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Quantum limit of different laser power stabilization schemes involving optical resonators

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Abstract. Three different laser power stabilization schemes are compared: a traditional power stabilization, a traditional one with subsequent optical resonator, and a power stabilization with the novel optical ac coupling technique. The performance of the schemes is evaluated using the theoretical quantum limit and the power stability achieved considering technical limitations. The scheme with optical ac coupling is superior to the other ones especially at high laser power levels that will be used in future interferometric gravitational wave detectors.

1. Introduction

Laser power stabilization is important in many high precision experiments. In particular gravitational wave detectors (GWDs) require one of the most demanding power stabilities for the laser beam to be injected into the interferometer at frequencies between a few hertz and a few kilohertz. Commonly photodetectors measuring the power fluctuations of a beam sample are used in a feedback control loop to suppress the fluctuations by means of appropriate actuators. The quantum limit of this traditional power stabilization scheme is at least 6 dB above the relative quantum power noise of the original beam [1]. In contrast the quantum limits of stabilization schemes involving optical resonators [2] are significantly closer than 6 dB to this quantum noise, and therefore a better power stability can be achieved.

In this article we derive the quantum limit of three different active power stabilization schemes: a traditional power stabilization, a traditional power stabilization with subsequent noise filtering by a resonator, and a power stabilization utilizing the optical ac coupling technique [2]. The performance of these schemes is compared with respect to their theoretical quantum limit and by evaluating the achievable stability in two representative examples at the 10 mW and 100 W power level taking technical limitations into account.

The power stabilization with optical ac coupling achieves the best stability for frequencies above the bandwidth of the resonator required, especially when considering technical limitations that arise, among others, from the limited power detection capability of the photodetectors. At low laser power, the performance difference to the traditional scheme is small and in some cases the additional experimental complexity due to the optical ac coupling might not be justified. However, at the high laser power levels that will be used, e.g., in future GWDs, the optical ac coupling scheme applied, e.g., at the power recycling cavity is significantly superior.

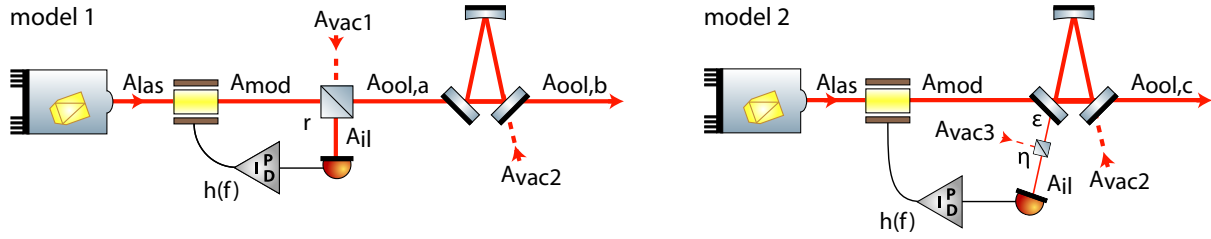


Figure 1. Models used to calculate the quantum noise limit of three different power stabilization schemes. Model 1 (left) showing the schemes A and B with OOL fields $A_{ool,a}$ and $A_{ool,b}$, model 2 showing scheme C with OOL field $A_{ool,c}$.

2. Theory

Two models are used to calculate the quantum limit of three power stabilization schemes (Fig. 1). The field A_{il} , called in-loop (IL) field, is used for the control loop sensor in order to compensate power fluctuations of the laser field A_{las} with an actuator. The required feedback loop is characterized by the frequency dependent loop gain $h(f)$. The fields $A_{ool,a}$ to $A_{ool,c}$ are the out-of-loop (OOL) fields of the three schemes A to C and are relevant for the actual control loop performance and for potential downstream experiments. The power reflectivity of the beam splitter r , the power transmission of the attenuator η in front of the IL detector in model 2, and the impedance matching of the resonator ϵ (power fraction ϵ^2 is reflected at the resonator) are important model parameters. Vacuum fluctuations A_{vac} couple into the system at open ports of the beam splitters and the resonator. Scheme A is a traditional power stabilization, scheme B is the same, but with subsequent resonator in the OOL beam, and scheme C is a stabilization with optical ac coupling [2].

The quantum limit of the stabilization schemes can be defined in terms of the relative power noise of the OOL beam. To determine this power noise, the spectral variance of the quantum mechanical photon number operator N for the different fields $A_{ool,a}$ to $A_{ool,c}$ has to be calculated, as described in more detail, e.g., in [3]. We assume that all but the vacuum fields have a large average amplitude α and only small fluctuations $\delta A(t)$, so that N can be expressed with linearized raising and lowering operators, A^\dagger and A :

$$A^\dagger(t) = \alpha + \delta A^\dagger(t), \quad A(t) = \alpha + \delta A(t), \quad N = A^\dagger \cdot A \approx \alpha^2 + \alpha \cdot (\delta A^\dagger + \delta A). \quad (1)$$

The equations for the operators $\delta A_{ool,a}$ to $\delta A_{ool,c}$ are deduced using the classical field equations and the canonical quantization. Assuming an optical resonator of high finesse, the following equations are yielded for model 1:

$$\begin{aligned} \delta A_{mod} &= \delta A_{las} + h(f)\delta A_{il}, & \delta A_{il} &= \sqrt{r} \delta A_{mod} + \sqrt{1-r} \delta A_{vac1}, \\ \delta A_{ool,a} &= \sqrt{1-r} \delta A_{mod} - \sqrt{r} \delta A_{vac1}, & \delta A_{ool,b} &= \frac{if\delta A_{vac2} + \delta A_{ool,a}}{1-if}, \end{aligned} \quad (2)$$

where f is the Fourier frequency in units of the resonator bandwidth. With the control loop closed, the first two equations can be combined to the steady state solution

$$\delta A_{mod} = \frac{\delta A_{las} + \sqrt{1-r} h(f) \delta A_{vac1}}{1 - \sqrt{r} h(f)}. \quad (3)$$

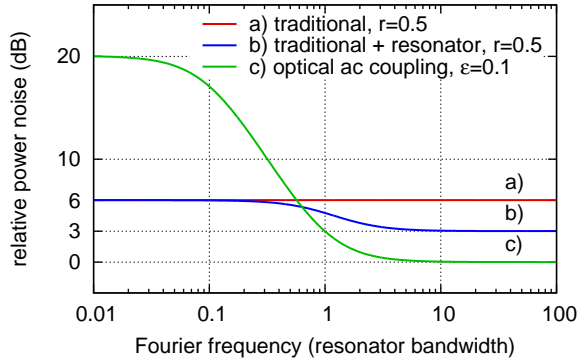


Figure 2. Quantum limit for the stabilization schemes. The limits are given in relation to the relative quantum noise of the original beam.

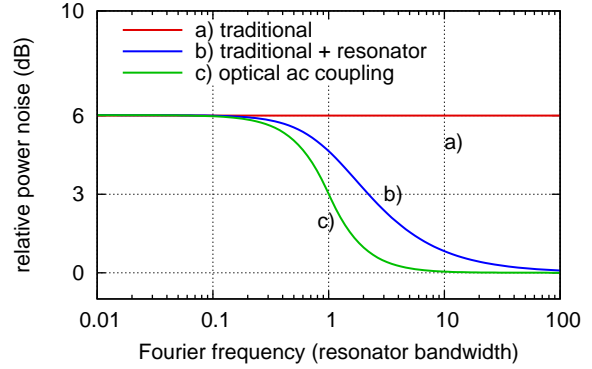


Figure 3. Minimal quantum limit for the three stabilization schemes, where the beam splitter reflectivity and the impedance matching were optimized for each Fourier frequency.

The double-sided power spectrum of the relative power fluctuations, P_s^2 , is the spectral variance of N normalized with $\langle N \rangle^2$ where $\langle N \rangle$ is its expectation value, which is the average photon flow α^2 :

$$P_{s,ool,a}^2 = \frac{\text{Var}(N_{ool,a})}{\langle N_{ool,a} \rangle^2} = \frac{\text{Var}(\delta A_{ool,a}^\dagger + \delta A_{ool,a})}{\langle N_{ool,a} \rangle} = \frac{(1-r)\text{Var}(\delta A_{las}^\dagger + \delta A_{las}) + |h(f) - \sqrt{r}|^2}{\langle N_{ool,a} \rangle \cdot |1 - \sqrt{r}h(f)|^2}$$

$$\stackrel{h \rightarrow \infty}{=} \frac{1}{\langle N_{ool,a} \rangle \cdot r} = \frac{1}{r \cdot (1-r)\alpha_{las}^2}, \quad (4)$$

$$P_{s,ool,b}^2 \stackrel{h \rightarrow \infty}{=} \frac{r f^2 + 1}{r(1-r)(f^2 + 1)\alpha_{las}^2}, \quad (5)$$

where α_{las}^2 is the average photon flow of the original laser beam. Furthermore, $\text{Var}(\delta A_{vac}^\dagger + \delta A_{vac}) = 1$ due to the commutation rule $[A, A^\dagger] = 1$ (see e.g. [3]). A high loop-gain $h \rightarrow \infty$ is assumed, which is the case in most power stabilization experiments. The single-sided linear spectral density $\sqrt{2 \cdot P_s^2}$ is often used in experiments and is used later-on in the case study.

In general the quantum noise of the original laser beam $P_s^2 = 1/\alpha_{las}^2$ is the fundamental limit for all schemes. In scheme A the relative power noise of the OOL beam (Fig. 2 a) does not depend on the laser noise δA_{las} due to the high-gain feedback loop and is frequency independent. The noise is minimal $P_{s,min,a}^2 = 4/\alpha_{las}^2$ for $r=0.5$ (Fig. 3 a) and thus is 6 dB above the quantum noise of the original beam ($1/\alpha_{las}^2$) or 3 dB above its own quantum noise ($2/\alpha_{las}^2$).

In scheme B the resonator has a high transmission at low frequencies and thus the resonator filtering effect is insignificant. The power noise is equal to the noise of a traditional stabilization $P_{s,ool,b}^2(f \rightarrow 0) = P_{s,ool,a}^2$. At high frequencies the reflectivity of the resonator and hence its filtering effect for noise at point $A_{ool,a}$ increases. However, the reflectivity for the field $A_{vac,2}$ increases as well such that the power noise is at these frequencies equal to the quantum noise of the OOL beam $1/[(1-r)\alpha_{las}^2]$, which in general is above the quantum noise of the original beam (Fig. 2 b).

The optimal reflectivity of the beam splitter $r_{\text{opt}}(f)$ and the minimal power noise $P_{\text{s,min,b}}^2(f)$ now depends on the frequency and is given by (Fig. 3 b)

$$r_{\text{opt}}(f) = \frac{\sqrt{f^2 + 1} - 1}{f^2}, \quad P_{\text{s,min,b}}^2(f) = \frac{f^4}{(f^2 + 1) \left(\sqrt{f^2 + 1} - 1\right)^2 \alpha_{\text{las}}^2}. \quad (6)$$

It should be noted that this minimal power noise cannot be achieved in the whole frequency band simultaneously since to our knowledge such a frequency dependent beam splitter does not exist. At low frequencies $r_{\text{opt}}(f \rightarrow 0) = 0.5$ and at high frequencies $r_{\text{opt}}(f \rightarrow \infty) \rightarrow 0$ yield the best stability.

The power noise of the OOL beam in model 2 is calculated in the same way as in model 1. The initial equations for model 2 are given by

$$\begin{aligned} \delta A_{\text{mod}} &= \delta A_{\text{las}} + h(f)\delta A_{\text{il}}, & \delta A_{\text{ool,c}} &= \frac{(-\epsilon + if)\delta A_{\text{vac2}} + \sqrt{1 - \epsilon^2} \delta A_{\text{mod}}}{1 - if}, \\ \delta A_{\text{il}} &= \frac{(\epsilon + if)\delta A_{\text{mod}} + \sqrt{1 - \epsilon^2} \delta A_{\text{vac2}}}{1 - if} \cdot \sqrt{\eta} + \sqrt{1 - \eta} \delta A_{\text{vac3}}. \end{aligned} \quad (7)$$

The IL detector is optical ac coupled and the OOL beam is filtered in addition by the resonator. The power spectrum of the relative power noise of the OOL beam is given by

$$P_{\text{s,ool,c}}^2 = \frac{1 + f^2 + (1 - \epsilon^2)(1 - \eta)/\eta}{\epsilon^2 + f^2} \frac{1}{(1 - \epsilon^2)\alpha_{\text{las}}^2}. \quad (8)$$

The attenuation controlled by η in front of the IL detector is integrated in the model only to be able to take technical limitations into account later on. Thus for the rest of this section we assume $\eta=1$. For low frequencies the resonator has a high transmission and the imprinted noise of A_{il} is dominant at the OOL field $A_{\text{ool,c}}$. For these frequencies the stabilization is comparable to a traditional power stabilization with a beam splitter reflection of $r=\epsilon^2$. The factor ϵ is determined by the impedance matching of the resonator. For small impedance mis-matches the OOL noise $P_{\text{s,ool,c}}^2(\epsilon \ll 1) \approx 1/\epsilon^2 \alpha_{\text{las}}^2$ is about a factor of $1/\epsilon^2$ above the quantum noise of the original beam A_{las} .

The reflection of the resonator increases for higher frequencies causing two effects: On the one hand the noise imprinted by the control loop onto A_{mod} is reduced. On the other hand the vacuum fluctuations A_{vac2} are mainly reflected by the resonator and dominate the power noise of $A_{\text{ool,c}}$. For $f \gg 1$ the relative power noise of $A_{\text{ool,c}}$ is only a factor of $1/(1 - \epsilon^2)$ above the quantum noise of the original beam $P_{\text{s,ool,c}}^2(f \rightarrow \infty) = 1/[(1 - \epsilon^2)\alpha_{\text{las}}^2]$. The power noise $P_{\text{s,ool,c}}^2$ is shown in Fig. 2 c for $\epsilon=0.1$.

The quantum limit of scheme C could in principle be optimized for a specific frequency by altering the impedance matching ϵ . The minimal noise $P_{\text{s,min,c}}^2$ is reached for (Fig. 3 c)

$$\epsilon(f) = \begin{cases} \sqrt{\frac{1 - f^2}{2}} & \text{for } f < 1 \\ \rightarrow 0 & \text{for } f \geq 1 \end{cases}, \quad P_{\text{s,min,c}}^2(f) = \begin{cases} \frac{4}{1 + f^2} \frac{1}{\alpha_{\text{las}}^2} & \text{for } f < 1 \\ \frac{1 + f^2}{f^2} \frac{1}{\alpha_{\text{las}}^2} & \text{for } f \geq 1 \end{cases}. \quad (9)$$

At low frequencies an impedance matching of $\epsilon^2=0.5$ is ideal. In this case and at these frequencies the resonator acts like a 50:50 beam splitter, which is as well optimal in the traditional power stabilization scheme. In contrast a nearly perfectly impedance matched resonator yields the best

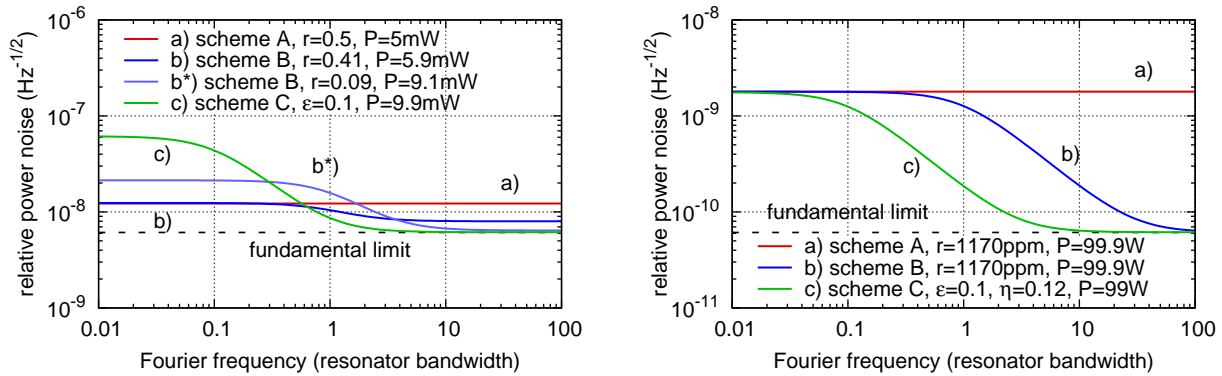


Figure 4. Out-of-loop achievable power noise for the different power stabilization schemes for laser beams of 10 mW (left) and 100 W (right). The model parameters chosen and the OOL available power are given.

stability at high frequencies and the minimal noise approaches the quantum noise of the original beam $P_{s,\min,c}^2(f \rightarrow \infty) = 1/\alpha_{\text{las}}^2$.

All in all scheme C involving the optical ac coupling is superior compared to the other schemes at frequencies $f > 1$ (Fig. 2 c). However, at low frequencies the traditional power stabilization scheme A and its modification, scheme B, achieve the lowest power noise.

A comparison of the minimal power noise (Fig. 3) shows that for a fixed Fourier frequency, the quantum limit of scheme C is at or below the quantum limit of the other schemes. As already mentioned this minimal power noise cannot be reached in the whole frequency band for scheme B and C since to our knowledge no beam splitter or resonator exists that allows to tune their reflectivity and impedance matching, respectively, with the required frequency dependence.

3. Case Study

So far the performance of the different power stabilization schemes were compared and evaluated using only the theoretical quantum limit. However, in real experiments several technical limitations have to be considered as well: The maximum photocurrent that can be detected with quantum-noise-limited noise performance at frequencies of interest for GWs is about 100 mA. This corresponds to a beam power of 117 mW at 1064 nm wavelength and photodiode quantum efficiency of 1. For frequencies above 10 Hz this is a typical value in experiments [4, 5, 6]. Furthermore, in current experiments the impedance matching has to be $\epsilon \geq 0.1$ so that additional technical noise sources caused by non-resonant modes do not limit the achievable stability with optical ac coupling [2]. This corresponds to a maximal carrier reduction down to $\epsilon^2 = 1\%$. In the following two realistic cases are described, where beams are stabilized for 10 mW ($\alpha_{\text{las}}^2 = 5.4 \times 10^{16}$ Hz) and 100 W ($\alpha_{\text{las}}^2 = 5.4 \times 10^{20}$ Hz).

The linear relative quantum noise of the laser beam with 10 mW power is 6.1×10^{-9} Hz^{-1/2}. The quantum limits of the three schemes are shown in Fig. 4. For scheme A $r=0.5$ is chosen and scheme B is optimized once for $f=1$ and once for $f=10$. The lowest quantum limit in scheme C is achieved for $\epsilon \rightarrow 0$ for frequencies $f \geq 1$. However, to account for technical limitations a realistic impedance matching of $\epsilon=0.1$ was chosen, but the power noise still gets as close as 0.04 dB to the fundamental quantum limit.

At low frequencies $f < 1$ the traditional power stabilization scheme A achieves the best stability, whereas at high frequencies $f > 1$ the optical ac coupling scheme C achieves a better one. Scheme B gives comparable results to scheme C at very high frequencies $f > 10$. All in all the OOL power noise differs only by at most 6 dB at high frequencies and thus the achieved

stabilities are close to each other. However the 5 mW beam power, available for downstream experiments, is in scheme A significantly lower compared to 9.9 mW in scheme C. If a power noise difference of 6 dB and a beam power difference of a factor of up to 2 is acceptable for the downstream experiment, a traditional power stabilization should be used since it is less complex and no optical resonator is needed.

The linear relative quantum noise of a 100 W beam is $6.1 \times 10^{-11} \text{ Hz}^{-1/2}$. Due to the technical limit of the IL photodetector in scheme A and B a reflectivity of $r=1.17 \times 10^{-3}$ has to be chosen. Scheme C is limited by the photodetector and as well by the impedance matching. Since even with $\epsilon=0.1$ the power on the IL detector is too high, the power needs to be attenuated with $\eta=0.12$ (see Fig. 1, model 2). The quantum limits of the three schemes are shown in Fig. 4, and in all cases most of the laser power is available in the OOL beam (99 W to 99.9 W).

The differences between the schemes are more significant at the high laser power of 100 W. The best stability is achieved with scheme C. Scheme B is at most frequencies a factor of 10 worse, but reaches the same power noise as scheme C at very low and very high frequencies. In the whole frequency band the traditional power stabilization achieves the worst performance. At this laser power level the higher experimental complexity of a power stabilization with optical ac coupling compared to a traditional stabilization might well be justified by the requirement of the downstream experiment.

4. Discussion

The quantum limits of three different power stabilization schemes are calculated, compared, and evaluated. The optimal stabilization parameters are determined for two different laser powers, 10 mW and 100 W, taking technical limitations into account.

In conclusion, the power stabilization scheme with optical ac coupling can achieve a better quantum limited performance compared to the other schemes, especially compared to the traditional scheme, at frequencies above the resonator bandwidth.

Ground-based GWDs will require even higher laser powers than a few 100 W in the future in order to improve the high-frequency sensitivity limited by shot-noise. With increasing laser power, the power stability requirements will become more stringent as well. Especially at these laser power levels, a stabilization with optical ac coupling has important advantages in contrast to traditional stabilization schemes.

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References

- [1] Taubman M S, Wiseman H, McClelland D E and Bachor H A 1995 *J. Opt. Soc. Am. B* **12** 1792–1800
- [2] Kwee P, Willke B and Danzmann K 2009 *Appl. Opt.* **48** 5423–5431
- [3] Bachor H A and Ralph T C 2004 *A Guide to Experiments in Quantum Optics* (WILEY-VCH) chap 8.3
- [4] Rollins J, Ottaway D, Zucker M, Weiss R and Abbott R 2004 *Opt. Lett.* **29** 1876–1878
- [5] Seifert F, Kwee P, Heurs M, Willke B and Danzmann K 2006 *Opt. Lett.* **31** 2000–2002
- [6] Kwee P, Willke B and Danzmann K 2009 *Opt. Lett.* **34** 2912–2914