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Test signal generation for analog circuits

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Abstract. In this paper a new test signal generation approach for general analog circuits based on the variational calculus and modern control theory methods is presented. The computed transient test signals also called test stimuli are optimal with respect to the detection of a given fault set by means of a predefined merit functional representing a fault detection criterion. The test signal generation problem of finding optimal test stimuli detecting all faults form the fault set is formulated as an optimal control problem. The solution of the optimal control problem representing the test stimuli is computed using an optimization procedure. The optimization procedure is based on the necessary conditions for optimality like the maximum principle of Pontryagin and adjoint circuit equations.

1 Introduction

Advances in EDA technology have increased the size and complexity of integrated circuits. As a result the test costs have become a key part of the overall manufacturing costs. Although the area of the analog part of a mixed-signal IC is much smaller than the digital one, the test costs are dominated by the analog part because of its more complex specifications. For this reason tools and efficient techniques for the generation of specific tests, which have the ability to reduce the test time and thus the test costs, are needed.

In transient testing (Gomes and Chatterjee, 1999; Variyam et al., 1999; Burdiek, 2001), the circuit under test (CUT) is excited with a transient test stimulus and the circuit response is sampled at specified time points for fault detection. In this paper a new test signal generation method based on control theory techniques like Pontryagin's maximum principle is presented. It should be noted that optimal control theory methods such as the maximum principle are based on the variational calculus. The proposed test generation approach formulated as an optimal control problem generates optimum transient test stimuli for a general analog circuit. A Lagrangian merit functional required for the optimal control problem serves as the fault detection criterion. The functional, which indirectly depends on the controls of the circuit representing the test stimuli, is based on the difference between the good test response and all faulty test responses. The solution of the control problem given by the optimal controls, is computed using an optimization procedure. The optimization procedure maximizes the merit functional with respect to the controls and thus enhances the fault detection capability. Since the procedure takes advantage from necessary optimality conditions such as the maximum principle, it does not need the gradient of the merit functional with respect to the controls. The solution of the optimal control problem must satisfy the necessary conditions for optimality like the maximum principle. Thus, all test stimuli, which do not satisfy Pontryagin's maximum principle, are not optimal and thus do not maximize the fault detection criterion. Therefore, they cannot be solutions of the optimal control problem.

In the next section the test generation problem is formulated as an optimal control problem. The optimization procedure and the necessary conditions for optimality, which have to be fulfilled by the optimal controls representing the test stimuli, are described in Sect. 3. Experimental results are presented in Sect. 4. A conclusion of the paper is given in Sect. 5.

2 Problem formulation

Let $\mathbf{F}(\dot{\mathbf{z}}(\mathbf{x}), \mathbf{x}, \mathbf{u}, t) = \mathbf{0}$ be the differential algebraic equations (DAE's) of the circuit under test (CUT). The DAE system, which models the circuit correctly, arises from the modified nodal analysis (MNA). The control vector $\mathbf{u}(t)$, with $\mathbf{u} \in U \subset R^p$, represents the controls of all independent voltage and current sources of the circuit. The state vector $\mathbf{x}$, with $\mathbf{x} \in R^n$, represents the node potentials $\mathbf{v}_n$ and the branch currents $\mathbf{i}_b$ of the modified nodal description. The vector function $\mathbf{z}(\mathbf{x})$ describes the charges of the voltage controlled capacitors and the fluxes of the current controlled inductors of the circuit. The variable vector of the adjoint network of the circuit (Director and Rohrer, 1969) is called the costate vector denoted by ψ.

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$$\mathbf{x} = \begin{bmatrix} \mathbf{v}_n \\ \mathbf{i}_b \end{bmatrix} \mathbf{u} = \begin{bmatrix} \mathbf{J} \\ \mathbf{E} \end{bmatrix} \psi = \begin{bmatrix} \hat{\mathbf{v}}_n \\ \hat{\mathbf{i}}_b \end{bmatrix} \mathbf{z} = \begin{bmatrix} \mathbf{q} \\ \phi \end{bmatrix} \quad (1)$$

The given fault list of the CUT containing k parametric and catastrophic faults is termed by the fault set $S_f = \{f_1, \cdots, f_k\}$. Throughout the paper good device is denoted with index g and the faulty devices with index f. Vectors and terms referring to the good and all faulty circuits are denoted with index a. All types of vectors used in this paper are shown in Eq. (1).

$$J_a(\mathbf{x}_a) = -\bar{J}_a(\mathbf{x}_a) \stackrel{!}{=} \min \quad (2)$$

$$\bar{J}_a(\mathbf{x}_a(t)) = \int_{t_0}^{t_f} \bar{f}_a\left(\mathbf{x}_g, \mathbf{x}_{f_1}, \cdots, \mathbf{x}_{f_k}\right) dt \quad (3)$$

$$m_j \le u_j(t) \le M_j \ , \ j = 1, \ldots, p \ , \ t \in [t_0, t_f] \quad (4)$$

$$\mathbf{F}_a(\dot{\mathbf{z}}_a, \mathbf{x}_a, \mathbf{u}, t) = \begin{bmatrix} \mathbf{F}_g(\dot{\mathbf{z}}_g, \mathbf{x}_g, \mathbf{u}, t) \\ \vdots \\ \mathbf{F}_{f_k}(\dot{\mathbf{z}}_{f_k}, \mathbf{x}_{f_k}, \mathbf{u}, t) \end{bmatrix} = \mathbf{0} \quad (5)$$

In Eqs. (2)–(5) the test generation problem is formulated as an optimal control problem. Without loss of generality we can describe the fault detection criterion $\bar{J}_a(\mathbf{x}_a(t))$ of the test generation problem by a Lagrangian merit functional. This is possible, since other types of functionals representing a fault detection criterion can be transformed into a Lagrangian functional. The argument of the merit functional is the state vector $\mathbf{x}_a$ containing the state vectors of the good and all faulty circuits $\mathbf{x}_a^t = (\mathbf{x}_g, \cdots, \mathbf{x}_{f_k})^t$. The functional $\bar{J}_a(\mathbf{x}_a(t))$, which only depends on the circuits states $\mathbf{x}_a$, is is based on the difference between the good test response and all faulty test responses. The functional $\bar{J}_a$ cannot depend on the controls $\mathbf{u}$, since only test response measurements can be used for fault detection. The optimal control problem defined in Eqs. (2)–(5) is to find a control vector $\mathbf{u}^*(t)$ form the set of admissible controls Eq. (4) which causes the DAE system Eq. (5) to follow an admissible trajectory $\mathbf{x}_a^*(t)$ that minimizes the merit functional $J_a(\mathbf{x}_a^*(t))$ in Eq. (2) and thus maximizes the fault detection criterion. Thus the solution of our problem is given by the optimal control vector $\mathbf{u}_a^*(t)$ and the optimal state vector $\mathbf{x}_a^*(t)$.

For a fixed $\mathbf{u}(t)$ the computation of the solution of the DAE system in Eq. (5) and the computation of the merit functional $J_a(\mathbf{x}_a(t))$ is performed by a fault simulation, simulating each fault of the fault set S_f sequentially. This can be done, since the DAE systems of the good and all faulty circuits are not coupled with each other.

3 Optimization procedure

In this section we describe the optimization procedure, which is used to solve the general test signal generation problem defined in Sect. 2–5. We first formulate the necessary conditions for the optimal controls $\mathbf{u}$ required for the optimization process. To minimize the merit functional $J_a(\mathbf{x}_a)$ from Eq. (2) under the constrains given by Eq. (4) and Eq. (5), we form an augmented merit functional L_a including the constraints. Using the concept of Lagrange multipliers we include the DAE systems of the good and all faulty circuits into an augmented merit functional $L_a(\mathbf{x}_a, \psi_a, \mathbf{u}, t)$ Eq. (6). The Hamiltonian function H_a resulting from L_a is given by Eq. (7).

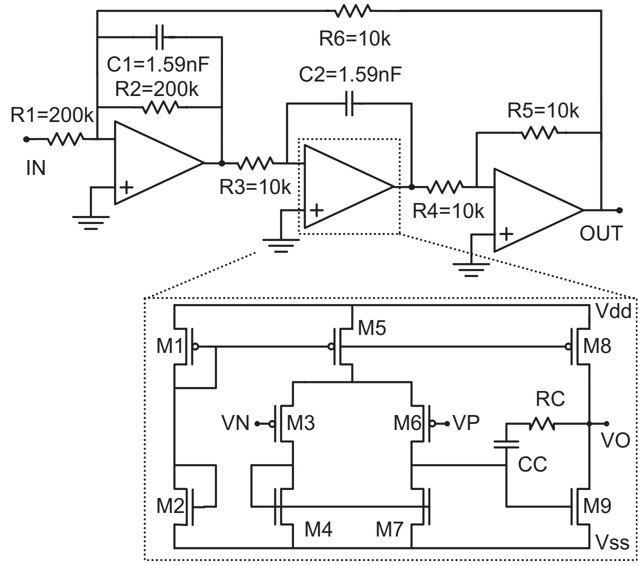

Fig. 1. Biquad filter (resonance frequency $f_r = 10$ kHz).

$$L_a = \int_{t_0}^{t_f} -H_a(\mathbf{x}_a, \psi_a, \mathbf{u}, t) dt \stackrel{!}{=} \min \quad (6)$$

$$H_a(\mathbf{x}_a, \psi_a, \mathbf{u}, t) = \bar{f}_a(\mathbf{x}_g, \mathbf{x}_{f_1}, \ldots, \mathbf{x}_{f_k}) +$$
$$\psi_g^t \mathbf{F}_g\left(\dot{\mathbf{z}}_g, \mathbf{x}_g, \mathbf{u}, t\right) + \sum_{i=1}^{k} \psi_{f_i}^t \mathbf{F}_{f_i}\left(\dot{\mathbf{z}}_{f_i}, \mathbf{x}_{f_i}, \mathbf{u}, t\right) \quad (7)$$

The costate vector ψ_a is composed of the costate vector ψ_g of the adjoint network of the good circuit and all costate vectors ψ_{f_i} of the faulty circuits. From the first variation of L_a we obtain the adjoint DAE system in Eq. (8) including the adjoint systems of the good and all faulty circuits.

$$\begin{bmatrix} \mathbf{S}_g & & \\ & 0 \ddots & \\ & & \mathbf{S}_{f_k} \end{bmatrix}^t \begin{bmatrix} \dot{\psi}_g \\ \vdots \\ \psi_{f_k} \end{bmatrix} - \begin{bmatrix} \mathbf{G}_g & & \\ & 0 \ddots & \\ & & \mathbf{G}_{f_k} \end{bmatrix}^t \begin{bmatrix} \psi_g \\ \vdots \\ \psi_{f_k} \end{bmatrix} = \begin{bmatrix} \frac{\partial \bar{f}_a}{\partial \mathbf{x}_g} \\ \vdots \\ \frac{\partial \bar{f}_a}{\partial \mathbf{x}_{f_k}} \end{bmatrix} \quad (8)$$

The matrices S and G are abbreviations for time dependent Jacobian matrices and arise from the storage and resistive elements of the circuit.

$$\mathbf{S}(t) = \left.\frac{\partial \mathbf{F}}{\partial \dot{\mathbf{z}}} \frac{\partial \mathbf{z}}{\partial \mathbf{x}}\right|_{\mathbf{x}(t)} \qquad \mathbf{G}(t) = \left.\frac{\partial \mathbf{F}}{\partial \mathbf{x}}\right|_{\mathbf{x}(t)} \quad (9)$$

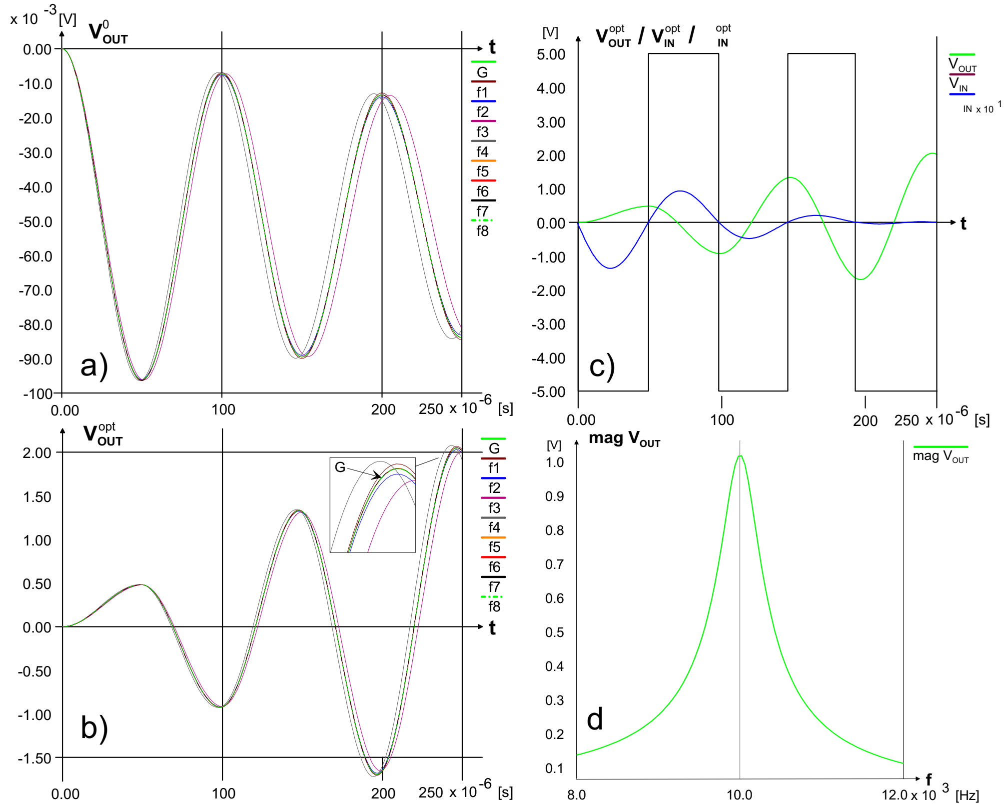

Fig. 2. **(a)** Test responses before optimization **(b)** Test responses after optimization **(c)** Test response V_{out} of the good device, optimal test stimulus V_{in} and its corresponding waveform $\psi_{s_{in}}$ **(d)** Frequency response of the biquad filter.

The classical methods of the variational calculus cannot be applied to problems with a closed set U as control region. For problems with bounded controls $\mathbf{u}$ a method known as Pontryagin's maximum principle (Pontrjagin et al., 1964) is used. In situations with constraints on the control variables the necessary condition for optimal control Eq. (10) is replaced with Eq. (11), which states the maximum principle.

$$\nabla_{\mathbf{u}} \; J_a = - \frac{\partial H_a}{\partial \mathbf{u}} = \mathbf{0} \tag{10}$$

$$\max_{\mathbf{u} \in U} \; H_a(\mathbf{x_a}(t), \psi_\mathbf{a}(t), \mathbf{u}(t), t) = H_a(\mathbf{x_a}, \psi_\mathbf{a}, \hat{\mathbf{u}}, t) \tag{11}$$

Suppose $\hat{\mathbf{u}}$ is an optimal control, which minimizes the merit functional of the control problem. Than Pontryagin's maximum principle Eq. (11) says, that $\hat{\mathbf{u}}$ and its corresponding pair $(\mathbf{x}_a, \psi_a)$ maximizes the Hamiltonian H_a for all $t \in [t_0, t_f]$ and for all admissible controls $\mathbf{u} \in U$. The maximum principle is a necessary condition for optimal control and is used as a calculation base for the optimal controls. In case of a free end point $\mathbf{x}(t_f)$ given with the test generation problem the optimal solution of the control problem $\mathbf{u}^*(t)$, $\mathbf{x}_a^*(t)$ must satisfy the system of the circuit equations (5) and their initial values $\mathbf{x}_a(t_0)$, the adjoint equations (8) and their final values $\psi_a(t_f) = \mathbf{0}$ and the maximum principle of Pontryagin. Since modified nodal equations are used for circuit description and these are linear in $\mathbf{u}$, the application of the maximum principle lead to the qualitative form of optimal test signals (Burdiek, 2002), given by Eq. (12). Only test signals of this form are possible candidates for optimal controls.

$$u_j(t) = \begin{cases} M_j\,, & \mathbf{b}^t(\psi_g + \cdots + \psi_{f_k}) \geq 0 \\ m_j\,, & \mathbf{b}^t(\psi_g + \cdots + \psi_{f_k}) < 0 \end{cases} \quad , \; 1 \leq j \leq p \tag{12}$$

Since the circuit equations (5) are coupled with the adjoint equations (8) an optimization procedure is needed for the calculation of the optimal controls. In the following the essential steps of the optimization procedure are explained. Starting with an initial control vector $\mathbf{u}^0(t)$ the solution $\mathbf{x}_a^0(t)$ of the DAE system in Eq. (5) is computed in the first step. The evaluation of the merit functional J_a^0 is performed during the simulation of Eq. (5). Using the state vector $\mathbf{x}_a^0(t)$ for the computation of the matrices S and G the solution vector $\psi_a^0(t)$ of the adjoint DAE system Eq. (8) is calculated in the next step. In the last step Pontriyagin's maximum principle is applied using Eq. (12) to obtain the control vector $\hat{\mathbf{u}}^0$. The next iterate $\mathbf{u}_a^1(t)$ is calculated with the aid of Eq. (13), whereby the stepsize α_l is optimal with $\alpha_l = 1$. The optimization procedure terminates when the minimal sequence $J_a^0 > J_a^1 > \ldots$ aborts.

$$\mathbf{u}^{l+1} = (1 - \alpha_l)\mathbf{u}^l + \hat{\mathbf{u}}^l \quad l = 0, 1\ldots \tag{13}$$

4 Experimental results

In this section the test generation method is applied to a biquad filter shown in Fig. 1. To demonstrate the approach the test generation procedure is applied to a small fault set of eight hard to detect parametric faults. For the fault detection criterion $\bar{J}_a$ we use the functional described by Eq. (14). The weighting factors w_{f_i} used to distinguish faults in $\bar{J}_a$ are determined from an initial fault simulation.

$$\bar{J}_a(\mathbf{x}_a(t)) = \sum_{i=1}^{k} w_{f_i} \phi_{f_i} \left(V_{out_g}, V_{out_{fi}}\right) \qquad (14)$$

$$\phi_{f_i} = \int_{t_0}^{t_f} \left| V_{out_g}(t) - V_{out_{fi}}(t) \right| dt \,, \quad \phi_a = \sum_{i=1}^{k} \phi_{f_i} \qquad (15)$$

The unit step function $1(t)$ is used as initial test stimulus for the procedure denoted by u^0. The optimal test stimulus u^{opt} (V_{in}^{opt}) shown in Fig. 2c was generated within 2 iterations. The simulation results and the parameters of the test generation procedure are listed in Table 1. The test responses V_{out} before and after the optimization of the CUT are shown in Fig. 2a and Figure 2b. As one can see form the simulation results the generated test stimulus $u^{opt}(t)$ significantly enhances the fault detection capability. The switching points (SP's) of the test stimulus computed with Eq. (12) are given by the isolated zeros of the waveform $\psi_{s_{in}}$ shown in Fig. 2c. Obviously, it is good a strategy to test the filter in the near of its resonance frequency $f_r = 10\,\text{kHz}$, which is shown in Fig. 2d. For this reason suitable test signals for the biquad filter have to contain a first harmonic, which is approximately f_r. This is the case for our generated test signal.

After the test generation process a fault simulation of 150 faults was carried out to determine the performance of the generated test stimulus. This resulted in a fault coverage of 97 percent. The test generation approach proposed in this paper has been implemented in a C++ program named TORAD (**T**est Generat**or** for **A**nalog **D**evices). The simulator TORAD supports several circuit analyses, like transient analysis and transient sensitivity analysis. The last one includes the ability to simulate the transient behaviour of adjoint networks.

5 Conclusion

In this paper a new test signal generation approach based on modern control theory methods such as the maximum principle of Pontryagin was presented. The proposed method, which was formulated as an optimal control problem, generates optimum transient test signals for general analog circuits. An optimization procedure was used for the computation of the solution of the control problem. The procedure takes advantage from necessary optimality conditions such as the maximum principle, so that it does not need the gradient of the merit functional with respect to the controls. Since the procedure generates optimal test signals, it is best suited for the detection of hard to detect faults in analog circuits.

Table 1. Results and parameters of the optimization procedure

Results of the optimization procedure:			
Faults	w_{fi}	Φ_{fi}^{0}	Φ_{fi}^{opt}
f_1 : F(p,r2,resistance,5%)	5.56	5.898e-08	1.092e-06
f_2 : F(p,r2,resistance,-5%)	5.08	6.438e-08	1.194e-06
f_3 : F(p,c2,capacitance,5%)	0.26	1.193e-06	2.155e-05
f_4 : F(p,c2,capacitance,-5%)	0.25	1.238e-06	2.255e-05
f_5 : F(p,m:op2:1,l,5%)	276	1.116e-09	7.952e-08
f_6 : F(p,m:op2:1,l,-5%)	361	8.536e-10	7.836e-08
f_7 : F(p,m:op1:1,l,5%)	267	1.827e-09	7.982e-08
f_8 : F(p,m:op1:1,l,-5%)	350	1.476e-09	8.030e-08
Φ_a^0 = 2.5602346e-06	Φ_a^{opt} = 4.6711551e-05	Φ_a^{opt} / Φ_a^0 = 18.3	
SPs of u^0 (t) = V_{in}^0(t) : { (0.00ms,1V) }			
SPs of u^{opt}(t) = V_{in}^{opt}(t) : { (0.0us, -5V) (49.4us, 5V) (98.1us, -5V) (146us, 5V) (193us, -5V) (250us, 5V) }			
Number of iterations l : 2			
Parameters of the optimization procedure:			
Start point of time t_0 : 0.0us	End point of time t_f : 250us		
Upper bound M_{in} of u_{in} : 5V	Lower bound m_{in} of u_{in} : -5V		
Minimum time distance between SPs : 25us			

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