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# On *hvLif*-like solutions in gauged Supergravity

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**Abstract** We perform a thorough investigation of Lifshitz-like metrics with hyperscaling violation (*hvLif*) in four-dimensional theories of gravity coupled to an arbitrary number of scalars and vector fields, obtaining new solutions, electric, magnetic, and dyonic, that include the known ones as particular cases. After establishing some general results on the properties of purely *hvLif* solutions, we apply the previous formalism to the case of $\mathcal{N}=2,\ d=4$ supergravity in the presence of Fayet–Iliopoulos terms, obtaining particular solutions to the $t^3$-model, and explicitly embedding some of them in Type-IIB string theory.

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## 1 Introduction

Gauge/gravity duality has been shown to be an instrumental tool to study strongly coupled systems near critical points where the system displays a scaling symmetry. Generically, conformal field theories provide consistent descriptions of certain physical systems near critical points. In the gauge/gravity avatar this means that the gravitational theory is living on a background which is asymptotically locally anti De Sitter (*aDS*). On the other hand, in many physical systems critical points are dictated by dynamical scalings in which, even though the system exhibits a scaling symmetry, space and time scale differently under this symmetry. A prototype example of such critical points is a hyperscaling-violating Lifshitz fixed point where the system is spatially isotropic and scale covariant, though there is an anisotropic scaling in the time direction characterized by a dynamical exponent, $z$, and hyperscaling violation characterized by the exponent $\theta$. More precisely the system is covariant under the following scale symmetry [1–23]:

$$x_i \to \lambda x_i\,, \quad t \to \lambda^z t\,, \quad r \to \lambda r\,, \quad \mathrm{d}s^2_{d+2} \to \lambda^{2\theta/d}\mathrm{d}s_{d+2}\,, \tag{1.1}$$

where $\lambda$ is a dimensionless parameter and $d$ is the number of spatial dimensions on which the dual theory lives ($i=1,\ldots,d$). The value $\theta=0$ corresponds to the standard scale-invariant theories dual to Lifshitz metric [24–29]. The values $z=1$ and $\theta=0$ correspond to conformally invariant theories dual to gravity theories on an *aDS* background. For other values of $z$ and $\theta$, the $d+2$-dimensional gravitational backgrounds are supported by metrics of the form

$$\mathrm{d}^2_{d+2} = \ell^2 r^{-2(d-\theta)/d}\left(r^{-2(z-1)}\mathrm{d}t^2 - \mathrm{d}r^2 - \mathrm{d}x_i\mathrm{d}x_i\right)\,, \tag{1.2}$$

where $\ell$ is the Lifshitz radius. As usual, we will refer to these metrics as hyperscaling-violating Lifshitz (*hvLif*) metrics.

The Lifshitz-type spacetimes are known to be singular in the IR. They suffer from a null singularity with diverging tidal forces [30–42]. Just like Lifshitz spacetimes, *hvLif* metrics are zero-temperature gravity solutions thought to represent a class of quantum critical points characterized by the two parameters $z$ and $\theta$ [4,6]. For holographic related applications, see [43–54].

It is interesting to obtain new gravitational solutions that may be used as duals of the corresponding field theories, if any. A first step on this direction, for $\mathcal{N}=2$, $d=4$ ungauged supergravity was taken in [55], where a complete analysis on the existence of such kind of solutions was performed. In this note we extend the systematic study to a general class of gravity theories coupled to scalars and vectors, up to two derivatives, in the presence of a scalar potential, in principle arbitrary, focusing later on $\mathcal{N}=2$, $d=4$ supergravity in the presence of Fayet–Iliopoulos terms.

The structure of the paper goes as follows: in Sect. 2 we dimensionally reduce the general action of gravity coupled to an arbitrary number of scalars and vectors in the presence of a scalar potential assuming a general static background which naturally fits the anisotropic scaling properties which correspond to *hvLif*-like solutions. In Sect. 3 we adapt the general formalism to the Einstein–Maxwell–Dilaton system. In Sect. 4 we focus on $\mathcal{N}=2$, $d=4$ supergravity in the presence of Fayet–Iliopoulos terms (which correspond to include a scalar potential in ungauged supergravity), were we exploit the symplectic structure of the theory in order to obtain further results. We also embed a particular truncation of the $t^3$-model in Type-IIB string theory compactified on a Sasaki–Einstein manifold times $S^1$. In Sect. 5 we perform an analysis of the properties of purely *hvLif* solutions for the general class of theories considered. In addition, we provide a general recip to obtain *hvLif*-like solutions of a particular class of Einstein–Maxwell–Dilaton systems, reducing the problem to the resolution of an algebraic equation. We apply the procedure to obtain explicit solutions, some of them embedded in string theory. In Sect. 6 we conclude.

## 2 The general theory

We are interested in Lifshitz-like solutions with hyperscaling violation (*hvLif*[1]) of the four-dimensional action

$$S=\int \mathrm{d}^4x\sqrt{|g|}\,\Big\{R+\mathcal{G}_{ij}\partial_\mu\phi^i\partial^\mu\phi^j+2I_{\Lambda\Sigma}F^{\Lambda}{}_{\mu\nu}F^{\Sigma\,\mu\nu} -2R_{\Lambda\Sigma}F^{\Lambda}{}_{\mu\nu}\star F^{\Sigma\,\mu\nu}-V(\phi)\Big\}, \tag{2.1}$$

that generalizes the action considered in Refs. [56,57] by including a generic scalar potential $V(\phi)$. We will take care of the constraints imposed by $\mathcal{N}=2$ supersymmetry on the field content, the kinetic matrices $(I_{\Lambda\Sigma}(\phi)<0,\ R_{\Lambda\Sigma}(\phi))$, the scalar metric $\mathcal{G}_{ij}(\phi)$, and the scalar potential $V(\phi)$ later on.

The idea now is to dimensionally reduce the action (2.1) using an appropriate ansatz for the metric. Since *hvLif* solutions are in particular static, a first step is to constrain the form of the metric to be

$$\mathrm{d}s^2=e^{2U}\mathrm{d}t^2-e^{-2U}\gamma_{\underline{m}\,\underline{n}}\mathrm{d}x^{\underline{m}}\mathrm{d}x^{\underline{m}},\quad \underline{m},\underline{n}=1,\ldots,3, \tag{2.2}$$

A sensible choice for $\gamma$ that fits the anisotropic scaling properties that we look for in a *hvLif* solution, is given by

$$\gamma=\gamma_{\underline{m}\,\underline{n}}\mathrm{d}x^{\underline{m}}\mathrm{d}x^{\underline{m}}=e^{2W}\left(\mathrm{d}r^2+\delta_{ab}\mathrm{d}x^a\mathrm{d}x^b\right),\quad a,b=1,2, \tag{2.3}$$

where $e^W$ is an undetermined function of the "radial" coordinate $r$. We now proceed to dimensionally reduce the Lagrangian (2.1) with the choice of metric given by Eqs. (2.2) and (2.3).

Assuming that all the fields are static and only depend on $r$, it is possible to show that the only non-vanishing components of the 1-form electric and magnetic vector fields are the time ones, and that the corresponding Maxwell equations can be integrated. This allows to completely eliminate the vector fields from the effective one-dimensional action $(' = \frac{\mathrm{d}}{\mathrm{d}r})$,

$$S=\int \mathrm{d}r\,e^W\,\Big\{2U'^2-2W'^2+\mathcal{G}_{ij}\phi^{i\,\prime}\phi^{j\,\prime} -2e^{2(U-W)}V_{\mathrm{bh}}+e^{-2(U-W)}V\Big\}, \tag{2.4}$$

which is obtained, following the same steps as in [56,58][2], by performing a dimensional reduction over the radial coordinate.

All the information related to the vector fields[3] and their coupling to the scalars gets thus encoded in the so-called

[1] We will understand for *hvLif* any non-trivial gravitational solution that presents some kind of Lifshitz limit with hyperscaling violation. *Purely hvLif* stands for metrics that are exactly Lifshitz with hyperscaling violation.

[2] A related procedure, used to obtain non-extremal $aDS_4$ black-hole solutions can be found in [59] and [60]. For related refences as regards solutions in gauged supergravity see [61,62].

[3] The form of the vector fields can be recovered following the dimensional-reduction procedure. The corresponding field strengths $F^{\Lambda}_{\mu\nu}$ are given by

$$F^{\Lambda}_{\underline{m}t}=-\partial_{\underline{m}}\psi^{\Lambda},\qquad F^{\Lambda}_{\underline{mn}}=\frac{e^{-2U}}{\sqrt{|\gamma|}}\epsilon_{\underline{mn}\tau}\Big[\left(I^{-1}\right)^{\Lambda\Omega}\partial_\tau\chi_\Omega -\left(I^{-1}R\right)^{\Lambda}_{\ \Omega}\partial_\tau\psi^{\Omega}\Big], \tag{2.5}$$

where $\Psi=(\psi^{\Lambda},\chi_{\Lambda})^T$ is a symplectic vector whose components are the time components of the electric and magnetic vector fields, $A^{\Lambda}$ and $A_{\Lambda}$. $\Psi$ is given by

$$\Psi=\int\frac{1}{2}e^{2U}\mathcal{M}^{MN}\mathcal{Q}_N\mathrm{d}\tau. \tag{2.6}$$

*black-hole potential* $V_{\rm bh}$, which is defined as in the asymptotically flat case by[4] [56,58,63]

$$V_{\rm bh}(\phi, \mathcal{Q}) \equiv 2\alpha^2 \mathcal{M}_{MN}(\phi)\mathcal{Q}^M \mathcal{Q}^N, \tag{2.7}$$

where $\alpha$ is a normalization constant for the electric and magnetic charges[5]

$$\left(\mathcal{Q}^M\right) = \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix}, \tag{2.8}$$

and where the symmetric matrix $\mathcal{M}_{MN}$ is defined in terms of $I \equiv (I_{\Lambda\Sigma})$ and $R \equiv (R_{\Lambda\Sigma})$ by

$$(\mathcal{M}_{MN}) \equiv \begin{pmatrix} I + RI^{-1}R & -RI^{-1} \\ -I^{-1}R & I^{-1} \end{pmatrix}. \tag{2.9}$$

The effective action Eq. (2.4) must be complemented by the *Hamiltonian constraint*, associated to the lack of explicit $r$-dependence of the Lagrangian:

$$2U'^2 - 2W'^2 + \mathcal{G}_{ij}\phi^{i\prime}\phi^{j\prime} + 2e^{2(U-W)}V_{\rm bh} - e^{-2(U-W)}V = 0. \tag{2.10}$$

The one-dimensional effective equations of motion are given by

$$e^{-W}\left[e^W U'\right]' + e^{2(U-W)}V_{\rm bh} + \frac{1}{2}e^{-2(U-W)}V = 0, \tag{2.11}$$

$$e^{-W}\left[e^W\right]'' + e^{-2(U-W)}V = 0, \tag{2.12}$$

$$e^{-W}\left[e^W \mathcal{G}_{ij}\phi^{j\prime}\right]' - \frac{1}{2}\partial_i \mathcal{G}_{jk}\phi^{j\prime}\phi^{k\prime} + e^{2(U-W)}\partial_i V_{\rm bh} - \frac{1}{2}e^{-2(U-W)}\partial_i V = 0, \tag{2.13}$$

to which we have to add the Hamiltonian constraint (2.10). The kinetic term for the scalars, as well as the scalar potential $V(\phi)$ and the black-hole potential $V_{\rm bh}(\phi, \mathcal{Q})$, can be solely expressed in terms of $U$ and $W$, i.e.,

$$\begin{aligned} V &= -e^{2U-2W}\left[W'' + W'^2\right], \\ V_{\rm bh}(\phi, \mathcal{Q}) &= -\frac{1}{2}e^{2W-2U}\left[2U'' + 2U'W' - W'' - W'^2\right], \\ \mathcal{G}_{ij}\phi^{i\prime}\phi^{j\prime} &= -2\left[-U'' - U'W' + U'^2 + W''\right]. \end{aligned} \tag{2.14}$$

Equations (2.14) are useful in order to obtain, given a particular metric, the behavior of $V(\phi)$ and $V_{\rm bh}(\phi, \mathcal{Q})$, or $\phi^i$ for models with small enough number of scalars, in terms of the coordinate $r$. These three quantities encode all the physical information of any solution (fitting in our ansatz for the fields) to the family of theories defined by Eq. (2.1).

### 2.1 Constant scalars: generalities

For constant scalars $\phi^i$, the potential $V(\phi)$ and the black-hole potential $V_{\rm bh}(\phi, \mathcal{Q})$ become constant quantities, the former playing the role of a cosmological constant and the latter of a *generalized* squared charge, magnetic and electric. In the case of constant scalars, Eq. (2.13) is not identically satisfied, but it becomes the following constraint:

$$e^{4(U-W)}\partial_i V_{\rm bh} = \frac{1}{2}\partial_i V. \tag{2.15}$$

We have two different options in order to fulfil Eq. (2.15).

**Constant scalars as *double* critical points:** $\partial_i V_{\rm bh} = 0$, $\partial_i V = 0$. Of course, the system of equations given by

$$\partial_i V_{\rm bh} = 0, \quad \partial_i V = 0, \tag{2.16}$$

is overdetermined. However, let us assume that a consistent solution to (2.16) exists and is given by

$$\phi^i = \phi^i_c(\mathcal{Q}, \phi_\infty), \tag{2.17}$$

*i.e.*, the values of the scalars are fixed in terms of the electric and magnetic charges, and we have included a dependence on $\phi_\infty$ to formally consider the existence of flat directions. We will see later on that, in fact, Eq. (2.16) occurs in $\mathcal{N} = 2$, $d = 4$ supergravity. The equations of motion reduce to

$$e^{-W}\left[e^W U'\right]' + e^{2(U-W)}V_{\rm bh} + \frac{1}{2}e^{-2(U-W)}V = 0, \tag{2.18}$$

$$e^{-W}\left[e^W\right]'' + e^{-2(U-W)}V = 0, \tag{2.19}$$

together with the Hamiltonian constraint

$$2U'^2 - 2W'^2 + 2e^{2(U-W)}V_{\rm bh} - e^{-2(U-W)}V = 0. \tag{2.20}$$

**Metric functions identified**: $e^U = \beta e^W$, $\beta \in \mathbb{R}^+$ **and** $2\beta^4\partial_i V_{\rm bh} = \partial_i V(\phi)$. In this case, the equations of motion imply

$$2\beta^4\partial_i V_{\rm bh} = \partial_i V, \quad 2\beta^4 V_{\rm bh} = V. \tag{2.21}$$

Assuming Eq. (2.21), there is a unique solution, which is precisely $aDS_2 \times \mathbb{R}^2$. Equations (2.21) can be understood as necessary and sufficient conditions for a gravity theory coupled to scalars and vector fields, up to two derivatives, to contain an $aDS_2 \times \mathbb{R}^2$ solution. Therefore, given a particular theory of such kind, with a specific potential $V(\phi)$ and black-hole potential $V_{\rm bh}(\phi)$, one only has to impose Eq. (2.21) in order to check the existence of an $aDS_2 \times \mathbb{R}^2$ solution. The parameter $\beta$ can always be found to be

$$\beta^4 = \frac{V}{2V_{\rm bh}}, \tag{2.22}$$

[4] It is important to stress that, in spite of its name, which is such because the dimensionally reduced actions in which it is often used are intended to build black-hole solutions, the black-hole potential should be understood as a very convenient generalization of a (negative) linear combination of squared electric and magnetic charges associated with the corresponding dimensionally reduced theory, regardless of whether the solution under consideration contains a black-hole spacetime or not.

[5] The canonical choice for $d = 4$ is $\alpha = \frac{1}{2}$.

and we are left with

$$\frac{1}{2}\partial_i \log V_{\rm bh} = \partial_i \log V. \tag{2.23}$$

Equation (2.23) is a system of $n_v$ equations for at least $n_v$ variables (the $n_v$ constant scalars), and hence in general it will be compatible and the theory will contain an $aDS_2 \times \mathbb{R}^2$ solution. Only in pathological cases the system (2.23) will be incompatible and the theory will fail to contain an $aDS_2 \times \mathbb{R}^2$ solution.

## 3 The Einstein–Maxwell–Dilaton model

Before we discuss the possible embeddings of Eq. (2.1) in gauged supergravity and string theory, let us consider the Einstein–Maxwell–Dilaton (EMD) system, whose action is characterized by the following choices, to be made in Eq. (2.1):

$$\begin{aligned} &F^{\Lambda\,\mu\nu} = F^{\mu\nu}, \quad I_{\Lambda\Sigma} = I = \frac{Z(\phi)}{2} < 0, \\ &R_{\Lambda\Sigma} = R = 0, \quad \phi^i = \phi, \quad G_{ij} = \frac{1}{2}. \end{aligned} \tag{3.1}$$

Hence, the EMD action reads

$$S_{\rm EMD} = \int \mathrm{d}^4x\sqrt{|g|}\left\{R+\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+Z(\phi)F^2-V(\phi)\right\}, \tag{3.2}$$

*i.e.*, we consider a single vector field and a single scalar field. Moreover, the coupling given by $R$ is taken to be zero, which greatly simplifies the black-hole potential $V_{\rm bh}(\phi, \mathcal{Q})$, which is therefore given by

$$V_{\rm bh}(\phi, \mathcal{Q}) = \frac{1}{4}\left[Z(\phi)p^2 + Z(\phi)^{-1}q^2\right], \tag{3.3}$$

where $q$ and $p$ are the electric and magnetic charges, respectively. The equations of motion take the form

$$\begin{aligned} &e^{-W}\left[e^W U'\right]' + e^{2(U-W)}\frac{1}{4}\left[Zp^2 + Z^{-1}q^2\right] \\ &\quad+\frac{1}{2}e^{-2(U-W)}V = 0, \end{aligned} \tag{3.4}$$

$$e^{-W}\left[e^W\right]'' + e^{-2(U-W)}V = 0, \tag{3.5}$$

$$\begin{aligned} &e^{-W}\left[e^W \phi'\right]' + e^{2(U-W)}\frac{\partial_\phi Z}{2}\left[p^2 - \frac{q^2}{Z^2}\right] \\ &\quad-e^{-2(U-W)}\partial_\phi V = 0, \end{aligned} \tag{3.6}$$

and the Hamiltonian constraint reads

$$\begin{aligned} &2U'^2 - 2W'^2 + \frac{1}{2}\phi'^2 + \frac{e^{2(U-W)}}{2}\left[Zp^2 + Z^{-1}q^2\right] \\ &\quad-e^{-2(U-W)}V = 0. \end{aligned} \tag{3.7}$$

For non-constant scalars, Eq. (3.6) is automatically satisfied if

$$V = -e^{2(U-W)}\left[W'^2 + W''\right], \tag{3.8}$$

$$\phi'^2 = 4\left[-U'^2 + U'W' + U'' - W''\right], \tag{3.9}$$

and $Z$ is such that

$$Z = \frac{1}{p^2}\left[\Upsilon \pm \sqrt{\Upsilon^2 - p^2q^2}\right], \text{ if } p,\ q \neq 0, \tag{3.10}$$

$$Z = \frac{2\Upsilon}{p^2} \quad \text{if } q = 0,\ p \neq 0 \tag{3.11}$$

$$Z = \frac{q^2}{2\Upsilon} \quad \text{if } p = 0, \tag{3.12}$$

where

$$\Upsilon = 2V_{\rm bh} = e^{2(W-U)}\left[-2U'W' + W'^2 - 2U'' + W''\right]. \tag{3.13}$$

Theories with conventional and sensible matter have to satisfy the null-energy condition (NEC) $n_\mu n_\nu T^{\mu\nu} \geq 0$, where $n_\mu$ is an arbitrary null vector and $T^{\mu\nu}$ is the correspondent energy-momentum tensor. This condition translates, for the EMD case, into the following constraints:

$$\Upsilon \leq 0,\ \phi'^2 \geq 0. \tag{3.14}$$

Hence, it is equivalent to the requirement of a semi-negative definite black-hole potential, and a semi-positive definite kinetic term for the scalar field, compatible with the condition $Z(\phi)$.

**Another coordinate system: $A - B - f$ coordinates.** There is another system of coordinates which we will use along this paper, and that will be useful for different purposes. It is related to the $U$–$W$ system of coordinates by the following identifications:

$$\begin{aligned} &\left(\frac{\mathrm{d}r}{d\tilde{r}}\right)^2 = f^{-1}(\tilde{r}), \quad e^{2U} = e^{2(A(\tilde{r})+B(\tilde{r}))}f(\tilde{r}), \\ &\quad e^{2W} = e^{4A(\tilde{r})+2B(\tilde{r})}f(\tilde{r}), \end{aligned} \tag{3.15}$$

giving rise to the metric

$$\mathrm{d}s_f^2 = \ell^2 e^{2A(\tilde{r})}\left[e^{2B(\tilde{r})}f(\tilde{r})\mathrm{d}t^2 - \frac{\mathrm{d}\tilde{r}^2}{f(\tilde{r})} - \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right], \tag{3.16}$$

which has proven to be useful (see e.g. [2,64]) in order to obtain solutions exhibiting *hvLif* asymptotics when $f(\tilde{r})$ is a function of $\tilde{r}$ that obeys

$$f(\tilde{r}_h) = 0,\ \ \tilde{r}_h \in \mathbb{R}^+ \qquad \lim_{\tilde{r}\to\tilde{r}_0} f(\tilde{r}) = 1. \tag{3.17}$$

The *hvLif* limit is, thus, assumed to be at $\tilde{r}_0$, whereas the horizon is at $\tilde{r}_h$. The equations of motion (3.4), (3.5) and

(3.6) can be rewritten accordingly as[6]

$$e^{-2A-B}\left[e^{2A+B}f\left[A'+B'+\frac{f'}{2f}\right]\right]' +e^{-2A}\frac{1}{4}\left[Zp^2+Z^{-1}q^2\right]+\frac{1}{2}e^{2A}V(\phi)=0 \tag{3.18}$$

$$e^{-2A-B}\left[f^{1/2}\left[e^{2A+B}f^{1/2}\right]'\right]'+e^{2A}V(\phi)=0 \tag{3.19}$$

$$e^{-2A-B}\left[e^{2A+B}f\phi'\right]'+e^{-2A}\frac{\partial_\phi Z}{2}\left[p^2-Z^{-2}q^2\right] -e^{2A}\partial_\phi V(\phi)=0, \tag{3.20}$$

where $' = \frac{\mathrm{d}}{\mathrm{d}\tilde{r}}$. The Hamiltonian constraint is given by

$$-2f\left[3A'^2+2A'\left[B'+\frac{f'}{2f}\right]\right]+\frac{f}{2}\phi'^2 +\frac{e^{-2A}}{2}\left[Zp^2+Z^{-1}q^2\right]-e^{2A}V(\phi)=0. \tag{3.21}$$

Again, for non-constant dilaton this set of equations is equivalent to[7]

$$V=\frac{e^{-2A}}{2}\Big[-3f'[2A'+B'] -2f\left[2A''+\left[2A'+B'\right]^2+B''\right]-f''\Big], \tag{3.22}$$

$$\phi'^2=4\left[-A''+A'B'+A'^2\right], \tag{3.23}$$

$$\Upsilon=-\frac{e^{2A}}{2}\Big[f'\left[2A'+3B'\right] +2f\left[2A'B'+B''+B'^2\right]+f''\Big]. \tag{3.24}$$

## 4 $\mathcal{N}=2$ Supergravity with FI terms

The action (2.1) has great generality and basically covers any possible theory of gravity coupled to Abelian vector fields and scalars up to two derivatives. However, in order to embed our results in string theory, it is convenient to focus on the bosonic sector of $\mathcal{N}=2,\ d=4$ supergravity, which is a particular case of (2.1). More precisely, we are going to consider gauged $\mathcal{N}=2,\ d=4$ in the presence of $n_v$ Abelian vector multiplets, where the gauge group is contained in the $R$-symmetry group of automorphisms of the supersymmetry algebra. Normally one refers to this theory as $\mathcal{N}=2,\ d=4$ supergravity with Fayet–Iliopoulos terms (from now on, $\mathcal{N}=2$ FI to abridge) [65]. The general Lagrangian of $\mathcal{N}=2$ FI is given by

$$S=\int \mathrm{d}^4x\sqrt{|g|}\Big\{R+2\mathcal{G}_{ij^*}\partial_\mu z^i\partial^\mu z^{*\,j^*} +2\Im\mathrm{m}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda\,\mu\nu}F^{\Sigma}{}_{\mu\nu}-2\Re\mathrm{e}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda\,\mu\nu\star}F^{\Sigma}{}_{\mu\nu} -V_{\rm fi}\left(z,z^*\right)\Big\}. \tag{4.1}$$

The indices $i,j,\ldots=1,\ldots,n_v$ run over the scalar fields and the indices $\Lambda,\Sigma,\ldots=0,\ldots,n_v$ over the 1-form fields. The scalar potential generated by the FI terms reads

$$V_{\rm fi}\left(z,z^*\right)=-3|\mathcal{Z}_g|^2+\mathcal{G}^{ij^*}\mathfrak{D}_i\mathcal{Z}_g\mathfrak{D}_{j^*}\mathcal{Z}^*_g, \quad \mathfrak{D}_i\mathcal{Z}_g=\partial_i\mathcal{Z}_g+\frac{1}{2}\partial_i\mathcal{K}\mathcal{Z}_g, \tag{4.2}$$

where $\mathcal{K}$ is the Kähler potential, $\mathcal{Z}_g$ is given by[8]

$$\mathcal{Z}_g\equiv\mathcal{Z}_g\left(z,z^*\right)=g_M\mathcal{V}^M=\mathcal{V}^Mg^N\Omega_{MN} =-g^\Lambda\mathcal{M}_\Lambda+g_\Lambda\mathcal{L}^\Lambda, \tag{4.3}$$

and the $g^M$ is a symplectic vector related to the embedding tensor $\theta_M$, which selects the combination of vectors that gauges $U(1)\subset R$-symmetry group, as follows[9]:

$$g_M=g\theta_M, \tag{4.4}$$

$g$ being the gauge coupling constant. The corresponding one-dimensional effective action and the Hamiltonian constraint are given, respectively, by

$$S=\int \mathrm{d}r\, e^W\left\{U'^2-W'^2+\mathcal{G}_{ij^*}z^{i\,\prime}z^{j^*\,\prime} -e^{2(U-W)}V_{\rm bh}+\frac{1}{2}e^{-2(U-W)}V_{\rm fi}\right\}, \tag{4.5}$$

$$U'^2-W'^2+\mathcal{G}_{ij^*}z^{i\,\prime}z^{j^*\,\prime}+e^{2(U-W)}V_{\rm bh}-\frac{1}{2}e^{-2(U-W)}V_{\rm fi}=0. \tag{4.6}$$

The black-hole potential takes the simple form

$$-V_{\rm bh}(z,z^*,\mathcal{Q})=|\mathcal{Z}|^2+\mathcal{G}^{ij^*}\mathfrak{D}_i\mathcal{Z}\mathfrak{D}_{j^*}\mathcal{Z}^*, \tag{4.7}$$

where

$$\mathcal{Z}=\mathcal{Z}(z,z^*,\mathcal{Q})\equiv\langle\mathcal{V}\mid\mathcal{Q}\rangle=-\mathcal{V}^M\mathcal{Q}^N\Omega_{MN} =p^\Lambda\mathcal{M}_\Lambda-q_\Lambda\mathcal{L}^\Lambda, \tag{4.8}$$

is the *central charge* of the theory.

**Constant scalars and supersymmetric attractors.** In Sect. (2) we studied the case of constant scalars in the general theory (2.1). We found that, besides the solution $aDS_2\times\mathbb{R}^2$, there was another possible solution, if Eq. (2.16) holds. We

[6] From now on, we will use always the symbol "$r$" to denote the "radial" coordinate, independently of which coordinate system we use, which will be specified by other means.

[7] Equations (3.10)–(3.12) hold.

[8] We assume the conventions of [66].

[9] Supergravity gaugings are originally electric, breaking therefore the symplectic covariance present in the ungauged case. The embedding tensor formalism allows to formally keep the theory simplectically covariant by introducing magnetic and electric gaugings.

will see now how this is always possible in $\mathcal{N}=2$ FI. The general theory of the attractor mechanism in ungauged $d=4$ supergravity proves that, for extremal black holes, the value of the scalars at the horizon is fixed in terms of the charges $\mathcal{Q}^M$, and given by the so-called *critical points* or *attractors*, *i.e.*, solutions to the system

$$\partial_i V_{\rm bh}\left(\mathcal{Q},\phi\right)_{|\phi_c}=0. \tag{4.9}$$

There might be some residual dependence in the value at infinity if the potential has flat directions. If the scalars are constant, they have to be given again by (4.9) in the extremal as well as in the non-extremal case. It can be proven that there is always a class of attractors, called supersymmetric, which obey

$$\partial_i\left|\mathcal{Z}\right|_{|\phi_c}=0,\quad\text{and}\quad \mathfrak{D}_i\mathcal{Z}_{|\phi_c}=0, \tag{4.10}$$

and therefore, given the definitions (4.2) and (4.7), they also obey (2.16) if $\mathcal{Q}^M\sim g^M$. Hence, setting the scalars to constant values given by the supersymmetric attractor points of the black hole potential is always a consistent truncation, provided that $g^M$ is identified with $\mathcal{Q}^M$, which besides fixes the value of the black-hole potential and the scalar potential exclusively in terms of the charges.

### 4.1 The $t^3$-model

In this section we consider a particular $\mathcal{N}=2$ FI model which can be embedded in string theory. In particular we start from Type-IIB string theory compactified on a Sasaki–Einstein manifold to five dimensions. This theory can be consistently truncated as to yield pure $\mathcal{N}=1$, $d=5$ supergravity with Fayet–Iliopoulos terms, which, due to the absence of scalars, introduce a cosmological constant. Further compactification on $S^1$ gives the desired four-dimensional theory, which is defined by [67–70]

$$\begin{aligned}&n_v=1,\quad F\left(\mathcal{X}\right)=-\frac{\left(\mathcal{X}^1\right)^3}{\mathcal{X}^0},\\ &g^0=g^1=g_0=0\Rightarrow V_{\rm fi}\left(t,t^*\right)=\frac{-\beta^2}{\Im{\rm m}t},\end{aligned} \tag{4.11}$$

where $\beta^2=g_1^2/3$, and we have defined the inhomogeneous coordinate on the Special Kähler manifold $\mathrm{SU}(1,1)/\mathrm{U}(1)$, by

$$t=\frac{\mathcal{X}^1}{\mathcal{X}^0}. \tag{4.12}$$

This theory is known as the $t^3$-model, and although the string theory embedding requires the gauging specified in Eq. (4.11), we are going to study it in full generality, particularizing only at the end.

The canonically normalized symplectic section $\mathcal{V}$ is, in a certain gauge,

$$\mathcal{V}=e^{\mathcal{K}/2}\begin{pmatrix}1\\ t\\ t^3\\ -3t^2\end{pmatrix}, \tag{4.13}$$

where the Kähler potential is

$$\mathcal{K}=-\log\left[i\left(t-t^*\right)^3\right]. \tag{4.14}$$

As a consequence, the Kähler metric reads

$$\mathcal{G}_{tt^*}=\frac{3}{4}\frac{1}{(\Im{\rm m}t)^2}, \tag{4.15}$$

and the central charge

$$\mathcal{Z}=\frac{p^0t^3-3t^2p^1-q_0-q_1t}{2\sqrt{2\Im{\rm m}t^3}}. \tag{4.16}$$

The period matrix $\mathcal{N}_{IJ}$ is, in turn, given by

$$\begin{aligned}{\rm Re}\,\mathcal{N}_{IJ}&=\begin{pmatrix}-2\Re^3 & 3\Re^2\\ 3\Re^2 & -6\Re\end{pmatrix},\\ {\rm Im}\,\mathcal{N}_{IJ}&=\begin{pmatrix}-(\Im^3+3\Re^2\Im) & 3\Re\Im\\ 3\Re\Im & -3\Im\end{pmatrix},\end{aligned} \tag{4.17}$$

where we use the notation $\Re\equiv\Re{\rm e}t$, $\Im\equiv\Im{\rm m}t$. The general expressions of $V_{\rm bh}$ and $V_{\rm fi}$, which can be obtained using Eqs. (4.2), (4.3), (4.7), and (4.8) read

$$\begin{aligned}V_{\rm bh}=&-\frac{1}{6\Im^3}\Big[3\Im^6p^{0\,2}+9\Im^4\left[p^1-p^0\Re\right]^2\\ &+\Im^2\left[q_1+6p^1\Re-3p^0\Re^2\right]^2\\ &+3\left[q_0+\Re\left[q_1+3p^1\Re-p^0\Re^2\right]\right]^2\Big],\end{aligned} \tag{4.18}$$

$$\begin{aligned}V_{\rm fi}=&-\frac{1}{3\Im}\Big[g_1^2+3g_1\left[g^1\Re+g^0\left[\Im^2+\Re^2\right]\right]\\ &+9\left[g_0\left[-g^1+g^0\Re\right]+g^{1\,2}\left[\Im^2+\Re^2\right]\right]\Big].\end{aligned} \tag{4.19}$$

Let us consider the truncation $\Re{\rm e}t=0$. In order to satisfy all the original equations of motion (those with $\Re{\rm e}t$ arbitrary) in such a case, we must impose the additional constraints

$$\partial_\Re V_{\rm bh}(\Re=0)=\partial_\Re V_{\rm fi}(\Re=0)=0. \tag{4.20}$$

These conditions explicitly read

$$3\Im p^0p^1-2\frac{p^1q_1}{\Im}-\frac{q_0q_1}{\Im^3}=0, \tag{4.21}$$

$$3g_0g^0+g_1g^1=0, \tag{4.22}$$

and are satisfied (without loss of generality in the functional form of the potentials) if we make

$$p^1=q_1=0;\quad g_0=g^1=0\vee g^0=g_1=0. \tag{4.23}$$

Thus, setting $\Re e t$ to zero in a consistent manner notably simplifies the expressions for the potentials

$$V_{\mathrm{bh}} = -\frac{1}{2}\left[\frac{q_0^2}{\Im^3} + {p^0}^2\Im^3\right], \tag{4.24}$$

$$V_{\mathrm{fi}}^{I} = -\left[\frac{g_1^2}{3\Im} + g_1 g^0 \Im\right],\ V_{\mathrm{fi}}^{II} = -\left[-\frac{3g_0 g^1}{\Im} + 3{g^1}^2\Im\right] \tag{4.25}$$

Making the redefinition $t \equiv \Re + ie^{-\frac{\phi}{\sqrt{3}}}$, the action is given by

$$S_{\Re=0}^{I} = \int \mathrm{d}^4x\sqrt{|g|}\left\{R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - 2e^{-\sqrt{3}\phi}\left(F^0\right)^2 + \frac{g_1^2}{3}e^{\frac{\phi}{\sqrt{3}}} + g_1 g^0 e^{-\frac{\phi}{\sqrt{3}}}\right\}, \tag{4.26}$$

$$S_{\Re=0}^{II} = \int \mathrm{d}^4x\sqrt{|g|}\left\{R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - 2e^{-\sqrt{3}\phi}\left(F^0\right)^2 - 3g_0 g^1 e^{\frac{\phi}{\sqrt{3}}} + 3{g^1}^2 e^{-\frac{\phi}{\sqrt{3}}}\right\}, \tag{4.27}$$

where we have already set $A^1_\mu$ to zero, in order to make the truncation consistent with the corresponding equation of motion.

**Embedding the $t^3$-model system in the EMD.** As it can be trivally verified, we have just obtained the action (3.2) with

$$Z(\phi) = -2e^{-\sqrt{3}\phi},\ \ q^2 = 4q_0^2,\ \ p^2 = {p^0}^2, \tag{4.28}$$

and the scalar potential of the EMD system (Eq. 5.31) given by

$$V(\phi) = c_1 e^{-\frac{\phi}{\sqrt{3}}} + c_2 e^{+\frac{\phi}{\sqrt{3}}};\ \ c_1^I = -g_1 g^0,\ \ c_1^{II} = -3{g^1}^2,\ \ c_2^I = -\frac{g_1^2}{3},\ \ c_2^{II} = 3g_0 g^1. \tag{4.29}$$

Hence, we find that our axion-free $t^3$-system with those particular choices of $Z$ and $V$ gets embedded in the EMD model and, for $g^0 = g^1 = g_0 = 0$, also in string theory in the way explained at the beginning of this section. In such a case, Eq. (4.26) clearly becomes

$$S_{ST} = \int \mathrm{d}^4x\sqrt{|g|}\left\{R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - 2e^{-\sqrt{3}\phi}\left(F^0\right)^2 + \frac{g_1^2}{3}e^{\frac{\phi}{\sqrt{3}}}\right\}. \tag{4.30}$$

## 5 *hvLif* solutions

In this section we are going to construct (purely and asymptotically) *hvLif* solutions to Eq. (2.1). After establishing some results on the properties of the solutions corresponding to the pure *hvLif* case in the general set-up of Eq. (2.14), we focus on the EMD system, obtaining the *hvLif* solutions allowed by the embedding of our *axion-free* supergravity model in this system. Then, we provide a recipe to construct asymptotically *hvLif* solutions to these theories in the presence of constant and non-constant dilaton fields, recovering and extending some of the results already present in the literature.

### 5.1 Purely *hvLif* solutions: general remarks

The *hvLif* metric in four dimensions, given by

$$\mathrm{d}s^2 = \ell^2 r^{\theta-2}\left(r^{-2(z-1)}\mathrm{d}t^2 - \mathrm{d}r^2 - \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right), \tag{5.1}$$

is recovered in our set-up for specific values of $U(r)$ and $W(r)$, namely

$$e^{2U(r)} = \ell^2 r^{\theta-2z},\quad e^{2W(r)} = \ell^4 r^{2(\theta-z-1)}. \tag{5.2}$$

For purely *hvLif* solutions, the equations of motion can be further simplified by direct substitution of (5.2)

$$(\theta-2z)(\theta-z-2) + 2r^{4-\theta}\ell^{-2}V_{\mathrm{bh}} + r^\theta\ell^2 V = 0 \tag{5.3}$$

$$(\theta-z-1)(\theta-z-2) + r^\theta\ell^2 V = 0 \tag{5.4}$$

$$r^{-2(\theta-z-1)}\left(r^{2(\theta-z-1)}\mathcal{G}_{ij}\phi^{j\,\prime}\right)' - \frac{1}{2}\partial_i\mathcal{G}_{jk}\phi^{j\,\prime}\phi^{k\,\prime} + r^{2-\theta}\ell^{-2}\partial_i V_{\mathrm{bh}} - \frac{1}{2}r^{\theta-2}\ell^2\partial_i V = 0. \tag{5.5}$$

The Hamiltonian constraint reads

$$(2-\theta)(3\theta-4z-2) + 2r^2\mathcal{G}_{ij}\phi^{i\,\prime}\phi^{j\,\prime} + 4r^{4-\theta}\ell^{-2}V_{\mathrm{bh}} - 2r^\theta\ell^2 V = 0. \tag{5.6}$$

Equation (2.14) can be also adapted to the purely *hvLif* case. We find

$$V = -\ell^{-2}\mathcal{X}_{(\theta,z)}r^{-\theta},\quad V_{\mathrm{bh}}(\phi,\mathcal{Q}) = \frac{1}{2}\ell^2\mathcal{Y}_{(\theta,z)}r^{\theta-4},\quad \mathcal{G}_{ij}\dot\phi^i\dot\phi^j = \frac{1}{2}\mathcal{W}_{(\theta,z)}r^{-2}, \tag{5.7}$$

where

$$\mathcal{X}_{(\theta,z)} = (\theta-z-2)(\theta-z-1), \tag{5.8}$$

$$\mathcal{Y}_{(\theta,z)} = (\theta-z-2)(z-1), \tag{5.9}$$

$$\mathcal{W}_{(\theta,z)} = (\theta-2)(\theta-2z+2). \tag{5.10}$$

Equations (5.3)–(5.6) are the general equations of motion that need to be solved in order to find a *hvLif* solution to any theory that belongs to the class defined by Eq. (2.1). Likewise, Eq. (5.7) provides the behavior of the black-hole potential and the scalar potential, in terms of the variable

$r$, for any *hvLif* solution consistent with the equations of motion. $\mathcal{G}_{ij}$ is positive-definite, therefore

$$\begin{aligned} &\mathcal{G}_{ij}\phi^{i\,\prime}\phi^{j\,\prime} \geq 0 \Leftrightarrow \mathcal{W}_{(\theta,z)} \geq 0, \\ &\mathcal{G}_{ij}\phi^{i\,\prime}\phi^{j\,\prime} = 0 \;\Leftrightarrow\; \phi^{i\,\prime} = 0\; \forall i, \end{aligned} \tag{5.11}$$

and hence we can establish the following result: all the scalar fields of any purely *hvLif* solution of any theory describable by Eq. (2.1) are constant *iff* $\theta = 2$, or $z = 1 + \theta/2$. In addition, $V_{\text{bh}}$ is, in our conventions, a negative definite function, hence $V_{\text{bh}} \leq 0 \Leftrightarrow \mathcal{Y}_{(\theta,z)} \leq 0$. These two conditions on the sign of $\mathcal{W}_{(\theta,z)}$ and $\mathcal{Y}_{(\theta,z)}$ are equivalent to imposing the null-energy condition (NEC) to our purely *hvLif* solutions, as we commented before, and they define a region of acceptable solutions in the $(\theta, z)$-plane, as we shall see.

It is possible to stablish some general results for the *hvLif* solutions of any theory describable by Eq. (2.1) attending to the vanishing of $V$, $V_{\text{bh}}$, and/or $\mathcal{G}_{ij}\dot{\phi}^i\dot{\phi}^j$. Let us proceed.

1. $\theta = 2$
   In this situation $\mathcal{G}_{ij}\dot{\phi}^i\dot{\phi}^j = 0$, and

$$V = -\ell^{-2} z(z-1) r^{-2}, \tag{5.12}$$

$$V_{\text{bh}} = -\frac{1}{2}\ell^{2} z(z-1) r^{-2}. \tag{5.13}$$

   The NEC imposes $z \in (-\infty, 0] \cup [1, +\infty)$, and we have the two special cases: $\theta = 2,\ z = 0$ (which corresponds to Rindler spacetime) and $\theta = 2,\ z = 1$ (which is Minkowski spacetime) for which $V = V_{\text{bh}} = 0$ as well.
2. $z = 1 + \frac{\theta}{2}$
   We have again $\mathcal{G}_{ij}\dot{\phi}^i\dot{\phi}^j = 0$, and

$$V = -\ell^{-2}\left(\frac{\theta}{2} - 3\right)\left(\frac{\theta}{2} - 2\right) r^{-\theta}, \tag{5.14}$$

$$V_{\text{bh}} = -\frac{1}{2}\ell^{2}\left(\frac{\theta}{2} - 3\right)\frac{\theta}{2} r^{\theta-4}. \tag{5.15}$$

   The NEC translates into $\theta \in [0, 6]$, and we have three more special cases: the Ricci flat one: $\theta = 6,\ z = 4$ corresponding to $V = V_{\text{bh}} = 0$ (this is a particular case of the general formalism developed in [55] for ungauged $\mathcal{N} = 2,\ d = 4$ supergravity); $\theta = 4,\ z = 3$, which corresponds to $V = 0$, $V_{\text{bh}} = -\ell^2$ (also in agreement with the results of [55]); and $\theta = 0,\ z = 1$, which is nothing but the $aDS_4$ spacetime in a conformally flat representation, and the only solution with vanishing black-hole potential, and constant (non-zero) scalar potential compatible with the equations: $V_{\text{bh}} = 0$, $V \equiv \Lambda = -\ell^{-2}6$.
3. $z = 1,\ \theta \neq 2,\ z \neq 1 + \frac{\theta}{2}$
   We have $V_{\text{bh}} = 0$, whereas

$$V = -\ell^{-2}(\theta - 3)(\theta - 2) r^{-\theta}, \tag{5.16}$$

$$\mathcal{G}_{ij}\dot{\phi}^i\dot{\phi}^j = \frac{1}{2}(\theta - 2)\theta\, r^{-2}. \tag{5.17}$$

   The NEC becomes now $\theta \in (-\infty, 0] \cup [2, \infty)$, and we have the limit case $\theta = 3,\ z = 1$, which will be a particular case of the family considered in the next paragraph.
4. $z = \theta - 2,\ \theta \neq 2,\ z \neq 1 + \frac{\theta}{2}$
   This situation imposes $V = V_{\text{bh}} = 0$, whereas

$$\mathcal{G}_{ij}\dot{\phi}^i\dot{\phi}^j = \frac{1}{2}(\theta - 2)(6 - \theta)\, r^{-2}. \tag{5.18}$$

   The NEC reads $\theta \in [2, 6]$. These will be solutions of the Einstein–Dilaton system for $\mathcal{G}_{ij} = \frac{1}{2}\delta_{ij},\ i = 1$, and

$$\phi = \phi_0 + \sqrt{(\theta - 2)(6 - \theta)}\,\log r. \tag{5.19}$$

5. $z = \theta - 1,\ \theta \neq 2,\ z \neq 1 + \frac{\theta}{2}$
   We have now $V = 0$, while

$$V_{\text{bh}} = -\frac{1}{2}\ell^{2}(\theta - 2)\frac{\theta}{2} r^{\theta-4}. \tag{5.20}$$

$$\mathcal{G}_{ij}\dot{\phi}^i\dot{\phi}^j = \frac{1}{2}(\theta - 2)(4 - \theta)\, r^{-2}, \tag{5.21}$$

   and the NEC becomes $\theta \in [2, 4]$.

Another particularly interesting case corresponds to the Einstein–Maxwell system with a cosmological constant: $\mathcal{G}_{ij}\dot{\phi}^i\dot{\phi}^j = 0$, $V \equiv \Lambda$. However, this could only be realized for $\theta = 0,\ z = 1$, which imposes the vanishing of $V_{\text{bh}}$. Hence, there is no purely *hvLif* solution (for non-vanishing vector fields) for such model.

### 5.1.1 Purely *hvLif* in the EMD

If we particularize now to the EMD system, we find

$$V = -\ell^{-2}\mathcal{X}_{(\theta,z)} r^{-\theta}, \tag{5.22}$$

$$\Upsilon = 2V_{\text{bh}} = \ell^{2}\mathcal{Y}_{(z,\theta)} r^{\theta-4}, \tag{5.23}$$

$$\phi = \phi_0 + \sqrt{\mathcal{W}_{(z,\theta)}}\,\log(r) \;\Rightarrow\; r = e^{\frac{\phi}{\sqrt{\mathcal{Z}}}}. \tag{5.24}$$

Therefore, $V$ and $V_{\text{bh}}$ written as functions of $\phi$, must take the form (Fig. 1)

$$V(\phi) = -\ell^{2}\mathcal{X} e^{-\frac{\theta\phi}{\sqrt{\mathcal{Z}}}}, \tag{5.25}$$

$$V_{\text{bh}}(\phi) = \frac{1}{2}\ell^{2}\mathcal{Y} e^{\frac{(\theta-4)\phi}{\sqrt{\mathcal{Z}}}}. \tag{5.26}$$

This means, on the one hand, that any EMD theory susceptible of containing *hvLif* solutions has a scalar potential which depends on $\phi$ through one single exponential [becoming a constant when $\theta = 0$, $\theta = 2$ or $z = 1 + \theta/2$ ($\phi = \phi_0$ in the last two cases)] [17]. On the other hand, the gauge coupling function is constant for $\theta = 4$, and again if $\phi = \phi_0$.

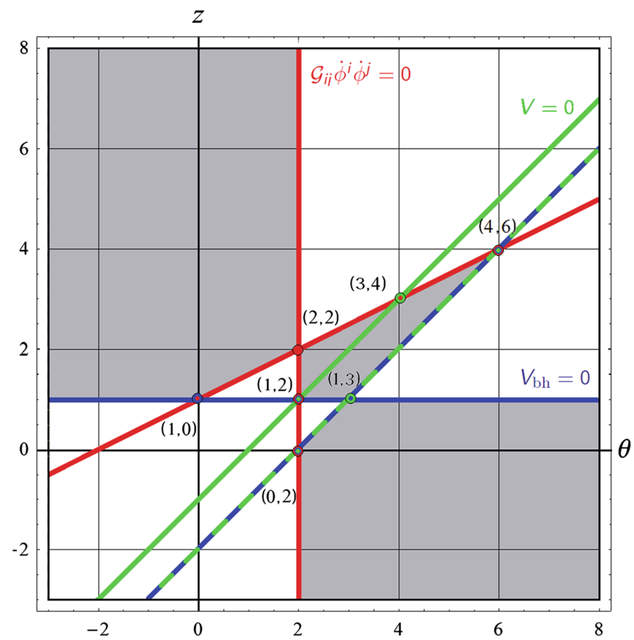


**Fig. 1** Purely hvLif $(\theta, z)$ plane. *Red lines* correspond to $\mathcal{G}_{ij}\dot\phi^i\dot\phi^j = 0$, the *blue* ones to $V_{\rm bh} = 0$, and those in *green* to $V = 0$. The *shaded regions* represent solutions which satisfy the NEC

*$t^3$-model*

Let us see now what the situation is for the truncation of the $t^3$-model considered in the previous section. In this case, $V^{I,II} = c_1 e^{-\phi/\sqrt{3}} + c_2 e^{\phi/\sqrt{3}}$ with $c_2 = 0 \Rightarrow c_1 = 0$ in the case I, and $c_1 = 0 \Rightarrow c_2 = 0$ in the case II. Since we can only keep one of the exponentials [in order to match $V$ with Eq. (5.25)], the only possibility is setting $g^0 = 0$ ($c_1 = 0$) in the case I (which leaves us with the string theory embedded model), and $g_0$ ($c_2 = 0$) in the case II. In both situations, $Z(\phi) = -2\, e^{-\sqrt{3}\phi}$. In I there exists one only solution, which is magnetic, and corresponds to $\theta = -2$, $z = 3/2$, $g_1^2 = 297/(4\ell^2)$ and $p^2 = 11\ell^2/4$. On the other hand, case II admits one only solution (magnetic as well) for $\theta = 1$, $z = 3$, $g^{1^2} = 4/\ell^2$, and $p^2 = 8\ell^2$. Both solutions satisfy the NEC, as was desirable, and have a running dilaton given by Eq. (5.24) with $\mathcal{Z} = 12$ and $\mathcal{Z} = 3$, respectively.

## 5.2 Asymptotically *hvLif* in the EMD

### 5.2.1 *Non-constant scalar field*

In order to construct new solutions with *hvLif* asymptotics, we switch now to $A$–$B$–$f$ variables. The required form for $A$ and $B$ is

$$e^{2A} = r^{\theta-2}, \quad e^{2B} = r^{-2(z-1)}. \tag{5.27}$$

With this election, Eq. (3.23) can be directly integrated, yielding

$$\phi = \phi_0 + \sqrt{(\theta-2)(\theta+2-2z)}\,\log(r). \tag{5.28}$$

$\Upsilon$ and $V$, in turn, become[10]

$$V = \frac{1}{2} r^{-\theta}\left[[1-\theta+z]\left[2\,[\theta-2-z]\,f + 3rf'\right] - r^2 f''\right], \tag{5.29}$$

$$\Upsilon = r^{\theta-4}\left[f\,[(\theta-2-z)(z-1)] - \frac{r}{2}\left[(1+\theta-3z)f' + rf''\right]\right]. \tag{5.30}$$

In order to tackle the problem of constructing asymptotically *hvLif* metrics, and taking into account the form of $V(\phi)$ and $Z(\phi)$ for our *axion-free* model (and others present in the literature), we can start by considering these functions to have the generic form

$$V(\phi) = c_1 e^{-s_1\phi} + c_2 e^{s_2\phi} + c_3, \tag{5.31}$$

$$Z(\phi) = d_1 e^{-t_1\phi} + d_2 e^{t_2\phi} + d_3. \tag{5.32}$$

The form of $V(\phi)$ is motivated by the expression of $V_{\rm fi}$ appearing in the *axion-free* $t^3$ model, as well as in other string theory truncations present in the literature (see, e.g., [71,72]). On the other hand, additional terms to the single-exponential gauge coupling have been introduced to mimic the quantum corrections appearing from string theory (see, e.g., [73]), in an attempt to cure the logarithmic behavior of the dilaton, which blows up in the deep IR, pointing out the non-negligibility of quantum corrections in this regime. The expressions for $V(\phi)$ and $Z(\phi)$ can be introduced in Eqs. (5.29) and (3.10), (3.11) or (3.12) (depending on whether we are searching for electric, magnetic or dyonic solutions) using Eq. (5.30). Once this is done, we are left with two second-order differential equations for $f(r)$ which can in general be converted into a first order equation plus a constraint that remains to be fulfilled. Obtaining the general solution in the presence of so many arbitrary parameters ($c_1$, $c_2$, $c_3$, $d_1$, $d_2$, $d_3$, $s_1$, $s_2$, $t_1$, $t_2$, $z$, and $\theta$) seems not to be possible and therefore we are forced to consider further simplifications, keeping in mind that the procedure does work for other set-ups in which $Z(\phi)$ and $V(\phi)$ are given by a different choice of the parameters in (5.31) and (5.32). Taking into account the form of the potentials obtained in the *axion-free* $t^3$ model, let us assume $s_1 = s_2$, $d_2 = d_3 = 0$ (we allow $t_1$ to be positive or negative) and we have

$$V(\phi) = c_1 e^{-s_1\phi} + c_2 e^{s_1\phi} + c_3, \tag{5.33}$$

$$Z(\phi) = d_1 e^{-t_1\phi}. \tag{5.34}$$

[10] Recall that $Z$ is given in terms of $\Upsilon$ in Eqs. (3.10)–(3.12) depending on the case.

The general form of the blackening factor, valid in all cases (electric, magnetic, and dyonic), reads

$$f(r)=\frac{c_3r^{\theta}}{D_3}+\frac{c_2r^{\theta+s_1\Delta}}{D_2}+\frac{c_1r^{\theta-s_1\Delta}}{D_1}+\frac{d_1p^2r^{4-\theta-t_1\Delta}}{2D_p}+\frac{q^2r^{4-\theta+t_1\Delta}}{2d_1D_q}+Kr^{2-\theta+z}, \tag{5.35}$$

where $\Delta=\sqrt{(\theta-2)(\theta-2z+2)}$, $K$ is an integration constant, and

$$D_1=(\theta-2)(2-2\theta+s_1\Delta+z), \tag{5.36}$$
$$D_2=(\theta-2)(2-2\theta-s_1\Delta+z), \tag{5.37}$$
$$D_3=(\theta-2)(2-2\theta+z), \tag{5.38}$$
$$D_p=(\theta-2)(2-t_1\Delta-z), \tag{5.39}$$
$$D_q=(\theta-2)(2+t_1\Delta-z). \tag{5.40}$$

As we said, there is an additional (non-trivial) constraint to be satisfied:

$$f''(r)-2r^{\theta-2}\Big[-c_3-c_1r^{-s1\Delta}-c_2r^{s_1\Delta}-\frac{1}{2}r^{-\theta}(\theta-z-1)\left[2(-2+\theta-z)f(r)+3rf'(r)\right]\Big]=0. \tag{5.41}$$

At this point, there are several ways to construct solutions. On the one hand, it is possible to impose values onto $z$ and $\theta$ and find the corresponding potentials admitting solutions for particular blackening factors. On the other hand, it is possible to fix the coefficients in the exponents of $Z$ and $V$ and find the blackening factors allowed by Eq. (5.41). We will proceed along the lines of the second possibility, looking for solutions embedded in the supergravity $t^3$ model. Before doing so, let us consider the general case in which the exponents in $Z(\phi)$ and $V(\phi)$ are such that $s_1=\theta/\Delta$, $t_1=(4-\theta)/\Delta$, and $c_2=q=0$. The result is a family of solutions for arbitrary values of $z$ and $\theta$ determined by

$$c_1=\frac{d_1p^2(\theta-z-1)}{2(1-z)}, \tag{5.42}$$
$$f(r)=\frac{d_1p^2}{2(1-z)(z-\theta+2)}\left[1-Kr^{2+z-\theta}\right], \tag{5.43}$$

which is well known (see, e.g., [54,64,72])

$$f(r)\sim 1-Kr^{2+z-\theta}. \tag{5.44}$$

The same family can also be found for electric solutions setting $s_1=\theta/\Delta$, $t_1=(\theta-4)/\Delta$, and $c_1=p=0$. In that case, the solution is given by

$$c_1=\frac{q^2(\theta-1-z)}{2d_1(1-z)}, \tag{5.45}$$
$$f(r)=\frac{q^2}{2d_1(1-z)(z-\theta+2)}\left[1-Kr^{2+z-\theta}\right]. \tag{5.46}$$

*$t^3$-model*

1. **Magnetic solutions.** As we saw, a consistent truncation of the $t^3$-model can be embedded in the EMD system for $s_1=1/\sqrt{3}$, $t_1=\sqrt{3}$, $c_3=0$. It turns out that setting $q=0$, it is possible to construct two families of solutions which, in the apropriate cases, asymptote to the purely *hvLif* ones constructed in the previous subsection. The first one is determined by

$$c_1=0,\ \theta=2\left(1-\frac{\Delta}{\sqrt{3}}\right),\ c_2=Ap^2, \tag{5.47}$$

where $A$ is a constant depending on $z$ and $\theta$. The blackening factor is given by

$$f(r)=Cp^2r^{\left(2-\frac{\Delta}{\sqrt{3}}\right)}+Kr^{\left(\frac{2\Delta}{\sqrt{3}}+z\right)}, \tag{5.48}$$

where $C$ is another $z,\ \theta$-dependent constant. Needless to say, the metric will not, in general, asymptote to a *hvLif* (with exponents $z,\ \theta$) as $r\to 0$ except for particular values of $\theta$ and $z$. However, if we choose $\theta=-2,\ z=3/2$, $c_2=-9p^2$, we find

$$f(r)=\frac{4p^2}{11}\left[1-Kr^{\frac{11}{2}}\right]. \tag{5.49}$$

The second family is characterized by

$$c_2=0,\ \theta=\left(2-\frac{\Delta}{\sqrt{3}}\right),\ c1=Ap^2, \tag{5.50}$$

where $A$ is another constant, and the blackening factor reads

$$f(r)=Cp^2r^{\left(2-\frac{2\Delta}{\sqrt{3}}\right)}+Kr^{\left(\frac{\Delta}{\sqrt{3}}+z\right)}. \tag{5.51}$$

If we set $\theta=1,\ z=3$, it becomes

$$f(r)=\frac{p^2}{8}\left[1-Kr^4\right] \tag{5.52}$$

which, as we will see in a moment, is a particular a case of a dyonic solution admitted by the model.

2. **Electric solutions.** Similarly, we can construct two families of electric solutions. The first one is characterized by

$$c_1=0,\ \theta=\left(2+\frac{\Delta}{\sqrt{3}}\right),\ c_2=Aq^2, \tag{5.53}$$

where, once more, $A$ is a constant depending on $z$ and $\theta$. The blackening factor is given by

$$f(r) = Cq^2 r^{\left(2+\frac{2\Delta}{\sqrt{3}}\right)} + K r^{\left(-\frac{\Delta}{\sqrt{3}}+z\right)}, \tag{5.54}$$

whereas for the second

$$c_1 = Aq^2,\ \theta = 2\left(1+\frac{\Delta}{\sqrt{3}}\right),\ c_2 = 0, \tag{5.55}$$

$$f(r) = Cq^2 r^{\left(2+\frac{\Delta}{\sqrt{3}}\right)} + K r^{\left(-\frac{2\Delta}{\sqrt{3}}+z\right)}. \tag{5.56}$$

In contradistinction to the magnetic cases, for no values of $(\theta, z)$ the above solutions take the form of Eq. (5.44). This is obviously connected to the fact that no purely *hvLif* electric solutions exist in this model for non-constant dilaton and scalar potential, as we saw before.

3. **Dyonic solutions.** It is possible to show that a dyonic solution does exist for $\theta = 1,\ z = 3,\ c_2 = 0$, and $c_1 = -3p^2/2$, with a blackening factor given by

$$f(r) = \frac{p^2}{8}\left[1 - Kr^4 + \frac{q^2}{p^2} r^6\right]. \tag{5.57}$$

The corresponding metric Eq. (3.16) reads (after the redefinitions $\mathrm{d}R^2 = 8\mathrm{d}r^2/p^2$, $\mathrm{d}T^2 = 8\mathrm{d}t^2/p^2$)

$$\mathrm{d}s_f^2 = \frac{L^2}{R}\left\{\left[1 - KR^4 + \frac{p^4 q^2}{512}R^6\right]\frac{\mathrm{d}T^2}{R^4} - \frac{\mathrm{d}R^2}{\left[1 - KR^4 + \frac{p^4q^2}{512}R^6\right]} - \mathrm{d}\vec{x}^2\right\}. \tag{5.58}$$

It asymptotes to a *hvLif* as $R \to 0$ with $\theta = 1,\ z = 3$, and to a different one as $R \to \infty$ with $\theta = 5/2,\ z = 3/2$ as can be seen by taking the limit in the previous expression, and defining $\rho \sim R^{-2}$

$$\mathrm{d}s_f^2 \overset{R\to+\infty}{\sim} \frac{L^2}{R}\left[R^2\mathrm{d}T^2 - \frac{\mathrm{d}R^2}{R^6} - \mathrm{d}\vec{x}^2\right], \tag{5.59}$$

$$\mathrm{d}s_f^2 \overset{[R\to+\infty,\ R^{-2}=\rho]}{\sim} L^2\rho^{1/2}\left[\frac{\mathrm{d}T^2}{\rho} - \mathrm{d}\rho^2 - \mathrm{d}\vec{x}^2\right], \tag{5.60}$$

which corresponds to $\theta = 5/2,\ z = 3/2$. The value of $K$ can be fixed in a way such that $\exists\ R_h \in \mathbb{R}^+/f(R_h) = 0$, or chosen to get a positive-definite metric in the whole spacetime.

In the previous section, we constructed two consistent truncations of this model (which we called "I" and "II"). The first one is such that $c_2 = 0 \Rightarrow c_1 = 0$, and hence the solution can be embedded in that model only for a vanishing $V_{\text{fi}}$ and magnetic charge. For the second, in turn, we get the conditions $g_0 = 0$, $\left(g^1\right)^2 = p^2/2$. It is interesting to investigate how the solution gets modified by turning off the electric or the magnetic charge. Obviously, setting $q = 0$ does not change the $R \to 0$ behavior, but does change the $R \to +\infty$ one. In such a case, the metric becomes

$$\mathrm{d}s_f^2 \overset{[R\to+\infty,R^{-1}=\rho]}{\sim} \rho\left[\mathrm{d}T^2 - \mathrm{d}\rho^2 - \mathrm{d}\vec{x}^2\right], \tag{5.61}$$

which is conformal to Minkowski, and corresponds to a *hvLif* with $\theta = 3,\ z = 1$. On the other hand, restoring $q$ and setting $p = 0$, imposes the vanishing of $V_{\text{fi}}$, and the solution is $\theta = 3,\ z = 1$ as $R \to 0$, and again $\theta = 5/2,\ z = 3/2$ as $R \to +\infty$.

It turns out that there exists another dyonic solution for $\theta = 5/2,\ z = 3/2$[11]. This is somehow "dual" to the previous one, as it presents the same IR and UV behavior but with both regimes interchanged. It is characterized by $c_1 = 0$, $c_2 = -\frac{3q^2}{8}$, and

$$f(r) = 2p^2\left[1 - Kr + \frac{q^2}{16p^2}r^3\right]. \tag{5.62}$$

In our "I" truncation, we have $c_2^I = -g_1^2/3 \Rightarrow g_1^2 = 9q^2/8$. Making the redefinitions $\mathrm{d}R^2 = \mathrm{d}r^2/(2p^2)$, $\mathrm{d}T^2 = \sqrt{2}p\mathrm{d}t^2$, it reads

$$\mathrm{d}s_f^2 = L^2R^{1/2}\left\{\left[1 - KR + \frac{pq^2}{4\sqrt{2}}R^3\right]\frac{\mathrm{d}T^2}{R} - \frac{\mathrm{d}R^2}{\left[1 - KR + \frac{pq^2}{4\sqrt{2}}R^3\right]} - \mathrm{d}\vec{x}^2\right\}. \tag{5.63}$$

As $R \to +\infty$, this becomes

$$\mathrm{d}s_f^2 \overset{[\text{UV},\ R=\rho^{-2}]}{\sim} \frac{L^2}{\rho}\left[\frac{\mathrm{d}T^2}{\rho^4} - \mathrm{d}\rho^2 - \mathrm{d}\vec{x}^2\right], \tag{5.64}$$

up to constants, which corresponds to a *hvLif* with $\theta = 1,\ z = 3$.

### 5.2.2 Constant scalar field

Let us consider now the case of a constant scalar field, $\phi' = 0$. As explained in Sect. 2, we consider

$$\partial_\phi V_{\text{bh}} = \partial_\phi V = 0. \tag{5.65}$$

In this case, the potential and the coupling become constant and we can write $V = \Lambda$, $Z = -Z_0^2$. When $Z$ and $V$ are

[11] One may wonder why we did not find a purely *hvLif* for these values of the exponents in the previous subsection. The reason is that for $\theta = 5/2,\ z = 3/2$ we have $\mathcal{X}_{(\theta,z)} = 0$, which implies the vanishing of $V_{\text{fi}}$ in the purely *hvLif* case. In fact, to recover the pure solution, we have to set $K = q = c_2 = 0$, and since we have already set $c_1 = 0$, this would make $V_{\text{fi}} = 0$.

given by Eqs. (5.32) and (5.31), (5.65) translates into

$$\partial_\phi V_{\rm bh}(\phi=0) = \partial_\phi Z \left(p^2 - \frac{q^2}{Z^2}\right)|_{\phi=0}$$
$$= (-t_1 d_1 + t_2 d_2)\left(p^2 - \frac{q^2}{(d_1+d_2)^2}\right) = 0, \quad (5.66)$$
$$\partial_\phi V(\phi=0) = (s_2 c_2 - s_1 c_1) = 0, \quad (5.67)$$

where we have imposed $\phi = 0$ to be a critical point of the potentials. We choose to fulfill the first condition demanding $(d_1+d_2)^2 = q^2/p^2$, which, when $d_1 = 0$, reads $d_2 = -|q/p|$. On the other hand, the second condition is $s_1 c_1 = s_2 c_2$, which becomes $c_1 = c_2$ when both exponents ($s_1$ and $s_2$) coincide. After imposing these constraints, $V$ and $Z$ become

$$V = c_2\left(\frac{s_2}{s_1}+1\right) + c_3 \equiv \Lambda (= 2c_2 + c_3 \text{ if } s_2 = s_1), \quad (5.68)$$
$$Z = -\left|\frac{q}{p}\right| \equiv -Z_0^2. \quad (5.69)$$

We have two cases: $z = 1 + \theta/2$ and $\theta = 2$ (and the one in the intersection: $z = 2, \theta = 2$).

1. $z = 1 + \frac{\theta}{2},\ \theta \neq 2$. In this situation, it is possible to find a solution which imposes no further constraints on $V$ and $V_{\rm bh}$. This reads

$$f(r) = -K r^{3-\theta/2} + \frac{\left[12 Z_0^2 r^{4-\theta} - 2\Lambda r^\theta\right]}{3(\theta-2)^2}. \quad (5.70)$$

The case $z = 1, \theta = 0$, in which we expect to recover $aDS_4$ asymptotically is a particularization of this. The blackening factor then reads

$$f(r) = -\frac{\Lambda}{6} - K r^3 + Z_0^2 r^4, \quad (5.71)$$

Assuming a negative cosmological constant, $\Lambda = -|\Lambda|$, this can be rewritten as

$$f(r) = \frac{|\Lambda|}{6}\left[1 - K r^3 + \frac{6 Z_0^2}{|\Lambda|} r^4\right]. \quad (5.72)$$

If we define $\mathrm{d}T^2 = |\Lambda| \mathrm{d}t^2/6$, $\mathrm{d}R^2 = 6\mathrm{d}r^2/|\Lambda|$, the metric Eq. (3.16) becomes

$$\mathrm{d}s_f^2 = \frac{L^2}{R^2}\left\{\left[1 - K R^3 + \frac{|\Lambda| Z_0^2}{6} R^4\right]\mathrm{d}T^2 - \frac{\mathrm{d}R^2}{\left[1 - K R^3 + \frac{|\Lambda| Z_0^2}{6} R^4\right]} - \mathrm{d}\vec{x}^2\right\}, \quad (5.73)$$

which, of course, asymptotes to $aDS_4$ as $R \to 0$, and is such that $\exists\ R_h \in \mathbb{R}^+ / f(R_h) = 0$ for $K > 0$. Similarly, the metric blows up as $R \to \infty$, behaving as a *hvLif* with $\theta = 4,\ z = 3$. Indeed,

$$\mathrm{d}s_f^2 \overset{R\to\infty}{\sim} \frac{L^2}{R^2}\left[R^4 \mathrm{d}T^2 - \frac{\mathrm{d}R^2}{R^4} - \mathrm{d}\vec{x}^2\right] \quad (5.74)$$

up to constants; if we make now the change $\rho \sim 1/R$

$$\mathrm{d}s_f^2 \overset{\rho\to 0}{\sim} L'^2 \rho^2 \left[\frac{\mathrm{d}T^2}{\rho^4} - \mathrm{d}\rho^2 - \mathrm{d}\vec{x}^2\right], \quad (5.75)$$

we find a *hvLif* metric with $\theta = 4,\ z = 3$ as we have said. If we plug these values $\theta = 4,\ z = 3$ in Eq. (5.70) we find a new solution, which behaves asymptotically as this one (with the IR and UV regions interchanged). Indeed, its blackening factor reads

$$f(r) = Z_0^2\left[1 - K r + \frac{|\Lambda|}{6 Z_0^2} r^4\right], \quad (5.76)$$

and with the redefinitions $\mathrm{d}R^2 = \mathrm{d}r^2/Z_0^2$, $\mathrm{d}T^2 = \mathrm{d}t^2/Z_0^2$

$$\mathrm{d}s_f^2 = L^2 R^2 \left\{\left[1 - K R + \frac{|\Lambda| Z_0^2}{6} R^4\right]\frac{\mathrm{d}T^2}{R^4} - \frac{\mathrm{d}R^2}{\left[1 - K R + \frac{|\Lambda| Z_0^2}{6} R^4\right]} - \mathrm{d}\vec{x}^2\right\}. \quad (5.77)$$

As $R \to 0$, it becomes a *hvLif* with $\theta = 4,\ z = 3$, and as $R \to \infty$,

$$\mathrm{d}s_f^2 = L^2 R^2 \left[\mathrm{d}T^2 - \frac{\mathrm{d}R^2}{R^4} - \mathrm{d}\vec{x}^2\right], \quad (5.78)$$

which we can rewrite as ($\rho = 1/R$)

$$\mathrm{d}s_f^2 = \frac{L'^2}{\rho^2}\left[\mathrm{d}T^2 - \mathrm{d}\rho^2 - \mathrm{d}\vec{x}^2\right], \quad (5.79)$$

which is $aDS_4$.

2. $\theta = 2$. This case imposes the constraint $Z_0^2 = -\frac{\Lambda}{2}$, and can be solved for any value of $z$. The general form of $f(r)$, which applies for $z \neq 2$ is now

$$f(r) = \frac{2 Z_0^2 r^2}{(z-2)^2} + r^z K_1 + r^{2(z-1)} K_2, \quad (5.80)$$

whereas for $z = 2$ we have

$$f(r) = 2r^2 \log(r) \left[K_2 + Z_0^2 \log(r)\right] + K_1 r^2. \quad (5.81)$$

For example, if we consider the case $\theta = 2$, $z = 1$, we immediately find the asymptotically flat metric (as $r \to 0$)

$$f(r) = 1 - Kr + 2Z_0^2 r^2, \quad (5.82)$$

$$\mathrm{d}s_f^2 = l^2 \left\{ \mathrm{d}t^2 \left[1 - Kr + 2Z_0^2 r^2\right] - \frac{\mathrm{d}r^2}{\left[1 - Kr + 2Z_0^2 r^2\right]} - \mathrm{d}\vec{x}^2 \right\}, \quad (5.83)$$

for which, once more $\exists\, r_h \in \mathbb{R}^+ / f(r_h) = 0$ for $K > 0$. As $r \to \infty$, up to constants, it behaves as

$$\mathrm{d}s_f^2 \overset{[R\to+\infty]}{\sim} l'^2 \left[e^{2R}\mathrm{d}t^2 - \mathrm{d}R^2 - \mathrm{d}\vec{x}^2\right], \quad (5.84)$$

where we defined $R = \log r$. This is nothing but $aDS_2 \times \mathbb{R}_2$. On the other hand, if we set $\theta = 2$, $z = 2$, from Eq. (5.81) we find

$$f(r) = 2r^2 \log(r) \left[-K + Z_0^2 \log(r)\right] + r^2 \overset{[R=\log r]}{=} e^{2R} \left[1 - KR + 2Z_0^2 R^2\right], \quad (5.85)$$

$$\mathrm{d}s_f^2 = l^2 \left\{ \mathrm{d}t^2 \left[1 - KR + 2Z_0^2 R^2\right] - \frac{\mathrm{d}R^2}{\left[1 - KR + 2Z_0^2 R^2\right]} - \mathrm{d}\vec{x}^2 \right\} \quad (5.86)$$

which is nothing but Eq. (5.83).

## 6 Conclusions

We have studied purely *hvLif* and *hvLif*-like solutions of the general class of theories defined by the Lagrangian (2.1), which covers any theory of gravity coupled to an arbitrary number of scalars and vector fields up to two derivatives. We have obtained the general effective one-dimensional equations of motion that need to be solved in order to obtain *hvLif*-like solutions.

The general analysis is intended to complete the case-by-case results present in the literature in a unified framework: given a particular kinetic matrix $(I_{\Lambda\Sigma}(\phi),\ R_{\Lambda\Sigma}(\phi))$, a scalar metric $\mathcal{G}_{ij}(\phi)$ and a scalar potential $V(\phi)$, the equations of motion of the theory follow trivially by plugging them into (2.11)–(2.13) and the Hamiltonian constraint (2.10).

For this broad family of theories, we have discussed the existence and properties of purely *hvLif* solutions attending to the presence (or absence) of non-constant scalar fields and non-vanishing black-hole and scalar potentials.

In the context of $\mathcal{N} = 2$ FI supergravity, we have studied the $t^3$-model, for which we have explicitly constructed two consistent *axion-free* embeddings in the EMD system, one of which is, in turn, embedded in Type-IIB string theory for a particular choice of embedding tensor $\theta_M$.

In addition, we obtained the general form of the $f(r)$ function (for the set of metrics determined by Eqs. (5.27) and (3.16)), up to a constraint, for a rather general family of (supergravity-inspired) scalar and black-hole potentials, and explicitly constructed some dyonic solutions for the $t^3$ truncations considered. We have provided a straightforward procedure to construct asymptotically *hvLif* solutions covered by Eqs. (5.27) and (3.16) for the family of theories specified by Eq. (5.33). This reduces the task to solving a single algebraic constraint, given by Eq. (5.41).

We have avoided, on purpose, the term *black hole* to denote the *hvLif*-like solutions obtained in this paper. The reason is that, although they look like black holes, a complete and rigorous proof (for example by constructing the corresponding Penrose–Carter diagram) is still missing. Therefore, any results obtained from them implicitly assuming that they do represent a black hole must be interpreted carefully, knowing that those would be yet to be proven statements.

As a final remark, we would like to point out that, thanks to the *BH-hvLif-Topological* triality (BHvTriality) discovered in [55], the new fascinating results that are being obtained in the context of static, spherically symmetric black holes in ungauged $\mathcal{N} = 2$, $d = 4$ supergravity [74–79] can also be applied to *hvLif*, giving therefore, the first examples of *hvLif*-like solutions in the presence of quantum corrections induced by Type-IIA string theory Calabi–Yau compatifications.

**Note added:** During the very last stage of this project, the very interesting Ref. [80] appeared, containing a minor overlap with our work.

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