Erratum: Ground-state properties of few dipolar bosons in a quasi-one-dimensional harmonic trap [Phys. Rev. A 81, 063616 (2010)]

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In the Appendix of our paper, we derive the effective one-dimensional (1D) dipole-dipole interaction (DDI) potential. The derivation and the result are incomplete, since the singularity of the three-dimensional (3D) DDI at zero distance is not properly accounted for. To see this, we introduce a small $\epsilon > 0$ in the denominator of the 3D DDI. The first steps of the calculation are analogous to those shown in the Appendix until one arrives at[1]

$$V_{\mathrm{dd}}^{(\epsilon)}(u) = \frac{1}{4}[1+3\cos(2\theta)]\frac{d^2}{l_\perp^3}\int_0^\infty dw\, w\, e^{-w^2/2} \times \frac{w^2-2u^2}{\sqrt{w^2+u^2+\epsilon^2}^5} \tag{1}$$

with $w = \rho/l_\perp$ and $u = x/l_\perp$. This is equal to

$$\begin{aligned} V_{\mathrm{dd}}^{(\epsilon)}(u) &= \frac{1}{4}[1+3\cos(2\theta)]\frac{d^2}{l_\perp^3}\left\{\int_0^\infty dw\, w\, e^{-w^2/2}\frac{w^2-2(u^2+\epsilon^2)}{\sqrt{w^2+(u^2+\epsilon^2)}^5} + 2\epsilon^2\int_0^\infty dw\, w\frac{e^{-w^2/2}}{\sqrt{w^2+u^2+\epsilon^2}^5}\right\} \\ &= -\frac{1}{8}[1+3\cos(2\theta)]\frac{d^2}{l_\perp^3}\left\{[-2v+\sqrt{2\pi}\,(1+v^2)\,e^{v^2/2}\,\mathrm{erfc}(v/\sqrt{2})] - \frac{8}{3}\delta_\epsilon(u)\right\} \end{aligned} \tag{2}$$

with $v = \sqrt{u^2+\epsilon^2}$ and

$$\delta_\epsilon(u) = \frac{3}{2}\int_0^\infty dw\, w\frac{\epsilon^2 e^{-w^2/2}}{\sqrt{w^2+u^2+\epsilon^2}^5}. \tag{3}$$

In the limit $\epsilon \to 0$, one finally obtains

$$\lim_{\epsilon\to 0} V_{\mathrm{dd}}^{(\epsilon)}(u) = -\frac{1}{8}[1+3\cos(2\theta)]\frac{d^2}{l_\perp^3}\left\{[-2u+\sqrt{2\pi}\,(1+u^2)e^{u^2/2}\,\mathrm{erfc}(u/\sqrt{2})] - \frac{8}{3}\delta(u)\right\}, \tag{4}$$

i.e., apart from the second term in square brackets, which is already given in the paper, there is an additional δ term present in the 1D DDI. This term is missing in Eqs. (2)– (4) of our paper. Let us briefly show that $\lim_{\epsilon\to 0}\int_{-\infty}^{\infty} du\, \delta_\epsilon(u) = 1$. One finds that

$$\begin{aligned} \int_{-\infty}^{\infty} du\, \delta_\epsilon(u) &= \frac{3}{2}\int_0^\infty dw\, \epsilon^2 w e^{-w^2/2}\left[\frac{u[2u^2+3(w^2+\epsilon^2)]}{3(w^2+\epsilon^2)^2\sqrt{w^2+u^2+\epsilon^2}^3}\right]_{u=-\infty}^{\infty} = 2\epsilon^2\int_0^\infty dw \frac{we^{-w^2/2}}{(w^2+\epsilon^2)^2} \\ &= \frac{1}{2}\epsilon^2 e^{\epsilon^2/2}\int_{\epsilon^2/2}^\infty dt\frac{e^{-t}}{t^2} = 1+\frac{1}{2}\epsilon^2 e^{\epsilon^2/2}\,\mathrm{Ei}(-\epsilon^2/2) \to 1 \quad \text{for } \epsilon \to 0. \end{aligned} \tag{5}$$

The third step follows from the substitution $t = (w^2+\epsilon^2)/2$, the fourth step from an integration by parts, and the last step from $\mathrm{Ei}(x) \approx \ln(x)$ and $x^2\ln(x) \to 0$ for $x \to 0$, where $\mathrm{Ei}(x)$ is the exponential integral.

It can be seen from Eq. (4) that the strength of the δ interaction depends on the dipole orientation with respect to the weak trap axis (the angle θ). This is different from the δ contribution, which stems from the point limit of a real (extended) dipole [1], where the strength depends on the relative orientation of the two interacting dipoles. In total there are three δ terms, which originate from the van der Waals interaction, the point limit of a real dipole, and the integration over the transverse directions.

In our paper, we study the effect of the 1D DDI without the δ terms and perform a sweep of the interaction strength. Such a sweep can be performed experimentally, when the strength of all δ terms is tuned to zero by means of a Feshbach resonance. Then, the strength of the interaction term in square brackets with respect to the level spacing can be tuned by changing the axial angular frequency, as described at the end of Sec. III.

[1]Equation numbers in this erratum do not reflect the numbering in the original paper.

As a test of Eq. (4) we finally calculate the interaction energy of N bosonic dipoles, which are oriented along the x axis ($\theta = 0$) and occupy the ground state of the axial harmonic oscillator $\phi_0(x) = e^{-x^2/(2l^2)}/\sqrt{l\sqrt{\pi}}$. The result is given by

$$E_{\rm int} = -\frac{N^2 d^2}{3\sqrt{2\pi} l l_\perp^2}\left[\frac{1+2\kappa^2}{1-\kappa^2} - \frac{3\kappa^2 \operatorname{artanh}(\sqrt{1-\kappa^2})}{\sqrt{1-\kappa^2}^3}\right] \tag{6}$$

with $\kappa = l_\perp/l$. This agrees with the second term of Eq. (5.8) in the review of Lahaye *et al.* [2] [note that $a_{\rm dd} = mC_{\rm dd}/(12\pi\hbar^2)$, $C_{\rm dd} = 4\pi d^2$, $a_{\rm ho} = \sqrt{\hbar/(m\bar{\omega})}$, $\sigma_\rho = l_\rho/a_{\rm ho}$, and $\sigma_z = l_z/a_{\rm ho}$]. The function in square brackets is 1 in the limit $\kappa \to 0$ and decreases monotonously to -2 in the limit $\kappa \to \infty$ with a zero crossing at $\kappa = 1$. In a cigar-shaped trap ($\kappa \ll 1$) the dipoles are mainly in a head-to-tail configuration, in which the DDI is attractive, and hence the interaction energy is negative, $E_{\rm int} < 0$. In a pancake-shaped trap ($\kappa \gg 1$) the dipoles are mainly in a side-by-side configuration, in which the DDI is repulsive, and hence the interaction energy is positive, $E_{\rm int} > 0$. In marked contrast to this behavior, the interaction energy would always have been negative if the δ term in Eq. (4) had been neglected.

[1] D. J. Griffiths, Am. J. Phys. **50**, 698 (1982).

[2] T. Lahaye, C. Menotti, L. Santos, M. Lewenstein, and T. Pfau, Rep. Prog. Phys. **72**, 126401 (2009).