

17th CIRP Conference on Modelling of Machining Operations

Improving technological machining simulation by tailored workpiece models and kinematics

V. Böß^{a,*}, B. Denkena^a, B. Breidenstein^a, M.-A. Dittrich^a, H.N. Nguyen^a

^a*Institute of Production Engineering and Machine Tools (IFW), Leibniz Universität Hannover, Germany*

* Corresponding author. Tel.: +49-511-762-2553; fax: +49-511-762-5115. boess@ifw.uni-hannover.de

Abstract

Geometric modelling is an established approach for gathering detailed knowledge about the chronological sequence of process conditions and for determining technological values of machining processes such as milling, turning, grinding or additive manufacturing. Performance and accuracy essentially depend on the chosen workpiece model and its parametrization. Furthermore, several influences on the investigated machine tool system lead to errors, which must be modeled separately. This paper shows approaches to increase performance and accuracy of the simulation by choosing an appropriate combination of different geometric representations of the workpiece and by considering possible errors within the kinematic model. Examples for different applications in metal cutting are given.

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Peer-review under responsibility of the scientific committee of The 17th CIRP Conference on Modelling of Machining Operations

Keywords: Geometric modelling; Simulation; Milling; Grinding;

1. Introduction

The production of parts with machining processes requires comprehensive planning. To ensure effectiveness and quality, detailed knowledge about the process conditions is necessary. Hence, the ability to predict these conditions has been focus of an extense area of research. A common approach in the literature, emerging with the increase of computer power, is the use of numerical simulation to determine the chronological progress of workpiece shape and engagement conditions. Here, frequent points of interest are cutting forces, torque, power, vibration amplitudes and dimensional surface errors on the part [1]. Although simulation of machining processes is already a common method for the detection of collisions and critical conditions, the use of more detailed simulation data for an advanced process planning is just emerging in industrial practice.

One reason for this is the rapid increase of computation time and memory usage with increasing level of detail. A second

reason is the high effort in preparing simulation tasks due to required parametrization.

If computation of data is even needed in real time, as in tasks of process monitoring or process control, simulation models that guarantee a short computation time are required. To find a matching compromise, varying models must be used according to locally differing simulation tasks. If, for example, a turning process is performed, it may be sufficient, to use a simple, high performant model to represent the major part of the workpiece, while for an additional boring process another model may be more suitable.

This paper introduces an approach to build up tailored models for locally differing requirements. In the first part, an overview of known application fields of geometrical engagement simulation is presented. The second part discusses the choice of different models due to the type of process. This is followed by the presentation and an example of a multi-model approach that uses overlaid models to locally match different demands. Finally, some aspects concerning the

modification of underlying machining kinematics to reproduce applying errors are discussed and examples are given.

2. Application fields of engagement simulation

Applications of engagement simulation are known for very different machining processes and with a wide range of level of detail. To characterize a process concerning simulation, the arrangement of linear and rotating axes has to be taken into account. Particularly, it must be considered, whether the tool is rotating or if there is a rotating workpiece. Another crucial factor is the aim of the simulation (e.g. material removal rate, process forces or surface topography). Hence, a simulation has to be characterized by the regarded processes, which are milling, grinding and turning. Recently appearing applications in additive manufacturing will not be discussed here.

2.1. Milling and drilling processes

The process of computing the engagement of tool and workpiece to examine milling processes has been in use for more than 30 years [2]. Nowadays, it is an established method to check collisions or critical conditions in five-axis machining processes and it functions as an enabler for complex machining as well. In these cases, the milling tool is considered to be rotational symmetric. The spindle speed and thus the speed of the cutting edges is much higher than the feed. Hence, the rotation of the tool can be ignored if focus lies on the shape of the resulting part or material removal rates. Additionally, simulation provides data about the shape of the contact zone and the currently removed material. This enables computation of values such as engagement width and depth or the angle of wrap and thus process forces, energy etc. Performance is relatively high in these applications, which even allows parallel computation of the workpiece shape while the milling process is running. [3]

A more detailed view is necessary if research or process development is focused on maximal process forces, machining limits, tool life or surface topography [4–6]. In these cases, the rotational motion of the cutting edge must often also be represented within the kinematics of the simulation.

Due to the higher level of detail, this kind of examination is much slower than the former. To examine a whole workpiece by this approach, generalized models are necessary which transfer results of few rotations to a larger area with similar engagement conditions. Concerning engagement simulation drilling can be regarded as a special case of milling with feed direction parallel to the rotation axis. Thus, an additional application for that approach is the examination of drilling tools.

2.2. Grinding

The application of engagement simulation in grinding processes is less common than in milling. Again, in most cases the tool is considered rotational symmetric to reduce the complexity of kinematic representation. Contrary to technological models of milling, the relative movement of the tool to the workpiece is less varying. On the other hand, the

contact zone is often very complex. This is why traditional models characterize grinding processes by the shape of the contact zone and the resulting forces and removal rates [7]. The process forces can be determined using empirical grinding force models. Conventional models consider flat grinding, round grinding and profile grinding separately with the additional distinction of face grinding and peripheral grinding [8]. Here, computation of the exact geometric shape of the engagement area by simulation allows a more detailed view on the distribution of characteristic indicators, e.g. specific removal rate, which can be used for process optimization. [9]

Similar to the consideration of cutting edges in milling, a more detailed view on the real shape of the tool can be realized with simulation [10]. In contrast to the well-defined geometric shape of a milling tool, a grinding tool consists of stochastically distributed grains of varying shape. To reproduce the shape of the grinding tool, the statistical distribution of grains has to be modelled separately or the real shape has to be measured with high effort [7]. Subsequently, removal volumes and engagement areas of each individual grain can be calculated. A possible application lies in optimization of grinding tools with defined grain distributions [11,12].

2.3. Turning

Given that in turning the rotating part is the workpiece, the tool is not rotational symmetric in most cases. If the workpiece is considered symmetric, this simplifies the simulation, because the model can be reduced to a longitudinal section. However, there may be simulation tasks, where the focus lies on workpiece divergence from the design. Moreover, many applications combine turning with other processes, such as drilling or milling e.g. mill-turning. In this case, usually less performant simulation models are used to ensure correct representation of the workpiece. Accordingly, new models with more details and hence higher effort are necessary which combine advantages of different available models.

3. Choosing an appropriate workpiece model

A simulation should pursue the objective, to be as simple as possible and as detailed as necessary. On this regard, a simulation task performed by the currently available means, makes the implementation simple for the user. Apart from that, the choice of an appropriate model according to the current demands also influences these factors. With more effort in simulation design the computation improves in performance and accuracy. While in some cases, it may be sufficient to raise computation time or power to reach a higher level of accuracy, there are applications where performance is a major criterion, e.g. a parallel simulation while the real process is performed to control process parameters.

In this chapter, different aspects affecting the choice of methods are discussed. Various models describe the workpiece and tool shape, differing in terms of display accuracy, memory requirements and computational effort. Analytical methods include the Constructive Solid Geometry (CSG) and the Boundary Representation (BRep) method [13]. Discrete methods are essentially dixel models [2], voxel models [14],

level curves [15] and contour line models [16]. An extensive overview of these models is given in [1].

A crucial difference among machining processes examined by engagement simulation is the kind of relative movement of the tool regarding the workpiece. In milling or flat grinding processes, Cartesian coordinates describe best the movement, which results from the regularly used linear axes in x- y- and z-direction. Rotational movement, in these cases, is used for changing the tool angle or performed by the tool spindle.

On the other hand, processes with rotating workpieces normally use the rotation as cutting motion or as feed motion. Although the workpiece is not rotational symmetric in every case, the reference for describing local properties is a cylindrical coordinate system. Thus, an important difference between rotating workpieces and linear moving workpieces is the angle of view. In the linear case, the reference directions are fix vectors or depend on the designed curvature of the surface. In the rotating case, axial or radial directions are used as reference.

Concerning this classification there are cases where local conditions require different views. A milling part, which includes a round hole with high quality demands, may be considered Cartesian in global view but rotational in local view. These local variance considerations will be described in more detail in section 4. Furthermore, models must be parametrized to fit the requirements of the simulation target while providing maximal performance. Next, the different factors that influence accuracy will be described.

3.1. Time discretization

As all simulation methods considered here are time discrete, i.e. use time slices to model chronological change, the size of these slices is an important parameter to consider. In fact, what is crucial to characterize a step it is not the time that passes, but the geometrical size of the tool movement in relation to the workpiece. Actually, there is no difference in accuracy if the tool moves at half speed while the length of the time slice is twice the size. Thus, the maximal step size of a moving point of the tool is the characteristic parameter. Moreover, the shape of the track passed by the tool (or each point of the geometric representation of the tool) is of high importance.

To account for the whole area that the tool traversed while cutting off parts of the workpiece, usually the so-called ‘sweep volume’ is computed [17]. If the movement itself follows a linear motion, the sweep is accurately represented by a linear connection of the tool position before and after the time step. If the movement of the tool includes rotational components, an error is made by calculating the sweep with a linear connection. This error ε can be estimated by the secant defined by the rotation angle α and the maximal distance d_{max} of all points of the tool to the axis of rotation:

$$\varepsilon = \left(1 - \cos \frac{\alpha}{2}\right) \cdot \frac{1}{2} d_{max}$$

To reach a higher accuracy, the stepsize and the timestep accordingly has to be reduced. Alternatively, the sweep can be computed with a number of substeps.

3.2. Tool representation

Another important adjustment for accuracy versus performance is the representation of the tool. Again, the choice of the representation type is important. The most accurate approach is to use the original CSG-description of the tool, as is provided by a CAD-System or by means of a mathematical description such as a cylinder. If simple shapes are used, high performance algorithms are available to compute engagement with the workpiece. For example, to determine the intersection of a ball end mill with the workpiece, it may be sufficient to calculate the distance to the tool center point. On the other hand, these specialized algorithms can only be used for a reduced set of tools. This means that either a specialized application is used or the user has to be an expert to configure the simulation.

An almost unlimited range of tool shapes can be applied using a mesh, e.g. a triangulation of the tool. This kind of geometric representation is very common as an interchange format for geometrical data. In practice, an appropriate format is usually available and there is a large number of algorithms for creating a mesh from geometrical data. As the original surface of the tool object is approximated by small plane faces, the resulting error is again the secant of the curved surface. This means that the maximal error can be estimated by the maximal size of the used faces and the curvature of the face. To raise accuracy, a higher number of smaller triangles has to be chosen. However, a higher number of triangles increases computation time [18]. As an appropriate solution the mesh can be created according to a given maximal secant error.

3.3. Workpiece representation

As described above, most available methods to represent the workpiece use a discretization in one or more dimensions. Here, the possible accuracy depends on the kind of model and its alignment concerning the applied process. The main characteristic concerning the accuracy of a simulation model is the density of discretization elements. To estimate resulting errors, the discrete elements can be considered as measuring points on the surface of a workpiece and thus, accuracy has to be determined according to measurement technology. The number of elements (space complexity) corresponds linearly to the number of comparisons (time complexity) and hence to the computation effort to calculate simulation results. Vice versa, a model, which needs a smaller number of elements, is more accurate with the same performance. Therefore, the complexity of a workpiece model is the main characteristic concerning its ratio of accuracy to computing time.

Just as in production, it is essential to classify workpieces differentiating between cuboid and rotational parts. According to the different geometric shape, the target of simulation also differs. Next, some aspects concerning the choice of an appropriate workpiece model are discussed.

A very common model for cuboid parts is the Cartesian multi-dexel model. Usually, the three chosen spatial directions of the dexels are equivalent to the main axes of a Cartesian coordinate system [19]. In all main directions we have parallel elements, which can be calculated with the maximum available accuracy of the underlying system. Depending on the demands,

floating point or double floating point has proven as effective. Nevertheless, resulting errors depend on the angle α between the dexels and the surface normal n of the workpiece. This can best be shown by examining a slice of the workpiece in the shape of a disc (figure 1). On the left, a disc with a single dixel model is shown. If a cutting operation is performed on the disc, the orientation of the dexels has a high influence on the estimation of the radius at the location where the tool cuts the workpiece.

At position a), the error is caused by the interpolation error between two neighboring, correctly computed values. At position b), the maximal error is the dixel distance. To avoid situations similar to position b), Cartesian multi-dixel models are used. Here, the worst case of angle α is $\alpha = 45^\circ$ within the plane (position c)).

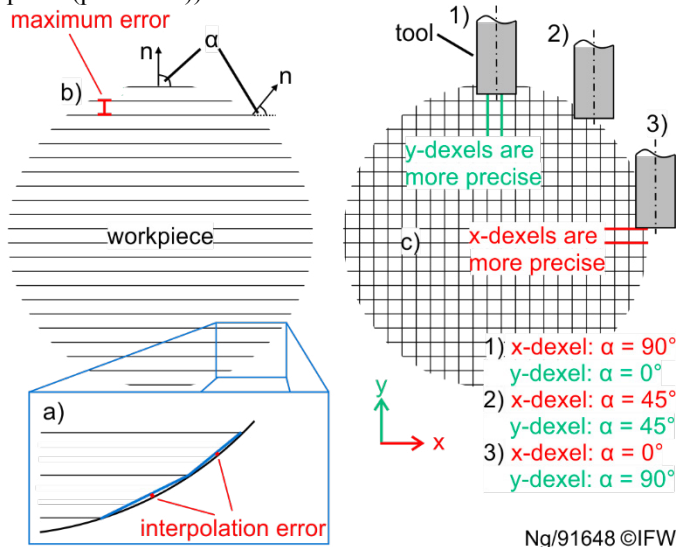


Fig. 1. (a) single dixel model; (b) multi-dixel model.

For the three-dimensional case, this maximal angle is given by the angle between the space diagonal and the reference axes:

$$\alpha_{max} = \arccos\left(\frac{1}{\sqrt{3}}\right) \approx 54.74^\circ$$

When examining rotational parts, the variance in accuracy results in a noise signal in the computed diameter when the workpiece is rotated. This leads to the demand for appropriate models that are independent of the rotation angle of the workpiece [20,21].

A solution is provided with the arrangement of the dixel models in cylindrical coordinates (figure 2). Within parallel workpiece slices, dexels are oriented in radial direction related to the axis of rotation (a). As the dimension of the part will usually not exceed initial size, the maximal distance D_{max} of two neighboring dexels is given by the number n of the dexels and the diameter d of the part:

$$D_{max} = d \cdot \sin\left(\frac{\pi}{2n}\right)$$

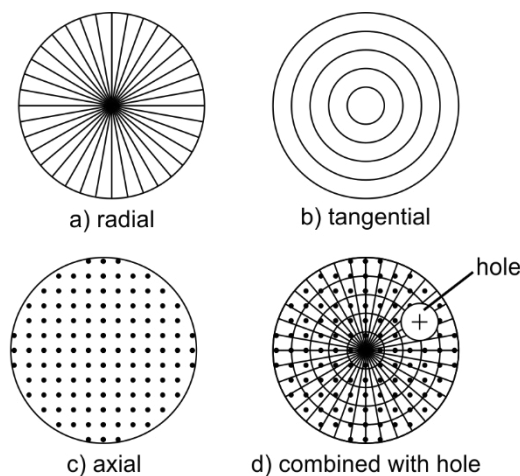
In case of radial measurements, the best possible accuracy is guaranteed. Again, situations may appear where the direction of measurement is perpendicular to the dixel direction. For

example, the surface of a hole in axial direction is not only measured in axial but also in tangential direction (d). The appropriate solution is to use another dixel grid in tangential direction, which forms a number of concentric rings (b). An additional dixel grid is oriented parallel to the axis of rotation (c).

A closer look shows that the axial, radial and tangential directions exactly fit the cylindrical coordinates. Thus, the transformation into Cartesian space can be easily performed by the common formulas and calculation of process parameters. Kimme et al. show an application of the cylindrical multi-dixel model for gear skiving and continuous gear grinding [22]. By using the additional radial dixel system it is possible to represent undercuts in the workpiece model.

As a rule of thumb, it can be summarized, that cuboid parts should be represented by Cartesian dexels and rotational parts by cylindrical dexels. However, processes like turning or cylindrical grinding usually produce workpieces that are axially symmetric. As a consequence, within a single slice of the workpiece, all radial dexels are of the same length, while tangential dexels are either of length 2π or of length 0. The field of axial dexels is also rotational symmetric. Therefore, the model to represent the workpiece can be reduced drastically without losing information. This leads to another workpiece model – the contour line model [16].

Actually, the contour line model was one of the first models that were used to represent the current shape of the workpiece in turning processes, for example on programming systems included in the machine control. It consists of a two-dimensional curve along the axis of rotation and represents the contour that has to be rotated to form the spatial object. The curve itself can be represented by a number of partially defined functions or – in most cases – by a discrete, one-dimensional field of nodes, i.e. a polygonal line. Engagement with a tool can easily be computed by the distance to the axis. Due to its simple representation, this model is very fast and has a very low need of memory. Raising the accuracy by increasing the number of nodes leads to linear increase of memory.



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Fig. 2. Dixel models in cylindrical coordinates.

4. Multi-model approach

In this section, a multi-model approach is presented, using the example of a rotational part with additional components (figure 3). The basic approach takes into account that the requirements of a geometric simulation vary locally depending on the demands of the different areas of the part to produce. Restrictions and quality requirements may be defined by the part specification in CAD or as additional data but in every case, set tolerances for the acceptance of technological values. In order to be able to build up a cutting simulation of rotational parts that is as accurate as necessary for predicting target values and as effective as possible, the part model is also divided into areas according to their geometric type and accuracy requirements.

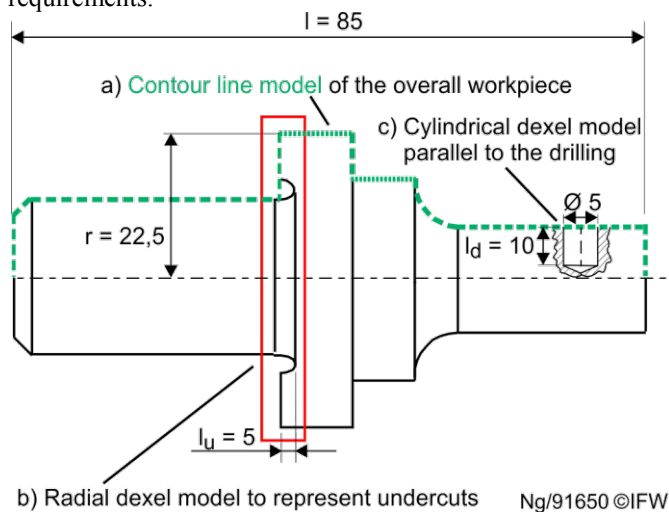


Fig. 3. Rotational part with locally assigned workpiece representations.

In the example of the rotational part, to give a numerical case study, the size of the part is chosen arbitrarily. Additionally, the space and time complexity of the method will be discussed. The general accuracy is referred to the contour of the overall part (figure 3 a)). As long as variation in roundness is not considered, the knowledge of the contour is sufficient. In area b), an undercut occurs. In our example we assume that in this area the simulation focusses on roundness verification. To represent this, an appropriate model is necessary. Area c) includes a hole that breaks the symmetry. Therefore, this area has to be regarded separately.

To identify the different areas of interest, an analysis of the initial shape and the design shape of the part is performed. Areas with similar properties concerning required accuracy are grouped and subdivided into superordinate stages, such as rotationally symmetrical, rotationally symmetrical with undercut, and asymmetrical. Currently this is conducted manually, but also design features, feature recognition or specialized algorithms may be used, e.g. considering the movement of the workpiece rotation axis.

Suitable model approaches are assigned for these different stages considering appropriate conditions. For the rotationally symmetric shape of area a), the contour line model is used. In area b), due to the undercut and a different focus, a model with radially arranged dexels is applied. For the round hole in area c) with highest requirement of surface roughness and small

tolerances, a cylindrical dixel model is applied. To avoid fragmentation of the areas, the implementation does not divide them where higher accuracy is needed but uses an overlay of the additional models. Each model is attached to a bounded area and arranged in a data structure with allocated priority. By this means, the contour of the rotational part is still calculated in the longitudinal interval of area c) while the shape of the hole is considered at the same time due to higher priority of the local model. For performance improvement, the less performance intensive model is always used first to detect remaining material by using the allocated priority the other way around. Only if material is present, the more accurate model with higher priority is applied.

Table 1 shows the calculated amount of memory usage for the different workpiece models referred to the workpiece areas of figure 3. The element distance for discretization is assumed to be $d = 0.1$ mm. Using conventional single type models, the whole workpiece would be represented by just one model type. Accordingly, the number of elements for a classical multi-dixel model is about 900,000. For the contour line model of the same area only 850 elements are necessary, with the disadvantage that only rotational symmetric areas are represented correctly. In area b) an additional amount of about 70,000 is allocated to increase the accuracy. For the hole, 18,000 more elements are needed. As a result, an overall number of less than 90,000 elements is necessary. Thus, the number of elements is reduced by factor 10. Actually, to be more precise the complexity is a better measure to compare the models. The last column of table 1 shows the space complexity which is identical to the time complexity in this case. Although the complexity for local models in area b) and c) is the same as for the single model approach, the number of elements n is much smaller and thus a drastically better performance can be assumed for the presented multi-model approach with the same element distance. Moreover, while the turning part of the process is simulated, the contour line model can be applied. Hence, only $O(n)$ comparisons have to be performed while a multi-dixel model would require $O(n^2)$ comparisons which results in performance factor 1,000.

For the implementation in a simulation system it is necessary that the simulation system is able to map different workpiece models simultaneously and to apply them to local, restricted areas. The work described here therefore uses the simulation platform IFW CutS. This system has a modular structure, so that extensions with new workpiece models are easy to realize [23]. On the one hand, the software offers determination of the final workpiece contour and on the other hand, calculation of technological process variables based on the cutting conditions between tool and workpiece. This enables the prediction of cutting forces, stability, tool wear etc. according to available technological models.

In addition to the local application of a more detailed model, the underlying models for technological effects can also be restricted to their area of interest. This especially applies to models in order to reproduce errors that appear due to external influences. An approach to deal with these is described in the following section.

Table 1. Calculated amount of memory usage.

Area Model	Overall workpiece	Undercut	Drilling	Comple- xity
Contour line	$\frac{l}{d} = 850$	$\frac{l_u}{d} = 50$	$\frac{l_d}{d} = 100$	$O(n)$
Radial dixel (rd)	$\frac{2\pi \cdot r}{d} \cdot \frac{l}{d}$ $\approx 1.2M$	$\frac{2 \cdot \pi r}{d} \cdot \frac{l_u}{d}$ $\approx 70K$	$\frac{2\pi r_d}{d} \cdot \frac{l_d}{d}$ $\approx 15K$	$O(n^2)$
Tangential dixel (td)	$\frac{r}{d} \cdot \frac{l}{d} = 191K$	$\frac{r}{d} \cdot \frac{l_u}{d} = 11K$	$\frac{r_d}{d} \cdot \frac{l_d}{d} = 2.5K$	$O(n^2)$
Axial dixel (ad)	$\frac{2\pi \cdot r^2}{d^2} \approx 13K$	$\frac{2\pi \cdot r_u^2}{d^2} \approx 13K$	$\frac{2\pi \cdot r_d^2}{d^2} = 675$	$O(n^2)$
Cylindrical multi- dixel	$rd + td + ad$ $\approx 1.4M$	$rd + td + ad$ $\approx 94K$	$rd + td + ad$ $\approx 18K$	$O(n^2)$
Cartesian multi- dixel	$\frac{2 \cdot 2rl + 4r^2}{d^2}$ $= 0.9M$	$\frac{2 \cdot 2r_l u + 4r_u^2}{d^2}$ $= 250K$	$\frac{2 \cdot 2r_d l_d + 4r_d^2}{d^2}$ $= 12500$	$O(n^2)$
Voxel	$\frac{l}{d} * \left(\frac{2r}{d}\right)^2$ $\approx 172M$	$\frac{l_u}{d} * \left(\frac{2r_u}{d}\right)^2$ $\approx 10M$	$\frac{l_d}{d} * \left(\frac{2r_d}{d}\right)^2$ $= 250K$	$O(n^3)$

5. Error kinematics

A common task of geometric simulation is to estimate errors that may appear due to faulty machine precision or external influences. To consider these, it is not sufficient to model the ideal process. Moreover, the used kinematics have to be expanded to represent resulting motions connected to possible disturbance values. Because these modifications depend on the target of simulation, a general approach cannot be given. Instead, some short examples are described to give an overview of this method.

5.1. Roundness of Tool

A recurring source of errors in machining is the insufficient roundness of the milling tool. This leads to divergent dimensions and a lack in surface quality. To avoid this kind of errors, the eccentricity of the tool and its tilt can be implemented in the kinematics of the simulation [24]. By performing multiple simulations with varying values, the effects can be reproduced and detailed knowledge of surface quality correlated. In much the same way, maximal tolerances for tool assembly can be defined using the simulation results.

5.2. Gear backlash in milling

Another source of error is the gears backlash. This is a particular effect if robots are used for high precision tasks such as milling. Accuracy depends on the direction of motion and thus leads to an error in positioning, which in turn causes surface errors that have to be avoided. By inserting additional axes at robot joints, the displacement can be reproduced while machining is simulated. If the size of the error is known, it can partly be fixed by modifying the trajectories and iteratively simulating the result [25].

5.3. Estimating corrective actions

If indefinite errors appear in machining, usually corrective actions will be carried out by experienced workers. The decision is usually based on plausibility and often leads to variances in other quality values. A more effective way lies in using simulations that reproduce the assumed error source and the resulting shape of the workpiece. If for example a helical profile grinding process produces shape errors in the cross section, the cause may be a displacement of one of the kinematic axes. By modifying the values of these axes, the shape of the cross section can be reproduced and compared to the measured cross section of the real process. By this means, characteristic properties of the occurring errors can be identified and appropriate actions that modify the corresponding axes can be defined.

6. Conclusion

This paper addresses the task of choosing appropriate models for geometrical-engagement simulation of machining processes. As demands differ locally due to requirements of product design, the advantages and disadvantages of different models for tool and workpiece have to be taken into account. An overview of the differences concerning the type of machining was given and available models were described. Additionally, it was discussed how appropriate models can be chosen according to the process characteristics and the target of the simulation. Moreover, parametrization concerning time steps, tool model and workpiece accuracy was analyzed.

To increase performance and accuracy of actual processes, we introduced a multi-model approach that uses locally-differing tailored workpiece models according to the requirements of the part area.

Finally, it was shown how modification of the kinematics can expand the prospects of geometrical simulation by reproducing possible machining errors and their impact on product quality.

The described work is part of a research project with the aim to build up a high performance simulation to support process monitoring in machining.

Acknowledgements

The scientific work has been supported by the DFG within the research priority program SPP 2086. The authors thank the DFG for this funding and intensive technical support.

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