

Supermembrane limit of Yang-Mills theory

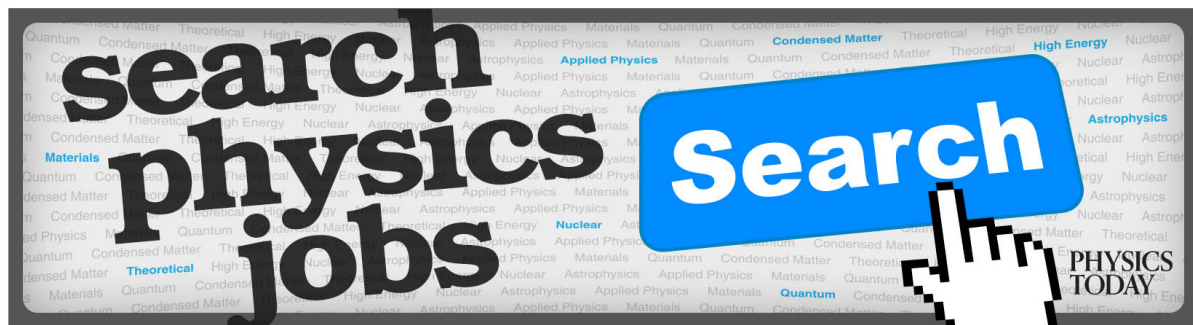
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Supermembrane limit of Yang-Mills theory

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We consider Yang-Mills theory with $N = 1$ super-translation group in eleven auxiliary dimensions as the structure group. The gauge theory is defined on a direct product manifold $\Sigma_3 \times S^1$, where Σ_3 is a three-dimensional Lorentzian manifold and S^1 is a circle. We show that in the infrared limit, when the metric on S^1 is scaled down, the Yang-Mills action supplemented by a Wess-Zumino-type term reduces to the action of an M2-brane. © 2016 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4942186>]

I. INTRODUCTION AND SUMMARY

The theory of membranes and supermembranes has been developed for a long time.^{1–9,24} Supermembranes are basic objects (M2-branes) of M-theory, which are needed for constructing an effective theory of multi-M2-branes.⁹ In this paper, we show that the action of supermembranes moving in $d = 11$ flat $N = 1$ extended superspace can be obtained from a Yang-Mills action functional on $\Sigma_3 \times S^1$ amended by a Wess-Zumino-type term when S^1 shrinks to a point.

Our construction is based on the adiabatic approach to differential equations (introducing “slow” and “fast” variables) which for a direct product manifold²⁵ $Z = X \times Y$ is equivalent to the introduction of a metric $g_X + \varepsilon^2 g_Y$ with a real parameter $\varepsilon \in [0, \infty)$ and a consideration of the limit $\varepsilon \rightarrow 0$.^{10,11,26} The adiabatic limit method has been applied to the description of the scattering of monopoles (i.e., constructing time-dependent solutions of the Yang-Mills-Higgs model), and it has been shown that in the limit $\varepsilon \rightarrow 0$, the scattering of monopoles is parametrized by geodesic motion on the moduli space \mathcal{M}_n of n -monopoles.^{15,16} In other words, the Yang-Mills-Higgs system on $\mathbb{R}^{3,1} = \mathbb{R}^{0,1} \times \mathbb{R}^{3,0}$ for “slow time” reduces to a sigma model on $\mathbb{R}^{0,1}$ (time axis) with \mathcal{M}_n as the target space.

In four dimensions, when $\dim Z = 4$, one has $\dim X = 1, 2$, or 3 and $\dim Y = 3, 2$ or 1 , respectively. In Ref. 10, the adiabatic method was applied to the Yang-Mills instanton equations on a direct product $X \times Y$ of two Riemann surfaces, and it was shown that instanton solutions on $X \times Y$ are in a *one-to-one correspondence* with holomorphic maps from X into the moduli space \mathcal{M} of flat connections on Y . In this case, the Yang-Mills action reduces to the action of a sigma model on X while Y shrinks to a point. The sigma-model target space is \mathcal{M} , and holomorphic maps $X \rightarrow \mathcal{M}$ are the sigma-model instantons. The same result for the Lorentzian signature with $X = \mathbb{R}^{1,1}$ and $Y = T^2$ (two-torus) was derived in Ref. 12: Yang-Mills theory on $\mathbb{R}^{1,1} \times T^2$ in the infrared limit $\varepsilon \rightarrow 0$ (the size of T^2 tends to zero) reduces to a sigma model on $\mathbb{R}^{1,1}$ whose target space is the moduli space of flat connections on T^2 . In Refs. 13 and 14, the same approach was applied to Yang-Mills theory²⁷ on $\mathbb{R}^{2,1} \times S^1$. It was shown that Yang-Mills theory on $\mathbb{R}^{2,1} \times S^1$ reduces to a sigma model on $\mathbb{R}^{2,1}$ whose target space is the space of vacua that arise in the compactification on S^1 . Finally, the adiabatic approach is natural and especially helpful in studying Yang-Mills instantons in more than four dimensions as it was shown in Refs. 11 and 17 (see also Ref. 18 and references therein).

To sum up, Yang-Mills theory on a manifold $X \times Y$ with metric $g_X + \varepsilon^2 g_Y$ flows in the infrared limit $\varepsilon \rightarrow 0$ to a sigma model on X whose target space is the moduli space \mathcal{M} of flat connections on Y when $\dim Y \leq 2$. In our short paper, we reverse this logic. For a given sigma model on X , we construct a Yang-Mills model on $X \times Y$ such that in the infrared limit $\varepsilon \rightarrow 0$, one gets back the

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initial sigma model. In Refs. 19 and 20, this algorithm was carried out for the bosonic string and for the Green-Schwarz superstring in a $d = 10$ Minkowski background. Here we apply this idea to the sigma model describing a supermembrane in a $d = 11$ Minkowski background^{5,7} and introduce a Yang-Mills model on $\Sigma_3 \times S^1$ whose low-energy limit recovers the supermembrane action on Σ_3 .

II. LIE SUPERGROUP \mathfrak{G}

We consider Yang-Mills theory on a direct product manifold $M^4 = \Sigma_3 \times S^1$, where Σ_3 is a three-dimensional Lorentzian manifold with local coordinates x^a , $a, b, \dots = 0, 1, 2$, and a metric tensor $g_{\Sigma_3} = (g_{ab})$, and on the circle S^1 of unit radius parametrized by $x^3 \in [0, 2\pi]$, we choose the metric $g_{S^1} = (g_{33})$ with $g_{33} = 1$. Then $(x^\mu) = (x^a, x^3)$ are local coordinates on M^4 with the metric tensor $(g_{\mu\nu}) = (g_{ab}, g_{33})$, $\mu, \nu, \dots = 0, \dots, 3$. Having in mind open membranes, we assume that Σ_3 has a Lorentzian boundary $\Sigma_2 = \partial\Sigma_3$. For closed membranes, Σ_2 is the empty set.

As the Yang-Mills structure group on M^4 , we consider the coset $G = \text{SUSY}(N = 1)/\text{SO}(10, 1)$ (cf., Ref. 7), where $\text{SUSY}(N = 1)$ is the super-Poincaré group in $d = 11$ dimensions. The coset G is the super-translation group in $d = 11$ auxiliary dimensions. Its generators span the Lie superalgebra $\mathfrak{g} = \text{Lie } G$,

$$\{\xi_A, \xi_B\} = (\gamma^\alpha C)_{AB} \xi_\alpha, \quad [\xi_\alpha, \xi_A] = 0, \quad [\xi_\alpha, \xi_\beta] = 0, \quad (1)$$

where γ^α are the gamma matrices in $d = 11$, C is the charge conjugation matrix, $\alpha = 0, \dots, 10$ and $A = 1, \dots, 32$. The coordinates on G are denoted by X^α and by the components θ^A of a Majorana spinor $\theta = (\theta^A)$, whose conjugate is $\bar{\theta} = \theta^\top C$. The one-forms

$$\Pi^\Delta = \{\Pi^\alpha, \Pi^A\} = \{dX^\alpha - i\bar{\theta} \gamma^\alpha \theta, d\theta^A\} \quad (2)$$

form a basis of (left-invariant) one-forms on G .^{5,7} On the superalgebra $\mathfrak{g} = \text{Lie } G$, we introduce the scalar product $\langle \cdot, \cdot \rangle$ such that

$$\langle \xi_\alpha, \xi_\beta \rangle = \eta_{\alpha\beta}, \quad \langle \xi_\alpha, \xi_A \rangle = 0 \quad \text{and} \quad \langle \xi_A, \xi_B \rangle = 0, \quad (3)$$

where $(\eta_{\alpha\beta}) = \text{diag}(-1, 1, \dots, 1)$ is the Lorentzian metric on $\mathbb{R}^{10,1}$.

III. ACTION FUNCTIONAL

Let us consider the gauge potential $\mathcal{A} = \mathcal{A}_\mu dx^\mu$ with values in \mathfrak{g} and the \mathfrak{g} -valued gauge field

$$\mathcal{F} = \frac{1}{2} \mathcal{F}_{\mu\nu} dx^\mu \wedge dx^\nu \quad \text{with} \quad \mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu + [\mathcal{A}_\mu, \mathcal{A}_\nu], \quad (4)$$

where $[\cdot, \cdot]$ is the commutator or anti-commutator depending on the Grassmann parity of its arguments. On $\Sigma_3 \times S^1$ we have the obvious splitting

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = g_{ab} dx^a dx^b + (dx^3)^2, \quad (5)$$

$$\mathcal{A} = \mathcal{A}_\mu dx^\mu = \mathcal{A}_a dx^a + \mathcal{A}_3 dx^3, \quad (6)$$

$$\mathcal{F} = \frac{1}{2} \mathcal{F}_{\mu\nu} dx^\mu \wedge dx^\nu = \frac{1}{2} \mathcal{F}_{ab} dx^a \wedge dx^b + \mathcal{F}_{a3} dx^a \wedge dx^3. \quad (7)$$

On $M^4 = \Sigma_3 \times S^1$, with its boundary $\partial M^4 = \partial\Sigma_3 \times S^1 = \Sigma_2 \times S^1$, the (super)group of gauge transformations is naturally defined as (see e.g., Refs. 21 and 22)

$$\mathcal{G} = \{g : M^4 \rightarrow G \mid g|_{\partial M^4} = \text{Id}\}. \quad (8)$$

This corresponds to a framing of the gauge bundle over the boundary. For closed membranes, we keep the framing over S^1 .

Employing the adiabatic approach,^{10,11,15,16,22,23} we deform the metric (5),

$$ds_\varepsilon^2 = g_{\mu\nu}^\varepsilon dx^\mu dx^\nu = g_{ab} dx^a dx^b + \varepsilon^2 (dx^3)^2, \quad (9)$$

where $\varepsilon \in [0, \infty)$ is a real parameter. This is equivalent to scaling the radius of our circle, replacing it with S_ε^1 of radius ε . Indices are raised by $g_\varepsilon^{\mu\nu}$, and we have

$$\mathcal{F}_\varepsilon^{ab} = g_\varepsilon^{ac} g_\varepsilon^{bd} \mathcal{F}_{cd} = \mathcal{F}^{ab} \quad \text{and} \quad \mathcal{F}_\varepsilon^{a3} = g_\varepsilon^{ac} g_\varepsilon^{33} \mathcal{F}_{c3} = \varepsilon^{-2} \mathcal{F}^{a3}, \quad (10)$$

where indices in $\mathcal{F}^{\mu\nu}$ have been raised by the non-deformed metric tensor components $g^{\mu\nu}$. In addition, we have $\det(g_\varepsilon^{\mu\nu}) = \varepsilon^2 \det(g_{\mu\nu})$.

We consider the Yang-Mills action functional with a cosmological constant Λ of the form

$$S_\varepsilon = \int_{M^4} d^4x \sqrt{|\det g_{\Sigma_3}|} \left\{ \frac{\varepsilon^2}{2} \langle \mathcal{F}_{ab} \mathcal{F}^{ab} \rangle + \langle \mathcal{F}_{a3} \mathcal{F}^{a3} \rangle + \Lambda \right\}. \quad (11)$$

For $\varepsilon = 1$ and $\Lambda = 0$, it coincides with the standard Yang-Mills action. The value of Λ will be fixed later.

IV. EULER-LAGRANGE EQUATIONS

For the deformed metric, the Yang-Mills equations take the form

$$\varepsilon^2 D_a \mathcal{F}^{ab} + D_3 \mathcal{F}^{3b} = 0 \quad (12)$$

$$\text{and} \quad D_a \mathcal{F}^{a3} = 0. \quad (13)$$

Allowing also the metric g_{Σ_3} on Σ_3 to vary, its Euler-Lagrange equations give the energy-momentum constraint

$$T_{ab}^\varepsilon = \varepsilon^2 (g^{cd} \langle \mathcal{F}_{ac} \mathcal{F}_{bd} \rangle - \frac{1}{4} g_{ab} \langle \mathcal{F}_{cd} \mathcal{F}^{cd} \rangle) + \langle \mathcal{F}_{a3} \mathcal{F}_{b3} \rangle - \frac{1}{2} g_{ab} (\langle \mathcal{F}_{c3} \mathcal{F}^{c3} \rangle + \Lambda) = 0. \quad (14)$$

In the adiabatic limit $\varepsilon \rightarrow 0$, our Equations (12)-(14) become

$$D_3 \mathcal{F}^{3b} \equiv \partial_3 \mathcal{F}^{3b} + [\mathcal{A}_3, \mathcal{F}^{3b}] = 0, \quad (15)$$

$$D_a \mathcal{F}^{a3} \equiv \sqrt{|\det g_{\Sigma_3}|}^{-1} \partial_a (\sqrt{|\det g_{\Sigma_3}|} g^{ab} \mathcal{F}_{b3}) + [\mathcal{A}_a, \mathcal{F}^{a3}] = 0, \quad (16)$$

$$T_{ab}^0 \equiv \langle \mathcal{F}_{a3} \mathcal{F}_{b3} \rangle - \frac{1}{2} g_{ab} (\langle \mathcal{F}_{c3} \mathcal{F}^{c3} \rangle + \Lambda) = 0. \quad (17)$$

V. MODULI SPACE

Let us recall how one considers the reduction of Yang-Mills theory from $\mathbb{R}^3 \times S_\varepsilon^1$ to \mathbb{R}^3 while S_ε^1 shrinks to a point for an ordinary compact Lie group G .^{13,14,28} First, one keeps in lagrangian (11) only the zero modes \mathcal{A}_3^0 in the Fourier expansion on S_ε^1 , which are nothing but the Wilson lines, whose moduli are parametrized by coordinates ϕ^α of the maximal torus in G . These moduli produce a term $\mathcal{F}_{a3} \mathcal{F}^{a3} = \delta_{\alpha\beta} \partial_a \phi^\alpha \partial^a \phi^\beta$ in the lagrangian. Second, for \mathcal{F}_{ab} smoothly depending on ε , the first term in lagrangian (11) vanishes. However, it was observed^{13,14} that for Dirac monopoles, the components \mathcal{F}_{ab} are related with the magnetic photon, having only one component $\tilde{\mathcal{A}}_3^0$ along S_ε^1 , via

$$\varepsilon_{abc} \mathcal{F}^{bc} = \varepsilon^{-1} \partial_a \tilde{\mathcal{A}}_3^0, \quad (18)$$

where the ε^{-1} appears from the metric dependence of the Hodge star operator. These monopole configurations correspond to 't Hooft lines around the circle S_ε^1 . They survive in the limit $\varepsilon \rightarrow 0$, yielding in lagrangian (11) an additional term proportional to $\delta_{\alpha\beta} \partial_a \psi^\alpha \partial^a \psi^\beta$, where ψ^α are coordinates on the Cartan torus in the dual group G^\vee .

In our case, the situation is different since our supermembrane moves in a noncompact superspace, namely, $G = \text{SUSY}(N = 1)/\text{SO}(10, 1)$. For any fixed $x^a \in \Sigma_3$, a generic framed \mathcal{A}_3 is parametrized by the moduli space

$$\Omega G = \text{Map}(S_\varepsilon^1, G)/G = LG/G, \quad (19)$$

i.e., the based loop group, and it can be written in the form

$$\mathcal{A}_3 = \hat{h}^{-1} \partial_3 \hat{h} = h^{-1} \mathcal{A}_3^0 h + h^{-1} \partial_3 h \quad \text{with} \quad \hat{h} = h_0 h \in \Omega G \quad \text{and} \quad \mathcal{A}_3^0 = h_0^{-1} \partial_3 h_0 \in \mathfrak{g}, \quad (20)$$

where $h \in \Omega G$ and $h_0 \in G \subset \Omega G$. Note that neither \hat{h} nor h belong to the gauge group. In fact, (20) defines a map $\hat{h} \mapsto h_0$ from ΩG to G . The Wilson lines \mathcal{A}_3^0 are parametrized by G . Since our aim is the supermembrane moving in G , we choose the magnetic photon component $\tilde{\mathcal{A}}_3^0$ to vanish. Furthermore, in the spirit of the adiabatic approach, it is assumed that all moduli of \mathcal{A}_3 are functions of $x^a \in \Sigma_3$, i.e., both functions h and h_0 depend on x^a via their moduli. We denote by \mathcal{N} the space of all \mathcal{A}_3 given by (20), and we define the projection $\pi : \mathcal{N} \rightarrow G$ since we want to keep only \mathcal{A}_3^0 in the limit $\varepsilon \rightarrow 0$.

VI. EFFECTIVE ACTION

The variable \mathcal{A}_3^0 , as introduced in (20), depends on $x^a \in \Sigma_3$ only via the moduli parameters $(X^\alpha, \theta^A) \in G$. Then the moduli of \mathcal{A}_3^0 define a map

$$(X, \theta) : \Sigma_3 \rightarrow G \quad \text{with} \quad (X(x^a), \theta(x^a)) = (X^\alpha(x^a), \theta^A(x^a)). \tag{21}$$

The map (21) is not arbitrary, it is constrained by Equations (15)-(17). The derivative $\partial_a \mathcal{A}_3$ belongs to the tangent space $T_{\mathcal{A}_3} \mathcal{N}$. With the help of the projection $\pi : \mathcal{N} \rightarrow G$ with fibres Q , one can decompose $\partial_a \mathcal{A}_3$ into two parts,

$$T_{\mathcal{A}_3} \mathcal{N} = \pi^* T_{\mathcal{A}_3^0} G \oplus T_{\mathcal{A}_3} Q \quad \Leftrightarrow \quad \partial_a \mathcal{A}_3 = \Pi_a^\Delta \xi_{\Delta 3} + D_3 \epsilon_a, \tag{22}$$

where $\Delta = (\alpha, A)$ and

$$\Pi_a^\alpha = \partial_a X^\alpha - i \bar{\theta} \gamma^\alpha \partial_a \theta \quad \text{and} \quad \Pi_a^A = \partial_a \theta^A. \tag{23}$$

In (22), ϵ_a are \mathfrak{g} -valued parameters ($D_3 \epsilon_a \in T_{\mathcal{A}_3} Q$), and the vector fields $\xi_{\Delta 3}$ on G can be identified with the generators $\xi_\Delta = (\xi_\alpha, \xi_A)$ of G .

On $\xi_{\Delta 3}$, we impose the gauge-fixing condition

$$D_3 \xi_{\Delta 3} = 0 \stackrel{(22)}{\implies} D_3 D_3 \epsilon_a = D_3 \partial_a \mathcal{A}_3. \tag{24}$$

Recall that \mathcal{A}_3 is determined by (20) and \mathcal{A}_a are yet free. In the adiabatic approach, one can naturally choose $\mathcal{A}_a = \epsilon_a$ (cf., Refs. 15 and 23), where ϵ_a are defined from (24). Then one obtains

$$\mathcal{F}_{a3} = \partial_a \mathcal{A}_3 - D_3 \mathcal{A}_a = \partial_a \mathcal{A}_3 - D_3 \epsilon_a = \Pi_a^\Delta \xi_{\Delta 3} \in T_{\mathcal{A}_3^0} G. \tag{25}$$

Substituting (25) into (15), we see that the latter is resolved due to (24). Plugging (25) into the action (11) with $\varepsilon \rightarrow 0$ and fixing $\Lambda = -1$, we obtain the effective action

$$S_0 = 2\pi \int_{\Sigma_3} d^3x \sqrt{|\det g_{\Sigma_3}|} \left(g^{ab} \Pi_a^\alpha \Pi_b^\beta \eta_{\alpha\beta} - 1 \right). \tag{26}$$

It coincides with the kinetic part of the supermembrane action.⁵ One may also show (cf., Ref. 19) that Equation (16) are equivalent to the Euler-Lagrange equations for (X^α, θ^A) following from (26). Finally, substituting (25) into (17), we arrive at

$$\Pi_a^\alpha \Pi_b^\beta \eta_{\alpha\beta} - \frac{1}{2} g_{ab} (g^{cd} \Pi_c^\alpha \Pi_d^\beta \eta_{\alpha\beta} - 1) = 0 \tag{27}$$

which may also be obtained from (26) by varying the metric.

From (27), it follows that

$$g_{ab} = \eta_{\alpha\beta} \Pi_a^\alpha \Pi_b^\beta, \tag{28}$$

and, after putting this back into (26), we get the standard Nambu-Goto lagrangian for the supermembrane. It is obvious that for $\theta = 0$, the bosonic membrane action remains.

VII. WESS-ZUMINO-TYPE TERM

The action (26) is not the full supermembrane action, since the latter needs also a Wess-Zumino-type term.^{5,7} Continuing our ‘‘reverse engineering’’ strategy, we look for an addition to the

Yang-Mills action (11) which in the infrared limit $\varepsilon \rightarrow 0$ will give us this Wess-Zumino-type term. This addition can be incorporated as follows. We extend Σ_3 to a Lorentzian 4-manifold Σ_4 with boundary $\Sigma_3 = \partial\Sigma_4$ and (local) coordinates $x^{\hat{a}}$, $\hat{a} = 0, 1, 2, 4$. On Σ_4 , one introduces the four-form^{5,7}

$$\Omega_4 = \langle \Pi \wedge \Pi \wedge \Pi \wedge \Pi \rangle = f_{\Delta\Lambda\Sigma\Gamma} \Pi^\Delta \wedge \Pi^\Lambda \wedge \Pi^\Sigma \wedge \Pi^\Gamma = \hat{d}\bar{\theta}\gamma_{[\alpha}\gamma_{\beta]} \wedge \hat{d}\theta \wedge \Pi^\alpha \wedge \Pi^\beta = \hat{d}\Omega_3 \quad (29)$$

for $\Pi := \Pi_{\hat{a}} dx^{\hat{a}} = \Pi_{\hat{a}}^\Delta dx^{\hat{a}} \xi_{\Delta}$, where $\hat{d} = dx^{\hat{a}} \partial_{\hat{a}}$. The explicit form of the constants $f_{\Delta\Lambda\Sigma\Gamma}$ and the three-form Ω_3 can be found in Refs. 5 and 7. Then one adds to the action (26) the term

$$S_{WZ} = \int_{\Sigma_4} \Omega_4 = \int_{\Sigma_3} \Omega_3, \quad (30)$$

which completes the M2-brane action. In the setup we investigate here, we take the direct product manifold $\Sigma_4 \times S^1$, extend the index a in (23) to $\hat{a} = 0, 1, 2, 4$ and introduce one-forms on Σ_4 ,

$$F_3 := \mathcal{F}_{\hat{a}3} dx^{\hat{a}}. \quad (31)$$

Adding (with a proper coefficient) the Wess-Zumino-type term

$$S_{WZ}^{YM} = \int_{\Sigma_4 \times S^1} f_{\Delta\Lambda\Sigma\Gamma} F_3^\Delta \wedge F_3^\Lambda \wedge F_3^\Sigma \wedge F_3^\Gamma \wedge dx^3 \quad (32)$$

to the action functional S_ε from (11) with $\Lambda = -1$, we obtain the gauge-field action which in the adiabatic limit $\varepsilon \rightarrow 0$ becomes the M2-brane action. This implies that features of Yang-Mills theory with the action (11) + (32) for $\varepsilon \neq 0$ can be reduced to properties of supermembranes by taking the limit $\varepsilon \rightarrow 0$.

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- ²⁴ See Refs. [8](#) and [9](#) for historical reviews and more references.
- ²⁵ The direct product structure is not necessary for the application of the adiabatic method. In general, it is enough if there is a fibration $Z \rightarrow X$ or if X is a calibrated submanifold of Z .
- ²⁶ In the physics literature this limit is called infrared or low-energy limit (see e.g., Refs. [12–14](#)).
- ²⁷ In fact, in Refs. [12–14](#) the authors considered $\mathcal{N} = 4$ and $\mathcal{N} = 2$ super-Yang-Mills theories but the restriction to the pure Yang-Mills subsector does not change the picture.
- ²⁸ For simplicity, we restrict ourselves to the pure Yang-Mills subsector of the supersymmetric theories in Refs. [13](#) and [14](#).