variance based uncertainty quantification are the Sobol indices, as described in section 2.1. The first order and total Sobol indices for both tools are visualized in figure 5. Independent of the tool, the torsional stiffness has the dominant influence on the unstable mode. This agrees with engineering judgement. This also explains the clear trend in figure 4a. The first order and total Sobol indices are almost identical. This signifies that there is no interaction between the uncertain parameters. E.g., the influence of the torsional stiffness on the damping is independent of the edgewise or flapwise stiffness.

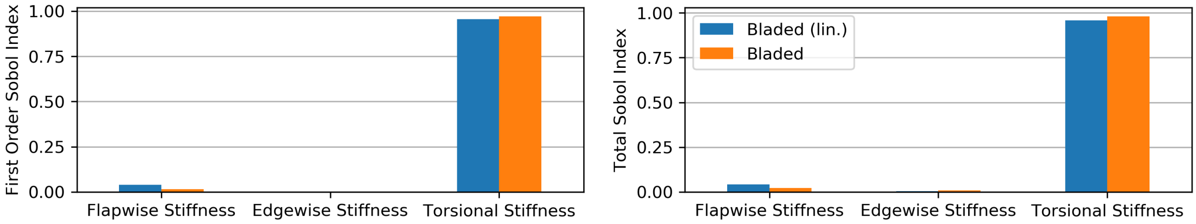


Figure 5: First Order (left) and Total (right) Sobol indices

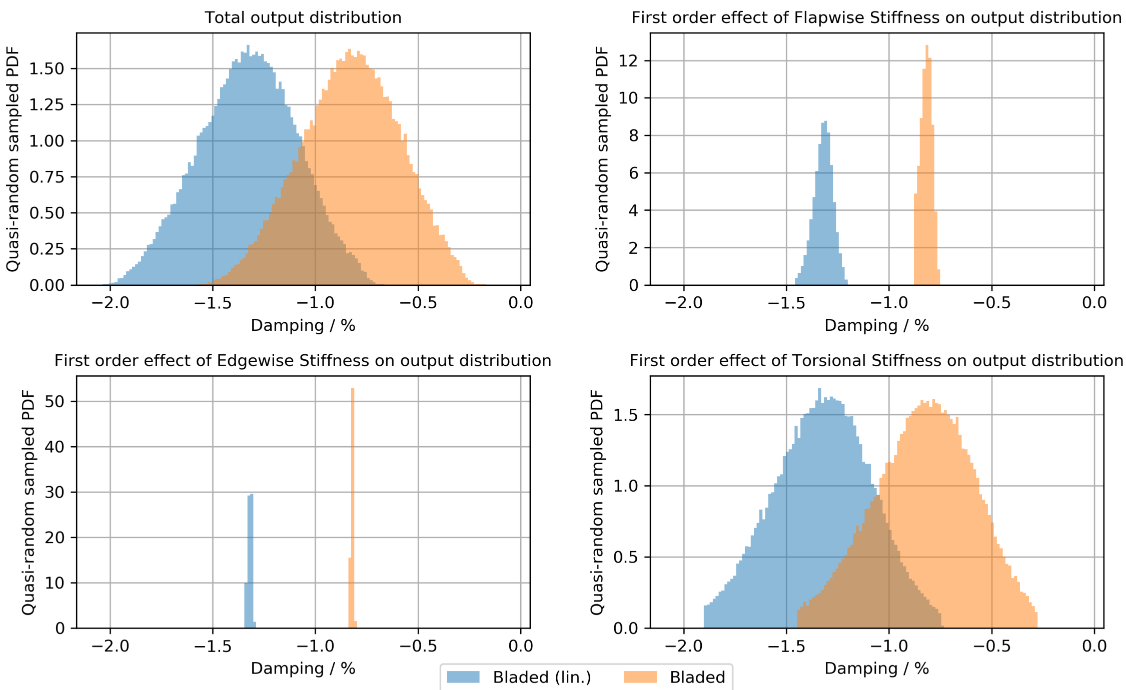


Figure 6: Total output distribution and individual first order effects on output distribution

The PCE model is resampled 100,000 times to obtain further statistical properties, as shown in figure 6. The top left plot shows the total output distributions. The offset of the mean value is again significant, but the shapes of the normal distribution are in excellent agreement between the two tools. The resampled mean for Bladed time domain is at a damping ratio of -0.825 %, with a standard deviation of 0.263 % damping. The resampled mean for Bladed (lin.) is at a damping ratio of -1.325 %, with a standard deviation of 0.261 % damping. The three remaining plots show the first order effects of each parameter individually on the output distribution. This is a visual representation of the conclusions drawn from the Sobol indices. The torsional stiffness has the dominant participation in the variance of the output parameter. In comparison, the flapwise and edgewise stiffness only have a negligible influence.

4. Conclusion

An open-source Python framework is presented with the goal to establish an interface between wind turbine aeroelastic simulation codes and publicly available uncertainty quantification tools. The non-intrusive, global, variance-based uncertainty quantification with a PCE surrogate model is done in *uncertainpy*. The framework can be applied to various research questions with different quantities of interest and different sets of uncertain parameters. An exemplary case study on the influence of structural blade properties on stability analysis results in Bladed has been used to demonstrate the requirements and outcomes of the framework. The case study showed that the uncertainty quantification on a linearized model and non-linear time domain simulation are in excellent agreement. The PCE surrogate model was successfully verified as approximation of the true model and the observed sensitivities agree with the engineering judgment expectations.

Acknowledgments

This work is a collaboration of three partners from research and industry in the frame of the German national research project *QuexUS*. This project is funded by the German Federal Ministry for Economic Affairs and Climate Action, grant no. 03EE3011A/B.

References

[1] van den Bos L M M and Sanderse B 2017 Uncertainty quantification for wind energy applications - Literature review Tech. Rep. SC-1701 Centrum Wiskunde & Informatica
[2] Rinker J M 2020 (*J. Phys.: Conf. Ser* vol 1618) (Delft, The Netherlands: TORQUE)
[3] Roberson A N, Shaler K, Sethuraman L and Jonkman J 2019 *Wind Energy Science* **4** 479–513 ISSN 2366-7443
[4] Fluck M and Crawford C 2016 (*J. Phys.: Conf. Ser* vol 753) (Munich, Germany: TORQUE)
[5] Ernst B, Schmitt H and Seume J R 2014 (*J. Phys.: Conf. Ser* vol 555) (Copenhagen, Denmark: TORQUE)
[6] Ziegler L and Muskulus M 2016 (*J. Phys.: Conf. Ser.* vol 753) (Munich, Germany: TORQUE)
[7] Lobitz D W 2005 *Journal of Solar Energy Engineering* **127** 538–543 ISSN 0199-6231
[8] Resor B and Paquette J 2011 *52nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference* (Denver, Colorado: American Institute of Aeronautics and Astronautics)
[9] Pourazarm P, Modarres-Sadeghi Y and Lackner M t
[10] Sudret B 2008 *Reliability Engineering and System Safety* **93** 964–979
[11] Le Gratiet L, Marelli S and B S 2017 *Metamodel-Based Sensitivity Analysis: Polynomial Chaos Expansions and Gaussian Processes* pp 1289–1325
[12] Caboni M, Carrion M, Rodriguez C, Schepers G, Boorsma K and Sanderse B 2020 (*J. Phys.: Conf. Ser.* vol 1618) (Delft, The Netherlands: TORQUE)
[13] Kumar P, Sanderse B, Boorsma K and Caboni M 2020 (*J. Phys.: Conf. Ser* vol 1618) (Delft, The Netherlands: TORQUE)
[14] Hirvoas A, Prieur C, Arnaud E, Caleyron F and Munoz Zuniga M 2021 *International Journal for Numerical Methods in Engineering* **122** 2528–2544 ISSN 1097-0207
[15] Hübler C, Gebhardt C G and Rolfes R 2017 *Renewable Energy* **111** 878–891 ISSN 0960-1481
[16] Hübler C, Gebhardt C G and Rolfes R 2019 29th European Safety and Reliability Conference (Hannover, Germany: ESREL)
[17] Müller F, Krabbe P, Hübler C and Rolfes R 2021 The 31st International Ocean and Polar Engineering Conference (Rhodes, Greece)
[18] Rinker J 2016 (*J. Phys.: Conf. Ser.* vol 753) (Munich, Germany: TORQUE)
[19] Li S and Caracoglia L 2019 *J. Phys.: Conf. Ser* **188** 43–60 ISSN 0167-6105
[20] Tennøe S, Halnes G and Einevoll G T 2018 *Frontiers in Neuroinformatics* **12** ISSN 1662-5196

[21] Feinberg J and Langtangen H P 2015 *Journal of Computational Science* **11** 46–57 ISSN 1877-7503

[22] Marelli S and Sudret B 2014 The 2nd International Conference on Vulnerability and Risk Analysis and Management (ICVRAM 2014) (University of Liverpool, United Kingdom) pp 2554–2563

[23] wtuq, Wind Turbine Uncertainty Quantification. `https://doi.org/10.5281/zenodo.6401223`

[24] Lüthen N and Sudret B 2019 *Book of Abstracts of the 90th Annual Meeting of the International Association of Applied Mathematics and Mechanics (GAMM 2019)* (Vienna, Austria: TU Verlag)

[25] Popko W *et al.* 2018 IWES Wind Turbine IWT-7.5-164. Rev 4 Tech. rep. Fraunhofer IWES Bremerhaven, Germany https://gitlab.cc-asp.fraunhofer.de/iwt/iwt-7.5-164, GPL v3

[26] Verdonck H, Hach O, Braun O, Polman J D, Balzani C, Müller S and Rieke J 2021 Code-to-code comparison of realistic wind turbine instability phenomena Presented at the Wind Energy Science Conference (WESC), `https://doi.org/10.5281/zenodo.5874658`

[27] Demo N, Tezzele M and Rozza G 2018 *The Journal of Open Source Software* **3** 530

[28] Le Clainche S and Vega J 2017 *SIAM Journal on Applied Dynamical Systems* **16** 882–925

[29] Hosder S, Walters R and Balch M April 23–26, 2007 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference (Honolulu, Hawaii: AIAA)