physics

Article

Key Experiment and Quantum Reasoning

Moritz Waitzmann [1], Kim-Alessandro Weber [2], Susanne Wessnigk [3] and Ruediger Scholz [4,*]

1 Institut fuer die Didaktik der Mathematik und Physik, Leibniz Universitaet Hannover, 30167 Hannover, Germany
2 Institut fuer Quantenoptik, Leibniz Universitaet Hannover, 30167 Hannover, Germany
3 QUEST Leibniz Research School, Leibniz Universitaet Hannover, 30167 Hannover, Germany
4 CRC 1227 Designed Quantum States of Matter, Leibniz Universitaet Hannover, 30167 Hannover, Germany
* Correspondence: r.scholz@iqo.uni-hannover.de; Tel.: +49-511-7622850

Abstract: For around five decades, physicists have been experimenting with single quanta such as single photons. Insofar as the practised ensemble reasoning has become obsolete for the interpretation of these experiments, the non-classical intrinsic probabilistic nature of quantum theory has gained increased importance. One of the most important exclusive features of quantum physics is the undeniable existence of the superposition of states, even for single quantum objects. One known example of this effect is entanglement. In this paper, two classically contradictory phenomena are combined to one single experiment. This experiment incontestably shows that a single photon incident on an optical beam splitter can either be reflected or transmitted. The almost complete absence of coincident clicks of two photodetectors demonstrates that these two output states are incompatible. However, when combining these states using two mirrors, we can observe interference patterns in the counting rate of the single photon detector. The only explanation for this is that the two incompatible output states are prepared and kept simultaneously—a typical consequence of a quantum superposition of states. (Semi-)classical physical concepts fail here, and a full quantum concept is predestined to explain the complementary experimental outcomes for the quantum optical "non-waves" called single photons. In this paper, we intend to demonstrate that a true quantum physical key experiment ("true" in the sense that it cannot be explained by any classical physical concept), when combined with full quantum reasoning (probability, superposition and interference), influences students' readiness to use quantum elements for interpretation.

Keywords: physics education; quantum theory; nature of science; scientific literacy; single photon experiments; quantum reasoning; key experiment

Citation: Waitzmann, M.; Weber, K.-A.; Wessnigk, S.; Scholz, R. Key Experiment and Quantum Reasoning. *Physics* **2022**, *4*, 1202–1229. https://doi.org/10.3390/physics4040078

Received: 10 June 2022
Accepted: 6 September 2022
Published: 8 October 2022

1. Intro: Beyond the Classical Horizon

"Few problems of physics have received more attention in the past than those posed by the dual wave-particle properties of light. The story of the solution of these problems is a familiar one. It has culminated in the development of a remarkably versatile quantum theory of the electromagnetic field. Yet, for reasons which are partly mathematical and partly, perhaps, the accident of history, very little of the insight of quantum electrodynamics has been brought to bear on the problems of optics. The statistical properties of photon beams, for example, have been discussed to date almost exclusively in classical or semi-classical terms. Such discussions may indeed be informative, but they inevitably leave open serious questions of self-consistency, and risk overlooking quantum phenomena which have no classical analogues" [1].

One of the central questions in the context of science education is: how should one overcome the conceptual barriers of quantum physics? Research shows that even major support from the quantum mechanical formalism would help only very little in overcoming these conceptual barriers [2]. Aiming to understand the puzzling quantum phenomena without converting them into quasi classical phenomena with a hybrid status between the quantum and the classical domain is something that requires consistent basic concepts and proper diction. Educational research shows that a quantum physics course focusing on an analysis, which is restricted to a limited number of well-considered phenomena and displays basic and exclusively quantum traits, promotes a deeper conceptual understanding [3].

Some topics from contemporary research on quantum optics are now mandatory course content for advanced quantum physics courses, while coherent optics and experimental approaches are developed for the undergraduate level [4–8]. Here, we discuss the impact of a key experiment combined with the rigorous usage of an appropriate argumentation derived from central quantum traits to obtain access to the quantum domain [8].

Interestingly enough, in and of itself, the specific and extremely successful language for formulating theoretical models in physics is mathematics. At the same time, to be a physical theory, a mindset has to be set up on a natural idea of the experiments, the measurements and the objects of nature to which the mathematical constructs refer. The formalism has to be interpreted upon the background of nature [9]. This vital need usually produces no problems in classical physics because the physical variables and their temporal development (trajectories in phase space) are manifest. Interpretations in quantum physics are much more challenging, because the theoretical constructs (quantum state, probability amplitude, superposition of states, quantum interference) are not just directly and conceptually connected to the real world of nature.

In a recent paper, Bitzenbauer et al. presented [10]. a new experiment-based quantum physics course for secondary schools, underlining the strong connection to contemporary applications of quantum optics in quantum information transfer The authors encourage the integration of single photon experiments and an interpretation of these experiments based on quantum theoretical fundamentals of light into the core concept of the course. As a consequence, one has to accept a significant distancing from Copenhagen-like quantum mechanics, such as atomic physics and stability of matter based on the discussion of Schrödinger's equation as an introductory concept.

As argued in [3], it is promising to minimize the number of axiomatic theoretical quantum features, absolutely necessary for achieving a deeper understanding of the quantum. Following this idea, we shall restrict ourselves to three basic traits: probability, superposition and interference (PSI). To allow for multivariant methods of access, we use the Dirac's bra-ket notation, complemented by a share of linear algebra formalism, which is adequate for school, and a geometrical pointer representation to get rid of the algebraic form of complex numbers directly using the imaginary unit, $\mathrm{i}^2 = -1$

The central idea of a conceptual change from classical to specific quantum reasoning is introduced in Section 2. Here, we present the conversion of the central traits (Wesenszuege) of quantum physics [3] into an appropriate quantum reasoning and the underlying physical picture of the photon. Section 3 presents a study which provides empirical evidence of a substantial interest to physics teachers in true quantum physical experiments without classical bonds. The results of this study strongly reflect the consequences of this epistemic requirement on classroom physics and teaching methods. The perspective of the learners is reflected in Section 4, where the idea of fostering a specific readiness of students for quantum reasoning as a consequence of the engagement with a quantum physical key experiment (single photon states interacting with optical beam splitters) is introduced.

A comparative look at the current literature shows a surprising result: Though educational research on teaching quantum physics has enjoyed increasing popularity and revealed numerous (and various) concepts of how to teach quantum physics over the last 20 or so years, one finds a lack of empirical research into student learning barriers for teaching strategies intended for a conceptual approach to the quantum domain (for a comprehensive review, see [11]). The analysis given in [11] shows that the main learning barrier is the switch from quantum physics to physical reality. The basic idea of the analysis given in the present contribution is strongly related to the historical judgement that pre-QED (quantum electrodynamics) models of the photon are anachronistic, only seemingly providing a simplification at the expense of deeper understanding [12]. Engagement with the key experiment thus aims to get rid of misleading dualistic wave-particle models.

Due to pandemic measures, this study was shrunk greatly to an explorative interview study (a small sample of 36 students and a questionnaire shortened to three questions) to test the instrument which investigates the impact of the key experiment. Assuming that the background knowledge of the first-term students is not far from the typical knowledge of high school graduates, the explorative investigation was conducted with university physics students.

2. Quantum Reasoning: The Contemporary Model of the Photon

2.1. The Dusk of Dualism

From a contemporary perspective, one should state that interference phenomena of massive quanta such as electrons are the result of quantum theory based on the Schrödinger's equation and deBroglie wavelength, while the interference of light is a matter of classical Maxwell theory. Switching to the physics of the interaction between light and matter, the semiclassical theory, which introduces quantized matter and classical light fields, works remarkable well for a wide range of phenomena (photoelectric effect, Compton effect, spectroscopy, nonlinear optics, etc.). Subtle phenomena, however, force us to move away from the classical model of light. Known examples are the almost-zero-time delay between the emission of photoelectrons and the incident of the light in the photoelectric effect and the Casimir effect (this is due to vacuum fluctuations of the electromagnetic field) and the two-photon interference shown by the Hong-Ou-Mandel effect [13,14]. These phenomena cannot be explained by theories based on classical light.

Since the early days of quantum physics, the puzzling interference phenomena has led to dualistic hybrid models mixing features of particles and waves of physical items.

Quoting [13]: "Dual conceptions of light, as wave and particle, have co-existed since antiquity. Quantum mechanics officially sanctions this duality, and puts both concepts on an equal footing . . . "

"Equal footing" seems to be a fairly cautious paraphrased way of getting rid of the dualistic "either–or" and "as-well-as" concepts of light. From this QED point of view, photons are neither waves nor particles. They are instead an intrinsic quantum optical entity that obeys the superposition principle. After interacting with an interferometer, it might be registered by a binary photodetector leading to a "click". As shown in the experiment (see Section 3), the accumulation of these detection events finally leads to an interference-like pattern of the counting rate (i.e., the detection probability). The idea of "equal footing" may be well suited to developing a mindset of the photon appropriate to get access to quantum reasoning beyond the classical horizon.

2.2. Conceptual Change to Quantum Reasoning

Today, the application perspective of the second quantum revolution fosters focussing on the conceptual understanding of quantum physics (e.g., [15]). These requirements are the background of the development of quantum physics as an often-compulsory part of upper secondary school curricula [3,16–21]. The move from traditional quantum mechanics to conceptual questions of quantum physics made students more attentive for fundamental differences between the explanations of the physical world and how physical reality is perceived [22]. Understanding phenomena of the quantum domain concerns the introduction of somewhat counterintuitive concepts such as probability, uncertainty, and superposition. A profound change in the mindset and corresponding educational concepts is needed [23–25]. More recently, some studies and educational proposals have given new stimuli to the discussion [26].

- Learning physics relies on conceptual understanding. It follows that misleading preconceptions or unwillingness to change a stable mindset inhibit physics learning.
- Learning quantum physics may be described as a change of reasoning not only scratching the surface but conceptually changing from classical reasoning to quantum rea-

soning, comparable to the changeover from the semi-classical Bohr's model to a full quantum model [27].
- Are there further theoretical approaches to understanding, and why do commonplace difficulties in quantum physics learning arise?

Some of these questions are touched on by diSessa's theory of Knowledge in Pieces (KiP) [28]. This theory describes learning physics as transforming the naïve sense of mechanism into a physical one. The term "sense of mechanism" refers to a cognitive heurism providing explanations for observed phenomena and arguments for the predictions of phenomena or possible events. The backbone of each sense of mechanism is a set of principles and laws, which have to be identified in the observation. The so-called "phenomenological primitives" (or "P-prims") are the superficial version of the physical sense of mechanism referring to the principles of children's concept of physics. P-prims are a result of the straightforward interpretation of the perception of everyday-phenomena. P-prims are primitive as they are self-explanatory and need no further justification. Thus, P-prims are strongly bounded to specific contexts, which allows different explanations for similar physical phenomena to exist [28].

To transform the naïve sense of mechanism into a deeper physical one, the role of p-prims has to be changed to a necessary set of components of a cognitive heurism to identify relevant physical laws and principles. For such a changeover of roles, it is necessary to decontextualize explanations. Decontextualization means that the same physical principles and laws always explain similar phenomena [28,29]. Because everyday contact with quantum physics is usually extremely rare, the existence of quantum physical p-prims are rather unlikely. Perhaps for this reason, learning quantum physics presupposes the transformation of senses of mechanism, from real-world-principles (such as local reality and determinism) to quantum principles (such as lost locality), an everlasting internal probability, a superposition principle and a specific Born's rule to re-enter the real world.

This paper deals with a role changeover from classical p-primes to a quantum sense of mechanism. This changeover is thought to be accomplished by decontextualizing a quantum reasoning based on PSI to explain the prototypical and thoroughly selected phenomena such as the single photon counter patterns from single photon experiments or from an electron diffraction tube.

2.3. Components of Quantum Reasoning

Following McNeill and Krajcik [30], three components constitute scientific explanation:

- The claim is an assertion or conclusion which addresses the problem or a phenomenon, to be explained. Quantum physical phenomena lead to quantum physical claims.
- The evidence supports the claim. It can be a set of scientific data, an observation or reading material. Thus, the evidence is always part of the real world. To support it, the evidence has to match the claim, according to the amount and quality of the data.
- The reasoning links claim and evidence, showing, why the evidence can be seen as supporting the claim, connecting the phenomena with the scientific laws and principles. Quantum claims need quantum features of reasoning.

Evidence in physics is always backed up by real-world experiments. It thus becomes understandable that reasoning which links the real-world evidence and phenomena from the quantum domain has to include a special design. Far from touching the deep logical questions of reasoning in quantum theory (e.g., [31]), the specific quantum reasoning proposed here turns out to be a mindset consisting of objects and elementary rules bijectively assigned to central terms of quantum theory (Wesenszuege [32]).

In this sense quantum reasoning is a mindset with the following characteristic features:

- It is designed to be consistent with a rigorously selected list of basic exclusive quantum theoretical axioms [33].
- It is well-suited to equally footing the features of quantum mechanics (states of massive quanta) and quantum electrodynamics (states of photons).

- It links claims from the quantum domain and evidence from the real world.

  Specific features of this rational are:

- Probability thinking, where properties of physical systems are completely incorporated into the quantum states. Principally different from classical physical states quantum states are just vectors in an abstract vector space. Each vector allows for the calculation of a probability to detect (or even measure) characteristics of the system. In quantum physics, we have to think in terms of probabilities derived from quantum states. One might state that there are no physical variables in quantum physics but solely probabilities for the assignment of specific values of the variable.
- Superposition thinking focuses on the superposition principle that is a core concept of quantum theory: The linear combination of quantum states again is a quantum state. Mathematically a superposition of two quantum states, $|\psi_1\rangle$ and $|\psi_2\rangle$, is nothing but the sum of these state vectors, weighted and phase shifted by specific amplitude factors, $|\psi\rangle = c_1 \cdot |\psi_1\rangle + c_2 \cdot |\psi_2\rangle$. The new quantum state $|\psi\rangle$ of the system must be distinguished from a *classical mixture* of ensemble states, where probability solely occurs because our exact knowledge of the components is incomplete. In the quantum superposition *each of the component states is always present*. In the weighted sum, $|\psi\rangle = c_1 \cdot |\psi_1\rangle + c_2 \cdot |\psi_2\rangle$, of the substates the coefficients c_1 and c_2 are closely related to an internal probability without any connection to the ability of the particular scientist.
- Interference thinking focusses on the detection of the superposition of states. A polarized spin-up-state $|u_z\rangle$ of atoms in the Stern-Gerlach apparatus confirms the superposition of a spin-up-state and a spin-down-state along the x-axis: $|u_z\rangle = (|u_x\rangle + |d_x\rangle)/\sqrt{2}$ directly measurable in the experiment. Generally, it is difficult to detect a superposition of states directly. To demonstrate a state superposition, $|\psi\rangle = c_1 \cdot |\psi_1\rangle + c_2 \cdot |\psi_2\rangle$, we are often forced to resort to demonstrating *interference effects*. Any phase difference between $c_1 \cdot |\psi_1\rangle$ and $c_2 \cdot |\psi_2\rangle$, temporally stable during the sampling time of the detector, may lead to interference patterns. One might say that there is no quantum interference without the superposition of states and that quantum interference is a safe indicator of a superposition of quantum states.

  See Appendix A for details of the pointer representation of quantum states.

3. Experimental Teaching in Classroom Quantum Physics

3.1. Theoretical Background

Science funding for digitalization and quantum technology (*The Quantum Flagship* [15]) is one of the most ambitious long-term European Commission research initiatives. The increasing popularity of quantum physics thus seems unsurprising. Our understanding of physics evolves hand in hand with advances in our understanding of the interaction of light and matter, as has been modelled since the 1950s by the modern version of the quantum electrodynamics. It seems, however, that the physical science of the last 100 years still represents a small share of classroom physics in German high schools, along with those in many other countries [10,11]. It looks as though quantum physics curricula worldwide are locked onto the historical development of quantum physics with some specific anachronisms (particle-wave-dualism).

The question of how to overcome these shortcomings leads to the corresponding teacher training requirements as follows.

- Following the major role of experiments in physics education, didactic aspects of quantum experiments and of science communication should play a substantial role in physics education [34–36].
- Interpreting quantum theory is a minor part of teachers' in- or pre-service training. Conceptual knowledge is often restricted to semiclassical concepts which ignore the basic concepts of quantum field theory, leading to the notorious problems which arise from the wave-particle dualism and localization of quanta.

- There is less information about students' views on quantum physics when compared to those of classical physics [37].
- The lack of pre-service teacher training on the educational and experimental efforts of quantum physics [38] is significant.
- Experiments such as single-photon and two-photon interferometry and current applications in quantum physics usually go beyond experimental and theoretical high school expertise.

To face these challenges, systematic training courses for teachers are required, which go beyond the semiclassical confinements as discussed by [9,39,40]. It is known that the courses have to comply with the real needs of teachers. First and foremost, the course must be in accordance with the particular curriculum, which assign quantum phenomena with single photons to a specific quantum reasoning (Table 1).

Table 1. Typical correspondence between quantum reasoning and quantum phenomena in hight school textbooks.

Phenomenon	Quantum Reasoning
Interference of single photons	Superposition of states
Double slit interferometer	Born's rule to determine the probability of an event
Mach-Zehnder-Interferometer	Nonlocality of single photon states in the interferometer
Blurring the interference pattern	Influence of measurement on system behaviour
Absence of coincidences at the optical beam splitter	Complementarity of anticorrelated events

3.2. An Empirical Study

The global aim of the project presented in this Section is to develop and offer a training course which fits these requirements. Questions such as the following turned out to be well-suited to structuring the complex research area.

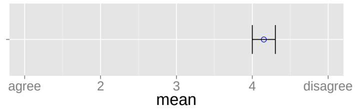
- How do teachers handle the problems which arise from the epistemological clash in quantum physics at school without experiments?
- How do teachers assess their pre-service education regarding the requirements of quantum physics at school?
- How can we design a teacher training program that offers university-level experimental and theoretical backgrounds suited to the real needs of teachers?

A suitable methodological tool for exploring such a complex field of attitudes and knowledge is the established Delphi-method [41–43]. The Delphi study presented here has been through three rounds between May 2014 and July 2015 (see Table 2 [43]). The first round started with $N = 84$ study participants in mid-2014, the second round with $N = 54$ at the end of 2014 and the third round with $N = 70$ in mid-2015. The difference in the number of participants is mainly explained by nonattendance during the second round and an increasing attendance in the third. The size of the sample is small; this shortcoming is compensated for with the high quality of the answers.

Table 2. Details of a Delphi study.

Details of the Design	Example	Bundled Result
The questionnaire of the first round mainly consists of open questions	Which topics are challenging for your students and how do you or what do you need to master these challenges?	A typical statement: "Due to absence of experiments in school, most of principles of quantum physics can only be believed and memorized."

Table 2. *Cont.*

Details of the Design	Example	Bundled Result
The goal of the second round is to get access to the position of each participant to disputable answers, main attitudes and motivational topics identified in the first round. The questions are more focussed in comparison to round 1, without losing the open character to permit critical discussions.	Most of principles of quantum physics can only be believed and memorized due to absence of experiments. Please position yourself to the statement above and describe especially the role of simulations and animations.	A typical statement: "Simulations enable the discussion of the principles of quantum-physics in my class, but often my students don't trust in simulations."
The question immediately arising asks for the generalizability of the positions identified in the first two rounds. In the third round, critical opinions and disputes from the previous two round have been identified. The group of experts have been asked to rate these on a 5-point Likert-scale	Do you agree (1), . . . , disagree (5)? My students do not believe in the results of computer simulations in quantum physics.	Most of the participants disagree with the position "My students don't trust in results from simulations." 

Condensing the results of the study into a course specification sheet, a few main implications for teacher training can be obtained (see Figure 1).

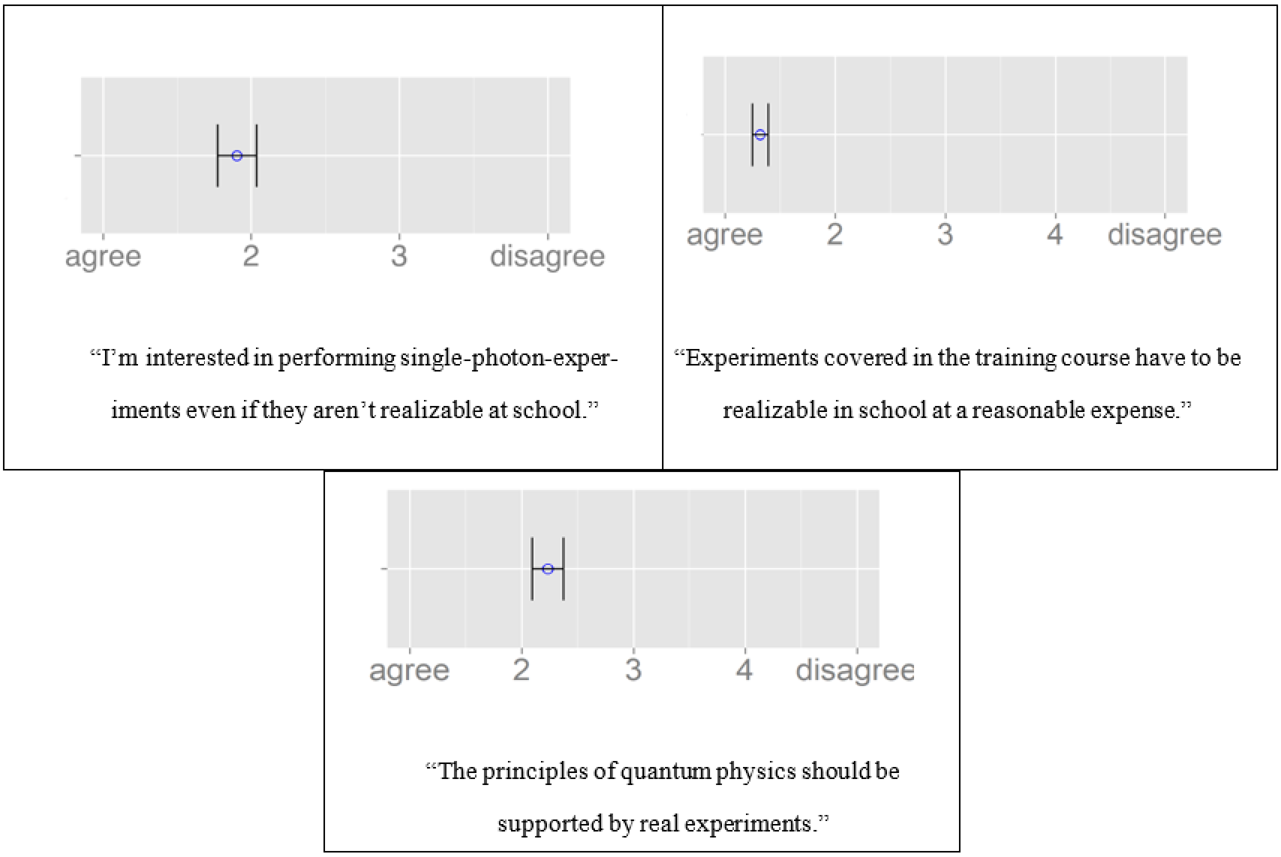


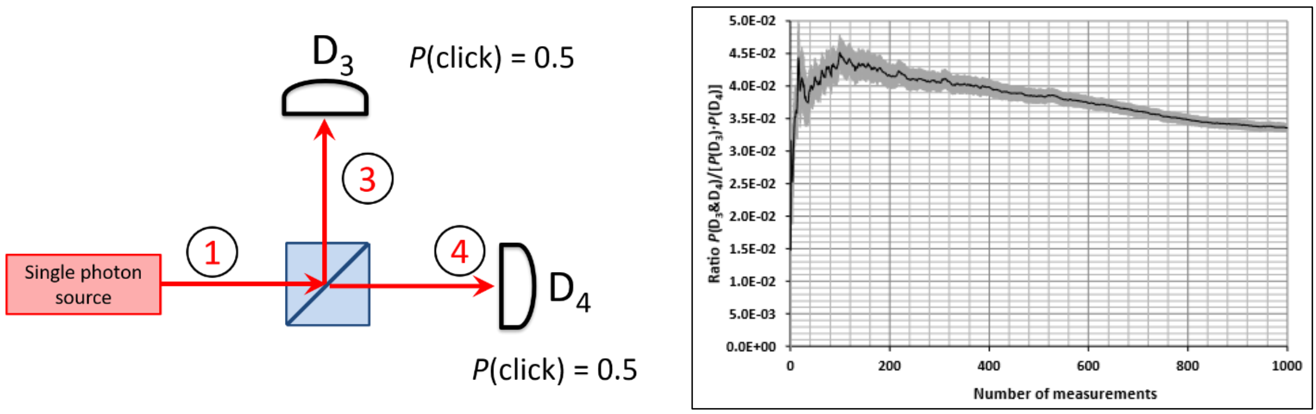
Figure 1. Teachers' basic needs [43]. The mean value and the standard deviation of the particular ratings is shown.

- Simulations and real (single-photon) experiments should be used in a mutually supportive manner.
- Focusing on experiments with interpretability does not depend on a formalism which goes beyond the scope of classroom mathematics.
- There should be an orientation towards principles of quantum reasoning.

The found results (Figure 1) well reflect the high level of classroom experiments:

3.3. A Quantum Physical Key Experiment

To address the consequences for teachers' training efforts, we conducted a study based on an experiment with heralded photons recently proposed for the preuniversity learning of quantum physics [34]. This experiment demonstrates two classically contradictory attributes of single photons: incompatibility of the beam splitter output states (absence of coincidences) and a $\cos^2$-dependence of the counting rate on the position of one of the mirrors in an interferometer set up (interference-like pattern of the counting rate). This emergence of maxima or minima of the measured photon counting rates (see Figure 2b) by changing the position of the mirror is quantum theoretically assigned to the difference of the phases of the substates of a state superposition: quantum interference.



(**a**) (**b**)

Figure 2. The number of coincident clicks of the detectors D_3 and D_4; (**a**) the setup (the blue square indicates the beam splitter, encircled numbers are assigned to the beam splitter modes; red lines indicate the direction of propagation of the light) and (**b**) experimental results: the mean value of 1000 measurements $\langle \alpha \rangle << 1$; the shaded area indicates the standard error. P denotes the probability of a detector click.

3.3.1. Part I: Specificities of Single Photon States

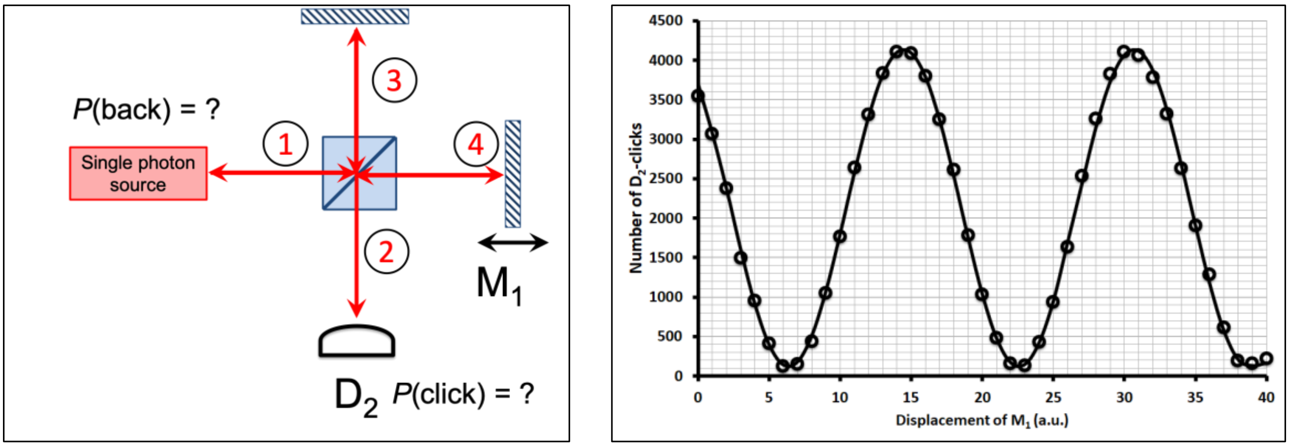
Consider the first part of the key experiment with single photon states (Fock states with $n = 1$) as the input of the beam splitter (Figure 2a). There are now two potential outcomes.

1. In any case, light behaves like classical electromagnetic fields and, therefore, the amplitude of the single-photon field (whatever that might be) will also just be split at the beam splitter. Depending on the physical properties of the light used in the experiment, clicks of D_3 and D_4 are more or less independent, the coincidences more or less accidental. For the joint probability of coincidences compared to the product of the marginals one finds $P(D_3 \,\&\, D_4) \geq P(D_3) \times P(D_4)$ [43].
2. The single-photon states of the light are made up of breakable particles. These particles behave like classical ones, and thus may burst, meaning that we have a finite probability $0 << P(D_3 \,\&\, D_4) \leq P(D_3) \times P(D_4)$ for coincident clicks.

Instead of the probability $P(D_3 \,\&\, D_4)$ itself, the ratio $\alpha = P(D_3 \,\&\, D_4)/[(P(D_3) \times P(D_4)]$ is shown. Classical fields (case (A)) would produce $\alpha \geq 1$. Breakable particles (Case (B)) would lead to $0 << \alpha < 1$. Figure 2b shows what happens instead. As can be seen, α and thus the mean value of the probability of coincidences is much smaller than expected from classical theories. The probability of coincident clicks from detector D_3 and D_4 almost vanishes and therefore too the probability of a splitting of the single photon state by the beam splitter.

3.3.2. Part II: Quantum Superposition

The interferometer setup is shown in Figure 3a. Again, the single photon state is the incident state of a beam splitter. Now however two mirrors are added (as shown in Figure 2a). After an argumentation based on inseparable single-photon-states, classical probability theory ignoring substates and their phases would lead to a constant probability, while the equally shared characteristic of the beam splitter would yield equal probabilities of 0.5 for D_2 clicking and for the possibility of the light being reflected back into the source, not dependent on the mirror's position.



(**a**) (**b**)

Figure 3. The number of single photon clicks of detector D_2; quantum interference: a displacement of the mirror M_1 produces a periodic variation of the counting rate even in the case of single photon states; visibility V = 93%; (**a**) the setup and (**b**) experimental results.

The experimental result is completely different. Depending on the position of mirror M_1 the number of output counts changes. Periodically alternating, the light reaches D_2 or is reflected back into the source (the latter is not shown here). This pattern is interpretable as an interference (Figure 3b). There is no classical physical explanation because there is neither an electromagnetic wave to produce interference fringes nor more than one photon to allow for some inter-photonic interaction. Quantum theory, however, is well suited to solving this conflict. The quantum interference phenomenon shown here experimentally is a consequence of the superposition of two quantum sub-states, one of them the result of a transmission process, the other state corresponding to the reflection. The superposition state leads to interference fringes in the final probability.

Combining both experiments, a puzzling situation concerning the concurrence complementary phenomena occurs: The bare beam splitter itself shows a lack of the coincidences one would expect for unbreakable radiation elements. With mirrors added, the experiment shows an interference-like dependency on counting rates with the same beam splitter. A nonlocal interpretation (including the complete interferometer), based on quantum theory resolves the conflict, implying that this combination experiment possibly acts as a "door opener" to the quantum world. The quantum theoretical interpretation.

- does not concern the splitting of anything by beam splitters,
- does not concern paths of quanta,
- tells us everything about the probabilities of detection eventualities,
- makes use of the superposition principle together with Born's rule to explain quantum interference,
- allows us to find the quantum state prepared by a beam splitter as a nonlocal superposition of single photon substates, and
- encourages us to rename beam splitters "quantum state preparators".

The key experiment might help to satisfy physics teachers' needs, if they are looking for real experiments which demonstrate true quantum physics which well demonstrate the failure of classical theory.

4. Impact of a Key Experiment on Quantum Reasoning

4.1. Research Goal

As has just been pointed out, the key experiment can be explained by using the basic components of quantum physical reasoning PSI. On the other hand, conducting and discussing the two parts of the experiment separately and independently risks stabilizing the well-established and attractive semiclassical wave–particle dualism. Closely combining these experiments, the wave-particle dualism does not well explain the concurrence of photon anti-correlation and interference in a single experiment. The experiment thus might function as a door opener to the quantum domain, motivating the development of a quantum reasoning which relies on PSI [8,9].

Closely related to the present contribution recent studies have reported on approaches to quantum physics that rely on student insights into the general relevance of quantum technology for everyday life [44]; that rely on the analysis of typical patterns of student difficulties in modern quantum physics in schools [45]; and that rely on a specific model of a learner's way of understanding quantum physics [46]. While real quantum experiments with single quanta have been available for many years, they are still not yet practicable in schools [34,47]. It therefore comes as no surprise that (at least to our knowledge) the impact of quantum key experiments on learning about quantum physics remains unknown. To obtain greater insight into the experiment's impact on students' readiness to use quantum terms in their interpretation of the experiment, it would be of interest to investigate.

3. in which regard the students' lines of argumentation move from classical reasoning (either-or) or a semiclassical dualism (as-well-as) to quantum reasoning (PSI);
4. which types of specific (quantum) reasoning (PSI; neither-nor) can be found for the explanation of quantum phenomena such as the key experiment after students had engaged with the key experiment.

4.2. Method and Sample

Unfortunately, due to the pandemic measures, it was not possible to get in touch with high school students. As a preliminary step, in order to test the pool of items and to see whether the idea functions in principle, a reserve sample was used.

For this purpose, 80 physics students (first term at university) in a one-group, pre-design and post-design were engaged with the key experiment. To gain empirical access to the experiment's influence on student reasoning, the empirical construct "use of quantum reasoning" was improved as explained above (usage of the PSI-line of argumentation) and a mixed-format test and an additional semi-structured interviews with a subgroup of 36 students were used. These interviews focussed on student reasoning for the explanation of classical contradictory phenomena of the key experiment. The findings are the topic of this section.

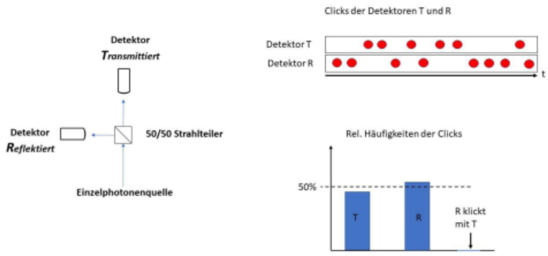
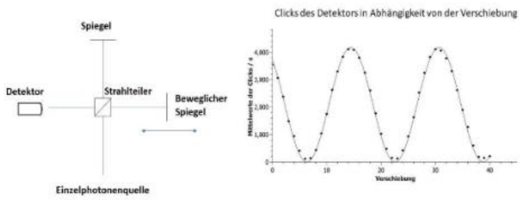
The interviews with a mean duration of 14.3 min (pre) and 12.0 min (post); standard deviation, SD = 3 min in both cases, were digitally conducted and recorded via an anonymous video conference. An interview guide with three tasks was developed in order to provide a basic structure; the guidelines were prepared according to [48] (see Table 3):

- Task 1: Basic understanding and description of the term photon.
- Task 2: The explanation of the absence of coincidences in the experiment with the naked beam-splitter.
- Task 3: The explanation of the counter pattern in the interferometer set up.

To visualise the key experiment, appropriate figures were provided in Tasks 2 and 3, displaying the absence of coincidences at the beam-splitter and the counter pattern with mirrors added. For students who were unable to answer the questions potential answers are made available, taking into account known misconceptions and appropriate responses.

As taken up in in the conclusion, the interview setup, separating the two components of the experiment into two tasks, was at risk of stabilizing the concept of the classical particle.

Table 3. Guidelines for the interviews.

<table>
<tr><th>Question</th><th>Hints and Instructions</th><th>Additional Material</th></tr>
<tr><td>Task 1
In this interview I would like to talk with you about the photon and its behaviour. Please describe your understanding of the photon.
Explain: What is youconception of photons?</td><td>What is your conception of particles/waves?</td><td>Answer options:
A photon is an undividable energy quantum of the light field:
1. a light particle, which is moved by the enveloping light wave. Depending on the experimental setup, the behaviour of the photon or the light wave occurs (the classical either–or argument).
2. existing as wave and particle concurrently. Depending on the experimental setup, it occurs as a wave or as a particle (the dualistic argument).
3. neither a particle nor a wave, but it behaves sometimes similar to a wave or a particle (the quantum neither–nor argument).</td></tr>
<tr><td>Task 2
Single photons interacting with a beam splitter:
Report the results of the experiment.
Explain the results</td><td>Hints regarding relative frequency of events:
What's about features of particles?
How does the result fits in with your concept of photons?
What can be said about the state of the light at the output of the beam splitter?
Try to give an explanation in terms of two different output modes of the beam splitter</td><td>
</td></tr>
<tr><td>Task 3
Single photons interacting with the Michelson interferometer:
Report the results of the experiment
Explain the results</td><td>Questioning the impact of the mirrors:
What's about features of waves?
How does the result fits in with your concept of photons?
What can be said about the state of the light at the screen?
Try to give an explanation in terms of the superposition two different output modes of the beam splitter.</td><td>
</td></tr>
</table>

Following Ref. [49], students' conceptions can essentially be placed into three categories: classical, dualistic and quasi-quantum physical and assigned to each task. Table 3 lists the questions of the interview guide translated from German and drastically shortened. For a detailed version, see Ref. [26]. The interview study was conducted in German. Semantic deviations from the original questionary due to the translation may have been unavoidable.

To evaluate the coherence of the student responses, the students were advised to link the results from Task 2 and Task 3 to the results from prior tasks.

4.3. Analysis

The interviews recorded were transcribed and subsequently paraphrased, thus reducing the number of utterances to the relevant aspects (i.e., a student's reasoning for the explanation of quantum phenomena). To analyse the students' responses, the method of structuring qualitative content analysis [50] was used, because it allows us to identify and conceptualise contend-related aspects in the material. Based on this conceptualization, the method allows for a systematic description of the material with respect to these content-related aspects [51]. The use of a deductively-inductively derived coding system is the method of choice for this analysis. To create this system a basic set of categories was obtained from the literature and enriched with sub-categories derived from an analysis of the interviews [50]. As basic categories, classical, dualistic and quasi-quantum categories were chosen.

In this way it was possible to assign the students' responses to categories even if they were given in more complex response patterns (see Table 4). To illustrate this process let us give an example for a typical dualistic reasoning:

Table 4. Categorial system used to analyse the interview data; the first column refers to the three basic tasks, the second and third columns show main categories and subcategories derived from paraphrasing the individual responses, paraphrases are described in the fourth column, the last column gives an example.

	Main Category	Subcategory	Description	Example
Describe the term photon	Classical		The photon is described as classical particle or is associated with the properties of a classical particle	Photons are light particles, energy packages of electromagnetic radiation. Photons are portions of light (h × f), which can be modelled in different ways, but mainly it is modelled as classical particle.
	Dualistic		Dualistic description of photons: Hybrids of waves and particles, like for example: a wave-particle, a particle with wave properties, complementary behaviour of photons	Light is composed of photons, which are waves as well as particles. Depending on the experiment one gets the one or the other. A photon is something small, which is a wave (wavy path) as well as a particle (straight direction of propagation). The experimental setup decides which property of a photon can be observed.
	Quasi-Quantum		An object which has wave like and particle like properties, but is neither a wave nor a particle; Attribution of probabilistic behaviour/ probability amplitudes	A photon is an object which has properties of a wave and a particle, but is neither a wave nor a particle. A photon is a small energy package, which can't be described by classical physics. It can be described by using probability amplitudes
	Other		Cannot be categorized	Photons are positive charged particles, which work as a current of light
Optical beam splitter	Particle		Explanation of the experimental results by using particle-based reasoning	
		Classical	Using properties of a classical particle (e.g., realistic arguments)	In this experiment a single photon can't be divided into two halves, because it can be at only one single position. At an optical beam-splitter, photons were reflected or transmitted with a probability of 50%, depending of the amplitude of the transporting wave. Due to particle properties, the photon can't be divided. With a probability of 50% photons were reflected or transmitted, but they will never be divided, because the photon has to choose one path. It cannot be said anything about wave or particle properties.
		Dualistic	Due to the experimental setup, the photons occur as particles.	Because the experimental setup allows to measure the photon's location it demonstrates particle behaviour.

Table 4. *Cont.*

	Main Category	Subcategory	Description	Example
		Quasi-Quantum	Photons behave like particles, but they are no particles	Due to the particle characteristics, the photon can't be divided into halves like a wave. Thus, we get "either-or"-results.
	Energy Quantum		Explaining the results by the indivisibility of the photon's energy.	With a probability of 50% single photons will be transmitted or reflected at the optical beam-splitter, but neither divided in two halves, because they are indivisible energy quanta. The photon decides whether to be reflected or transmitted in probabilistic ways.
	Probability		The experimental results will be explained by using a probabilistic reasoning.	
		Non-localisation	The experimental results need a probabilistic explanation, because the photon position is not determined until the photon's detection.	The path taken by the photon, is unknown until the photon's detection. However, the detection is arbitrary, with a probability of 50%. Single photons will be reflected or transmitted with a probability of 50%, but neither divided into two halves. This looks like a particle property, but the behaviour can only be described by probability amplitudes. Until the photon's detections it is not determined whether the photon is reflected or transmitted.
		(Non-localisation + superposition	Explaining the experimental results by describing the final state as a superposition of the substates reflected and transmitted.	Photons will be reflected or transmitted with a probability of 50%. Until the photon's detection the photon's path is not determined, but it can be described as a probability amplitude. By detecting the photon, the superposition of probability amplitudes will be destroyed.
	Choice		A provided explanation is chosen by the student.	Choose Explanation1, because the photons show up as indivisibleenergy quanta.
	Other		The explanation cannot be sorted into one of the categories.	No explanation, but the photons are distributed randomly.
Interferometer	Wave interference		Explaining the results like the interference of waves.	
		Wave characteristics	Due to the wave characteristics of photons, the interference occurs.	Interference can be observed in this experiment, because of the wave characteristics of photons. By moving the mirror, a difference in the path length is realized. Particle characteristics would lead to a constant number of counts.

Table 4. *Cont.*

Main Category	Subcategory	Description	Example
	Dualism	The photon shows up as a wave/the experimental setup determines the photon as a wave	Intensity minima and maxima can be observed because of the interference. Due to the superposition of wave and particle, unbreakable particles show up as waves in this experiment, because the experiment demonstrates the interference as a classical wave property. The diagram shows a sinusoidal click distribution, because the wave properties are observed in this experiment. Thus, the photon cannot be regarded as a particle/localized object. It is in a subordinated state. Depending on the superposition of the amplitudes, constructive or destructive interference can be observed. It becomes understandable, how the photon's properties are determined by the experimental setup.
Probability interference		The observed interference is a probability interference.	
	Non-localisation	A probability interference is observed, because the photon's position cannot be determined. → Basic probabilistic reasoning	In this experiment constructive and destructive interference can be observed, because reflected or transmitted photon 50% probability is reflected on the beam-splitter again, by the mirrors. Thus, it is impossible to determine whether the photon is reflected or transmitted and probabilities will interfere. This experiment demonstrates a wave characteristic.
	Superposition of probability amplitudes	The photon must be described by probability amplitudes and the superposition of the probability amplitudes causes the interference. → Advanced probabilistic reasoning	In this experiment the probability amplitudes of the both, possible paths the photon could take, are superimposed. By moving the mirror, a difference in the paths is realized and the inference pattern changes from constructive to destructive interference of the probability. Thus, a wave characteristic is attributed to the photon. In this experiment constructive and destructive interference can be observed, because the probability distributions of the both possibilities interfere. For each possibility, reflection or transmission, a probability distribution exists, which can be regarded as waves in the arms of the interferometer. Thus, by moving the mirror a phase difference is created.
Wave behaviour		The experiment demonstrates the photon's wave properties but no interference	More or less photons are detected, because depending on the distance between mirror and beam-splitter more or less photons can be registered. This must be the wave property, because a particle will be detected with a property of 50%.
Other		The explanation cannot be categorized.	Not an explanation, but the chose option 2 or 3. The experiment allows no opportunity to talk about the indivisibility, because it can't be measured whether the photon is reflected or transmitted. And the energy of the photon changes by increasing the distance between beam-splitter and mirror, due to air friction.

Table 4. *Cont.*

Main Category	Subcategory	Description	Example
No explanation		Student choose an option without any further explanation.	

This interview can be paraphrased: Photons are light particles, transporting and releasing energy (energy quanta). However, they are also electromagnetic waves and have characteristics of waves.

This type of content-related analysis always risks subjective rating, leading to reliability issues of the results. To adhere to due diligence obligations, we checked the categorization by two independent raters. Due two different priorities (single statements vs. more complex patterns) we found Cohen's kappa [50] M(κ) = 0.6/SD(κ) = 0.2, mean valued over all three tasks/pre and post. After clarifying the reason for the discrepancy, we reached full interrater agreement. A piloting test of the questionnaire ensured the students ability to understand the technical language and to edit the questionnaire.

Student: Eh, I see the photon as a tiny particle which transports a portion of energy as a light quantum, thus, a particle, energy and it is also an electromagnetic wave which has particle properties.

Interviewer: Ok, now you have mentioned three different aspects. I want to take a closer look at them. You said, photons are something like particles, did not you?

S: Yes

I: Please explain what it means to you.

S: As I already explained, (they are) portions of energy, light as a particle. Light may be presented as a particle, able to release energy just like a particle. e.g., as in the case of the photoelectric effect, where the light particles transfer their energy to the metallic plate and the electron will be emitted.

I: Ok, as a second aspect you mentioned light quanta and as a third aspect you mentioned electromagnetic waves. How would you further specify these aspects?

S: Ehm, light as an electromagnetic wave has wave properties, like for example diffraction on surfaces or slits or light refraction.

I: If I understood you correctly, light has properties of waves as well as properties of particles?

S: Yes, that is what think about it.

I: Please try to go into greater detail.

S: Do you mean the wave-particle dualism of light, that it can be both, particle and wave.

I: It is irrelevant what I am thinking about it. I just try to find out what you think about it (laughing).

S (laughing): I think there is a duality and light have both features, features of particles and waves as well.

I: Simultaneously?

S: Yes, simultaneously.

4.4. Results

4.4.1. Description of a Photon

During the "pre"-status, more than half of the students describe a photon as a dualistic wave-particle-hybrid (Figure 4): Photons are light particles, which transport and emit energy. However, they are also concurrently electromagnetic waves.

37% of the students described a photon in a quasi-quantum physical way: Photons are small bundles of energy, which cannot be well categorised as waves or particles. Depending on the situation they show wave or particle behaviour.

9% of the students described a photon as a classical particle. A wavy behaviour emerged as a feature of larger samples: Light is composed of photons, which are portions of energy. The photons are considered as particles. However, a bunch of photons will behave like a wave. Depending on the experimental situation I'm observing the characteristics of the wave or of the particles.

For the post-interviews an increase in the quasi-quantum physical description (53%) and a decrease in the dualistic description (33%) could be recognized. The classical description of a photon has been stabilized; we found an increase for the classic position (+2 people/5%).

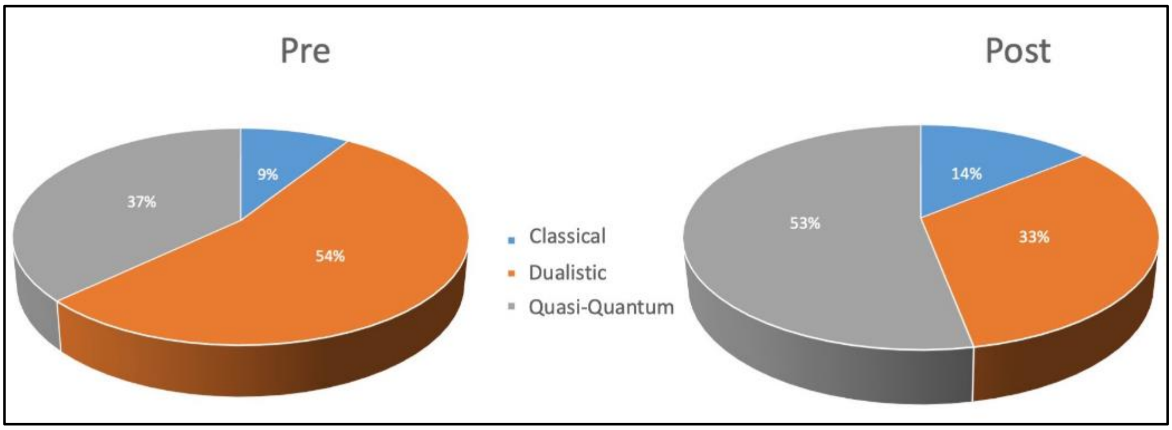


Figure 4. Students' understanding of a photon, before ("pre") and after ("post") they were engaged with the experiment.

4.4.2. Beam-Splitter Experiment

For the beam-splitter experiment (Figure 5), one can see that in the pre-interviews, 50% of the students preferred a particle argument, in which students talk about classical particle, dualistic particle occurrence or something like a particle behaviour of a rather obscure object; 25% came up with a probability argument, such as: Photons are indivisible energy quanta, for which only probability distributions can be formulated behind the beam-splitter. Only the measurement will lead to a certain result.

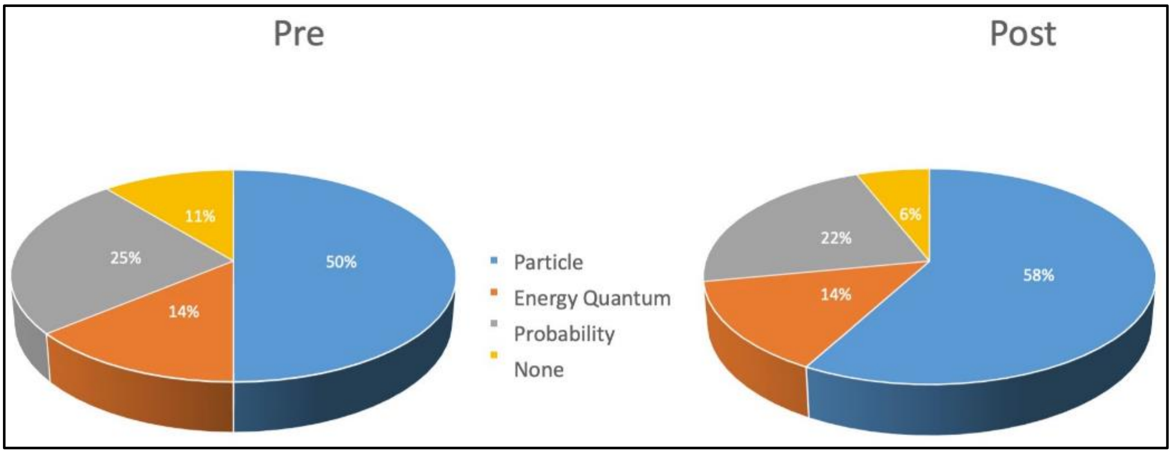


Figure 5. Students' reasoning for the explanation of the beam-splitter experiment, before ("pre") and after ("post") they were engaged with the experiment.

One student used the principle of superposition (subcategory of probability arguments).

A group of 14% used energy quantization as an argument to explain the results, such as: Photons will be reflected or transmitted with a probability of 50% and will produce a click in one of the detectors, with a certain probability. The photons do not hit both of the detectors, because they are indivisible energy quanta.

A group of 11% cannot be assigned to any category, because either the argumentation was entirely wrong (like one person who mixed-up photons and protons).

For the post interviews, a slight decrease in the probability arguments (−1 person using superposition) and a significant reduction in the non-categorizable paraphrase can be seen, while there is no change in arguments with quantized energy. Here, we find a visible increase in the particle position to 58% (+3 persons).

To get deeper insight into the different particle arguments, Figure 6 shows the distribution in classical, dualistic and quasi-quantum physical particle arguments in absolute numbers. In the pre-interviews four people used a classical particle argument: Photons will be reflected or transmitted with the same probability (50%).

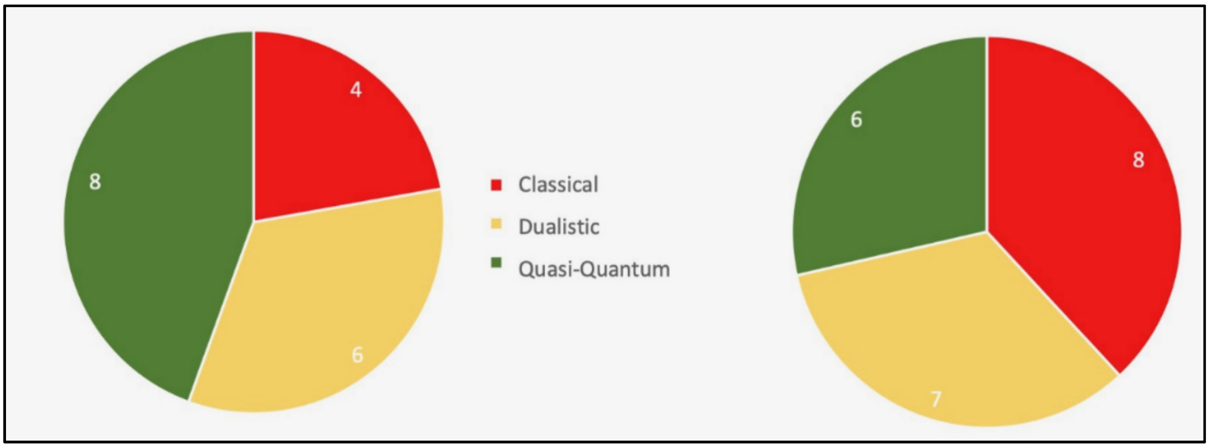


Figure 6. Particle arguments in detail (**left**) before the students were engaged with the experiment and (**right**): after the students were engaged with the experiment. The absolute numbers of the students are indicated.

This fosters the idea of photons as localized hard particles: Because particles could only be at one point at the same time, waves, however, could be transmitted and reflected concurrently.

Additionally, eight people used a quasi-quantum particle argument: The experiments demonstrate the elementary particle character of a photon. This is fostered by the inseparability.

Six persons used a dualistic argument: The photons will either be completely reflected or transmitted, but not divided into halves, because then photons occur as particles, which can only be completely reflected or transmitted.

For the post-interviews, an increase in classical particle arguments (+4 persons) and the number of dualistic-particle arguments (+1 person) can be seen, while the number of quasi-quantum-particle arguments decreased (−2 persons).

4.4.3. Michelson Interferometer

For the Michelson interferometer, one can see that 42% of the students who use wave arguments for their explanation (which can be specified as 53% of the group), say photons behave like waves due to a basic wavy character (Figure 7):

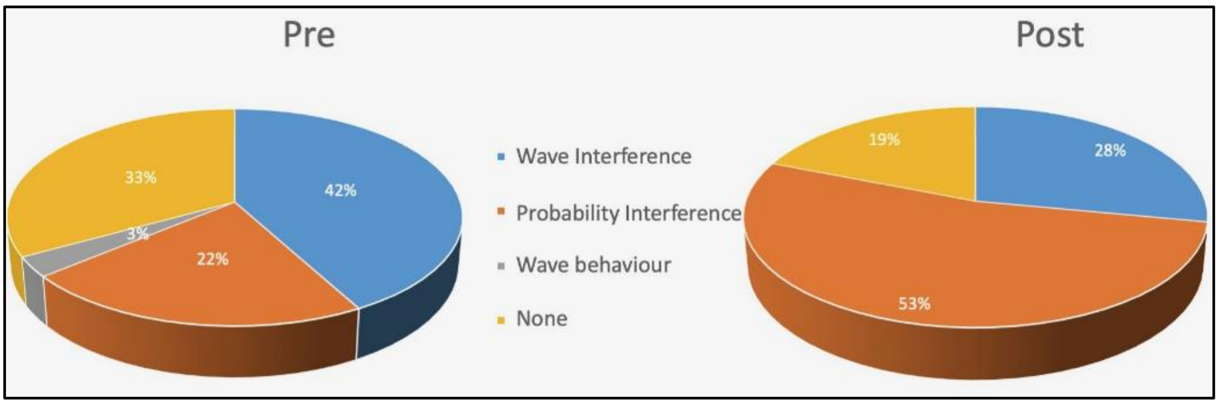


Figure 7. Students' reasoning for the explanation of single photon interference.

Destructive and constructive interference can be observed in this experiment, due to the wave character of photons. Moving the mirror produces a phase difference. A particle character, however, would produce a constant rate of counts.

A total of 47% use a dualistic reasoning: Constructive and destructive interference can be measured, because the experimental setup does not allow us to measure whether the photon is reflected or transmitted. The wave character therefore occurs. By moving the mirror, the waves were displaced against each other and as a result the detector measures the photon's wave.

A quarter of the students used probability arguments, while half of the group has already used the principle of superposition: In this experiment the wave behaviour of photons is demonstrated. Against classical waves, however, photons are not divided into two halves at the optical beam-splitter, but there is a superposition of all the possible paths a photon could take to the detector. Moving the mirror now produces a phase difference in the superposition and a single photon cannot be detected. Thus, more photons are need.

Nevertheless, 13 persons/33% of the students do not recognize an interference pattern, though at least 3% argued, that the result must be a wave behaviour: The experiment demonstrates the wave character of photons, because depending on the mirror's position, more or fewer photons will be detected. Due to the photon's position on the light wave, the reflexivity of the beam-splitter will change.

When analysing the post interviews, it can be seen that the amount of non-categorized paraphrases (19%) and wave arguments decreases (28%), while probability arguments increase (53%). Going into further details, it can be seen that 76% of these students used the superposition of probability amplitudes for their explanation. Nevertheless, 19% of the students interviewed did not recognize the interference pattern, although they analysed it in the key experiment.

4.5. Conclusions

The analysis of the interviews shows that engaging students with the key experiment seems to challenge dualistic conceptions/reasoning. For the photon description, an increase in quasi-quantum physical descriptions and a decrease in dualistic descriptions can be seen. Nearly the same can be recognized for the explanation of the interference.

By contrast an increase in a classical particle conception and reasoning can also be observed. The beam-splitter part of the key experiment seems to foster particular classical reasoning. Here, an increase in classical particle arguments can be seen (+4 persons), while the use of the principle of superposition is slightly increasing (+3 persons). It can therefore be concluded that the students did not understand the idea of superposition. This conclusion is fostered by the explanation of the interference. Here, 39% of the students explained it by using the superposition of probability amplitudes (~6% in pre-design). It can thus be concluded that the students deem superposition necessary only in the context of interference, but not as a fundamental principle for the explanation. A deficiency in the organisation of the interview study may additionally support this result. Due to the fact that the two parts of the experiment could not be presented as a closed unit, the particle concept has been supported.

4.6. Limitations

The analysis of the interviews gave valuable insights into the key experiment's impact on the change in rationales for the interpretations of a quantum phenomenon, uncovering the possible need for change of the concept and the organisation of the study. Some limitations of the study should be underlined.

- The size and composition of the sample of the present test are not satisfactory for obtaining robust results which answer the research questions. However, we received strong suggestions regarding a redesign of the questionnaire (items concerning the argumentation with the superposition of states must be improved).

- Perhaps the most important limitation is the design of this study. Because of the missing control group, the observed effect cannot be attributed to the treatment without uncertainty. Thus, only evidence-based suppositions about the key experiment's impact on students learning quantum reasoning can be derived [52]. On the other hand, one group designed studies seem to be suitable for gathering fruitful hypotheses about a treatment's effect and can therefore be starting points for subsequent studies (see, e.g., [53]).
- Due to pandemic conditions requiring social distancing, the students had no chance to really engage with the experiment. Instead, they were reliant on a digital version of the set up. It is to be expected that the impact of the experiment was drastically lowered due to this shortcoming. For this reason, its comparability with other studies is assumed to be quite limited and thus omitted in the present paper.
- Finally, the sustainability of the effect can be assumed to be low, due to the singularity and shortness of the intervention (length ~4 h). However, the aim was to get insights into the experiment's effect on learning quantum reasoning. Based on these results, implications can be made for teaching strategies based on key experiments, especially for gaining a more sustainable effect. More research in this field is necessary.

5. Discussion

Quoting [54]: "The universe revealed by modern research on the foundations of quantum mechanics is a strange and wonderful place ... As a matter of fact, our suspicion is that this [how to explaining it, Author 4] will prove to be impossible. For surely, to explain something is to reduce it to what is already known. But it may turn out that we will never be able to reduce the quantum universe to our customary ways of thinking. Perhaps we will have to adjust our ways of thinking to it. Perhaps years from now, people will think in new and unfamiliar ways, ways in which the quantum universe is no longer a challenge, but rather simple everyday reality."

In this paper, we present a rigorous formulated concept for an access to fundamentals of basic quantum phenomena. The argumentation solely relies on a quantum reasoning directly corresponding to a contemporary quantum traits approach to quantum physics [10]: probability, superposition and interference. The mathematics could be restricted to secondary school linear algebra and a straightforward pointer representation for the addition and multiplication of complex numbers. Our concept makes no use of classical argumentation and thus might show that there ought to be no resemblance between the classical and the quantum domain. We were able to show that, if one wants to understand quantum physics one has to be ready for a lane change [8] and that it is not impossible to make that change.

Author Contributions: Conceptualization, R.S. and S.W.; methodology, K.-A.W., M.W. and S.W.; validation, K.-A.W., M.W. and S.W.; formal analysis, R.S. and S.W.; investigation, R.S., S.W., K.-A.W. and M.W.; resources, R.S..; data curation, K.-A.W. and M.W.; writing—original draft preparation, R.S.; writing—review and editing, R.S.; visualization, R.S.; supervision, R.S.; project administration, R.S. and S.W.; funding acquisition, R.S. All authors have read and agreed to the published version of the manuscript.

Funding: We acknowledge financial support from Deutsche Forschungsgemeinschaft/DFG through collaborative research centre/CRC 1227 (Designed Quantum States of Matter/DQ-*mat*), project Ö, Leibniz Universität Hannover, Hannover, Germany.

Institutional Review Board Statement: Ethical review and approval were waived for this study, due to the fact that the study was in accordance with the Local Legislation and Institutional Requirements: Research Funding Principles (https://www.dfg.de/en/research_funding/principles_dfg_funding/research_data/index.html, (accessed on 5 September 2022)), and General Data Protection Regulation (https://www.datenschutz-grundverordnung.eu/wp-content/uploads/2016/04/CONSIL_ST_5419_2016_INIT_EN_TXT.pdf (accessed on 5 September 2022)).

Informed Consent Statement: Informed consent was obtained from all subjects involved in the study.

Data Availability Statement: The data can be obtained upon request from one of authors (M.W.).

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. Pointer Algebra of Quantum States

First let us emphasize that our analysis accepts one limitation that is apparent in high school physics textbooks: The difficile epistemic differences between quantum mechanics (physics of atoms, molecules, solid state bodies) and quantum field theory (physics of many body systems, physics of creation and annihilation of quanta) are mainly ignored. Instead, we have restricted ourselves to an equal footing strategy, relying on a set of basic theoretical traits similarly valid in any case. As explained in Section 2, we have chosen the triple probability-superposition-quantum interference. For more details of quantum theory and a representation of the basic ideas and for the mathematical formalism we refer to standard quantum physics textbooks [55–57].

Appendix A.1. The Geometrical Interpretation of the Phasor q(ϕ)

The phasors which occur as q-factor constitute a concept of invaluable importance in science and technology. Figures A1–A3 picture a geometrical representation of electrical oscillations by a rotating phasor. Figure A2 demonstrates how to get the usual trigonometrical term from the projection on the abscissa. The electrical field strength of an oscillating field, $E_0 \times \cos \omega t$, can be derived from two opposite rotating phasors, E^+ and E^-:

$$E(t) = E_0 \cdot \cos \omega t = \frac{1}{2}\left(E^+ + E^-\right). \tag{A1}$$

To facilitate the solution of problems in the physics of oscillations and waves, phasors are used instead of the trigonometrical functions. A generalization of the idea draws phasors of length $|q| = 1$ and a phase angle ϕ (Figure A3), depending on the system under observation. It is straightforward to deduce calculation rules for $q(\phi)$ with $|q(\phi)| = 1$ (e.g., from the definition $q(\phi) = \exp(\mathrm{i}\,\phi)$) (Table A1).

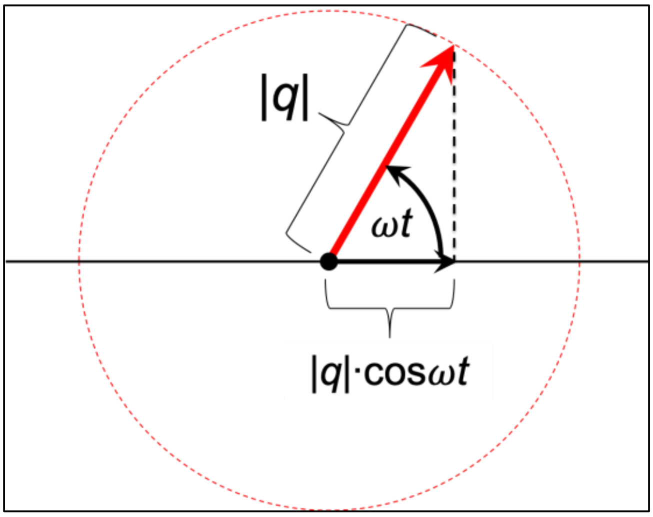


Figure A1. Phasor representation of an oscillation $q \cdot \cos\omega t$.

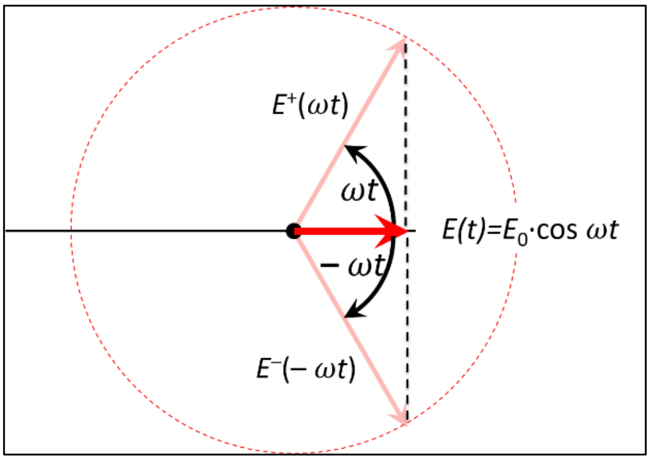


Figure A2. Phasor representation of an oscillating field.

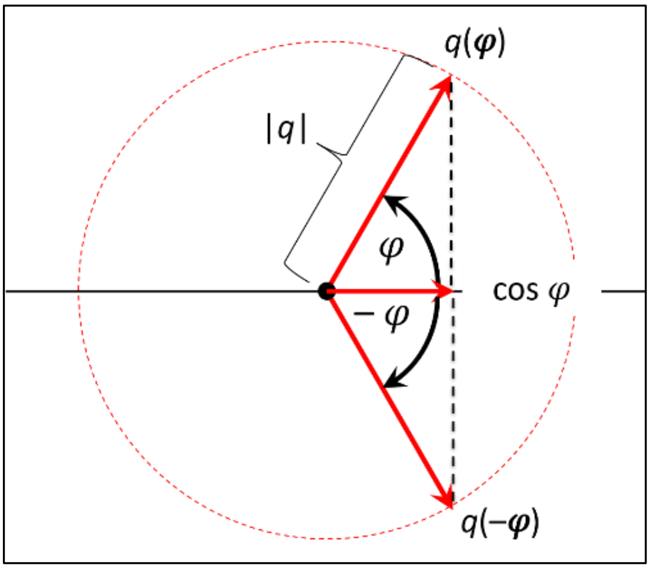


Figure A3. Generalized phasors.

Table A1. Calculation rules for the phasors $q(\phi)$.

Operation	$q(\phi)$-algebra
Multiplication of $q(\phi)$ (geometrical interpreted as a pointer rotation)	$q(\varphi_1)\cdot q(\varphi_2) = q(\varphi_1+\varphi_2)$; $q^2(\varphi) = q(2\varphi)$
Some special values of $q(\phi)$	$q(0) = 1$, $q(\pi) = -1$
The absolute squared value	$\lvert q(\varphi)\rvert^2 = q(\varphi)\cdot q(-\varphi) = q(0) = 1$
The values for negative phase angles	$q(-\varphi) = 1/q(\varphi)$
Addition of $q(\phi)$	$q(\varphi_1)+q(\varphi_2) = 2\cdot\cos((\varphi_1+\varphi_2)/2)\cdot\cos((\varphi_1-\varphi_2)/2) \Rightarrow$ $q(\varphi)+q(-\varphi) = 2\cos(\varphi)$ and $q(\varphi)-q(-\varphi) = 2\cdot q(\pi/2)\cdot\sin(\varphi)$

Appendix A.2. Pointers and Quantum States

In the quantum domain the factor $q(\phi)$ carries the complete phase information of the quantum state. As stated above we have $q(\phi) = \exp(\mathrm{i}\,\phi)$ and thus $q(\phi)$ can be viewed as the algebraic version of the pointer representation of $\exp(\mathrm{i}\,\phi) = \cos\phi + \mathrm{i}\cdot\sin\phi$. In this sense Equation (A1) is close to Feynman's pointer representation of quantum electrodynamics and has proven extremely useful for an educational approach to the physics of the interaction of light and matter [58]. Furthermore, the angle ϕ makes it possible to give an illustrative interpretation of the quantum theoretical scattering amplitude of photons by atoms. The probability amplitude of detecting the photon by a detector placed at distance d from the atom and with c as the velocity of light ([55], p. 158):

$$f(d,t) \propto \exp\left[-i\omega\left(t - \frac{d}{c}\right)\right]. \quad \text{(A2)}$$

Here, we demonstrate the application of pointers for our key experiment.

- Quantum states are represented by pointers. Positive real numbers by pointers with phase 0 (3 o'clock position) negative ones with phase π. Phases between 0 and π belong to numbers with an imaginary part.
- Here, photons are basic energy quanta populating the energy states of physical systems. The evolution of the quantum states of photons is modelled by the rotation of the pointers (Equation (A2)).
- The length of the pointer is a measure of the expected number of clicks of the detection set up (considered proportional to the number of incident photons); the phase of the state is identical to the phase angle of the pointer.
- Pointer rules (how to add and multiply them) and the calculation of the area of the square, sided by the pointer length, transform Born's rule (see [59]) into the pointer domain.
- The pointer length corresponds to $\sqrt{p}$. The square area sided by the pointer length thus gives p, the probability for a de-tector click.
- The algebraic representation of the pointer is given by $\sqrt{p} \cdot q(\varphi) \rightarrow \text{pointer}(|\psi\rangle)$

Appendix A.3. Counting Single Photons—Coincident Clicks

One of the most important optical components of quantum optics is the optical beam splitter (Figure A4). Textbooks on quantum optics discuss it widely (see, e.g., [60]). Here, we underline the "key model perspective": Optical beam splitters can be understood with our reduced model and suitable for performing experiments to uncover the quantum character of single photon states.

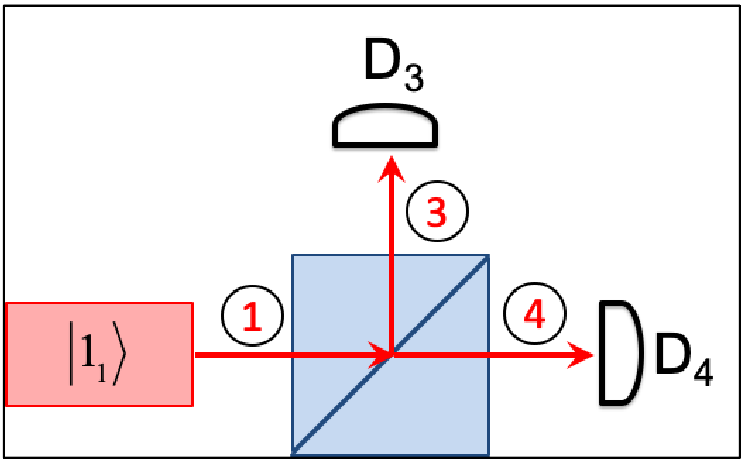


Figure A4. The optical beam splitter. $|1_1\rangle$ describes the Fock state at the entrance mode of the beam splitter. See Figure 2a for more details.

The binary detection scheme is realized, using quite low intensities of light. The binary detectors will only produce a voltage pulse, a "click" or not. For the analysis one usually estimates click probabilities from the numbers of clicks detected, N_i. The temporal length of a total measurement cycle may be denoted by T (e.g., 1 s); Δw (e.g., 5 ns) giving the temporal width resolving the minimum time between two different clicks (the so-called coincidence window) and, leading to a maximum number of counts $N_{\max} = T/\Delta w$ (e.g., 2×10^8 counts). From the Laplacian definition we find the counting probabilities:

$$P_3 = \frac{N_3}{N_{\max}} = N_3 \cdot \frac{\Delta w_c}{T};\ P_4 = \frac{N_4}{N_{\max}} = N_4 \cdot \frac{\Delta w_c}{T};\ P_c = \frac{N_c}{N_{\max}} = N_c \cdot \frac{\Delta w_c}{T} \Rightarrow$$
$$\alpha = \frac{P_c}{P_3 \cdot P_4} = \frac{N_c}{N_3 \cdot N_4} N_{\max} = \frac{N_c}{N_3 \cdot N_4}\left(\frac{T}{\Delta w_c}\right). \tag{A3}$$

where N_c is the number of coincident clicks of detectors D_3 and D_4. As above we introduced the ratio $\alpha = P(D_3 \,\&\, D_4)/[(P(D_3) \times P(D_4)]$.

Classical light with constant intensity I_0 is split into two beams, each carrying half of the energy (equal probability beam splitter). The detector will thus register $I_0/2$ from the incident irradiance I_0. Let us assume, the probability of a "click" being proportional to the light intensity (for small intensity) and to the sampling time Δt of the detector:

$$P_i = \eta_i \cdot I \cdot \Delta t,\ i = 3,\ 4. \tag{A4}$$

The probability of the two detectors clicking coincidentally is then given by

$$P_c = \eta_3\eta_4 \cdot I^2 \cdot (\Delta t)^2 \Rightarrow \alpha = \frac{P_c}{P_3 P_4} = \frac{\eta_3\eta_4 \cdot I^2 \cdot (\Delta t)^2.}{\eta_3 \cdot I \cdot \Delta t \cdot \eta_4 \cdot I \cdot \Delta t} = 1. \tag{A5}$$

Appendix A.4. Single Photons Interacting with Beam Splitters

Experimentally one finds (Figure 2/Section 3) that the probability of coincident clicks from detector D_3 and D_4, $P(D_3 \,\&\, D_4)$, vanishes and thus the substates are incompatible [8]. Quantum reasoning is now used for an explanation of this result.

Appendix A.4.1. Preparation of the Quantum States

The physical system of a single photon populating the output mode of a beam splitter can be described by a two-dimensional Hilbert space with the natural basis of a photon at beam splitter mode ③ and non at the mode ④ (see Figure A4): $|1_3\rangle = |1_3, 0_4\rangle$ and vice versa: $|1_4\rangle = |0_3, 1_4\rangle$. For the transformation between input and output we have to take into account two different possibilities, transmission and reflection, each with a particular probability and phase. This gives the recipe for preparing the output state.

The impact of a symmetrical beam splitter is described by one probability coefficient for the reflection, $r = \sqrt{p_r}q(\varphi_r)$, and another for the transmission, $t = \sqrt{p_t}q(\varphi_t)$, with $p_{r,t}$ as real probabilities and the phase factors $q(\phi)$ taking into account any phase jumps due to reflection or transmission. The input state is transformed by the beam splitter into a superposition of the substates {single photon state at beam splitter mode ③)} and {single photon state at beam splitter mode ④)}

$$|\psi_{\text{in}}\rangle \to |\psi_{\text{out}}\rangle = (\sqrt{p_r}q(\varphi_r) + \sqrt{p_t}q(\varphi_t))|1_3\rangle + (\sqrt{p_t}q(\varphi_t) + \sqrt{p_r}q(\varphi_r))|1_4\rangle. \tag{A6}$$

Checking for orthonormality fixes the meaning of p_r and p_t as probabilities and the values of the phase jumps. With $q(\phi_r - \phi_r) = q(\phi_t - \phi_t) = q(0) = 1$ we get $\phi_r - \phi_t = \pi/2$ [8]. Here, we use $\phi_r = \pi/2$ and $\phi_t = 0$. Inserting further conditions of an equal probability beam splitter ($p_r = p_t = 0.5$) we get the quantum states of the beam splitter experiment (see Table A2).

Figure A5 illustrates this transformation for the single photon incident in mode ①. The matrix $\hat{B}$ symbolizes the transformation by the beam splitter.

Table A2. Preparation of the quantum state at the bare beam splitter. See text and Figure A4 for details.

Operation	System State
Input: single photon state at the input mode ①	$\vert\psi_{\text{in}}\rangle = \vert 1_1, 0_2\rangle = \vert 1_1\rangle$
Output: a superposition of the output modes ③ and ④	$\vert\psi_{\text{out}}\rangle = \frac{1}{\sqrt{2}}(q(\pi/2)\vert 1_3\rangle + \vert 1_4\rangle)$

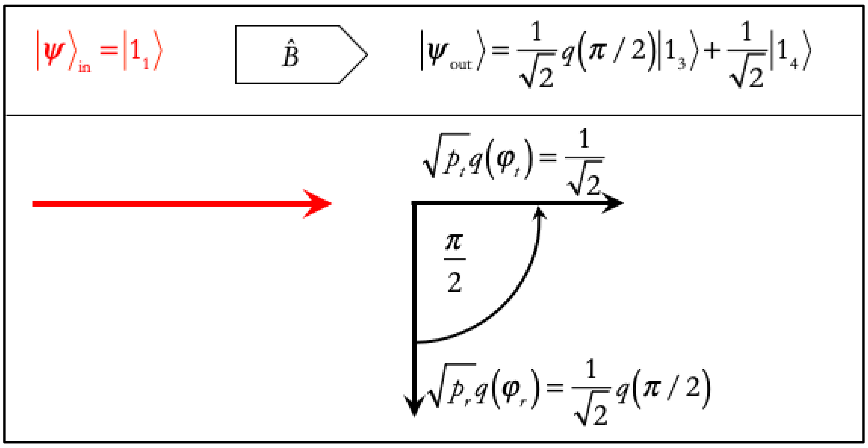


Figure A5. Pointer representation of output state superposition $|\psi_{\text{out}}\rangle$; the transformation $|\psi_{\text{in}}\rangle \rightarrow |\psi_{\text{out}}\rangle$ is mediated by the matrix $\hat{B}$.

Appendix A.4.2. Detection

The probabilities to find the photon in the output substates $|1_3\rangle$ or $|1_4\rangle$ are given by [59]:

$$\begin{aligned} P(|1_3\rangle) &= |\langle 1_3|\psi_{\text{out}}\rangle|^2 = \tfrac{1}{2}\left|q\left(-\tfrac{\pi}{2}\right)\langle 1_3|1_3\rangle + \langle 1_3|1_4\rangle\right|^2 = \tfrac{1}{2}\left|q\left(-\tfrac{\pi}{2}\right)\right|^2 = \tfrac{1}{2}, \\ P(|1_4\rangle) &= |\langle 1_4|\psi_{\text{out}}\rangle|^2 = \tfrac{1}{2}\left|q\left(-\tfrac{\pi}{2}\right)\langle 1_4|1_3\rangle + \langle 1_4|1_4\rangle\right|^2 = \tfrac{1}{2}. \end{aligned} \tag{A7}$$

The interpretation is straightforward: one finds $P(|1_3\rangle) + P(|1_4\rangle) = 1$; the input photon will be detected from D_3 or D_4. This explains why coincident counts are not to be expected. This result, a lack of coincident counts, given by $P(D_3 \,\&\, D_4) = 0$, is shown in Figure 2b. The result indicates that the source produces single-photon states.

Appendix A.5. Single Photons Interacting with a Michelson Interferometer

In Equation (A7) a phase difference of $\pi/2$ is noticeable between the two superposed states $|1_3\rangle$ and $|1_4\rangle$. A phase difference between superposed substates will lead to interference fringes if one removes the spatial/temporal separation of the substates and detecting both states simultaneously superposed on one detector. Two further mirrors help.

The result is shown in Figure 3: Depending on the position of the mirror M_1 one gets interference fringes. To minimize noise, we measured the number N_{G2} of coincidence clicks of detector D_2 and a single photon trigger detector D_G, thus ensuring that only a single photon state is incident on the beam splitter (D_G is not shown in Figure 3). The visibility of the interference pattern is convincingly high. Inserting experimental data, for the visibility we get

$$V = \frac{N_{G2}(\max) - N_{G2}(\min)}{N_{G2}(\max) + N_{G2}(\min)} = \frac{3982}{4234} = 0.94. \tag{A8}$$

Appendix A.5.1. Preparation of the Quantum States

The operator procedure demonstrated in Equation (A6) is used here too. Again, the starting point is a single photon state at mode ① of the beam splitter. The impact of the beam splitting is thought to be the same as before. We thus can write down the total transformation chain for deriving the output state (see Table A3):

Table A3. Preparation of the quantum state at the interferometer.

Operation	System State	
Input	$\vert\psi_{\text{in}}\rangle = \vert 1_1\rangle$	
Beam splitting 1-step	$\vert\psi_{\text{out}/1}\rangle = \frac{1}{\sqrt{2}}\left(q\left(\frac{\pi}{2}\right)\vert 1_3\rangle + \vert 1_4\rangle\right)$	
Phase shift ϕ_M due to mirror 1 and mirror 2	$\vert\psi_{\text{out}/2}\rangle = \frac{1}{\sqrt{2}}\left(q\left(\frac{\pi}{2}\right)q(\varphi_{M2})\vert 1_3\rangle + q(\varphi_{M1})\vert 1_4\rangle\right)$	
Beam splitting 2-step	$\vert\psi_{\text{out}}\rangle = \frac{1}{2}\left[\begin{array}{c}(q(\pi)q(\varphi_{M2}) + q(\varphi_{M1}))\vert 1_1\rangle + \\ q\left(\frac{\pi}{2}\right)(q(\varphi_{M2}) + q(\varphi_{M1}))\vert 1_2\rangle\end{array}\right]$	(A9)

Appendix A.5.2. Detection

Again we find the probability to detect photons at mode ① and mode ② (see Figure A6) using Born's rule,

$$\begin{aligned} P(\text{mode }(2)) = |\langle 1_2|\psi_{\text{out}}\rangle|^2 = & \ \frac{1}{4}\left|\begin{array}{l}(q(\pi)q(\varphi_3)+q(\varphi_4))\underbrace{\langle 1_2|1_1\rangle}_{0}+ \\ q\left(\frac{\pi}{2}\right)(q(\varphi_3)+q(\varphi_4))\underbrace{\langle 1_2|1_2\rangle}_{1}\end{array}\right|^2 = \frac{1}{4}[(q(-\varphi_3)+q(-\varphi_4))][(q(\varphi_3)+q(\varphi_4))] \\ & = \frac{1}{2}(1+\cos(\varphi_3-\varphi_4)) = \cos^2\left(\frac{\varphi_3-\varphi_4}{2}\right), \\ P(\text{mode }(1)) = |\langle 1_1|\psi_{\text{out}}\rangle|^2 = & \ \ldots = \sin^2\left(\frac{\varphi_3-\varphi_4}{2}\right). \end{aligned} \tag{A10}$$

We assume the interferometer is adjusted so that the state evolution between beam splitter and the mirrors leads to $\omega \times d/c$ =2π (see Equation (A2)). The displacement of mirror 1 adds a phase $\Delta\phi$, thus ϕ_4 = 2π + $\Delta\phi$:

This results expectedly gives the quantum interference pattern from Figure 3b. Quantum interference means, interference without waves. The experiment demonstrates transparent experimental evidence of the phase sensitivity of the probability amplitudes. The total probability, summed over the output modes equals one, as it should be. The pointer representation of Equation (A9) (Figure A6) illustrates this result: Adding the pointers gives the total quantum state, calculating the square of the pointer length gives the probability.

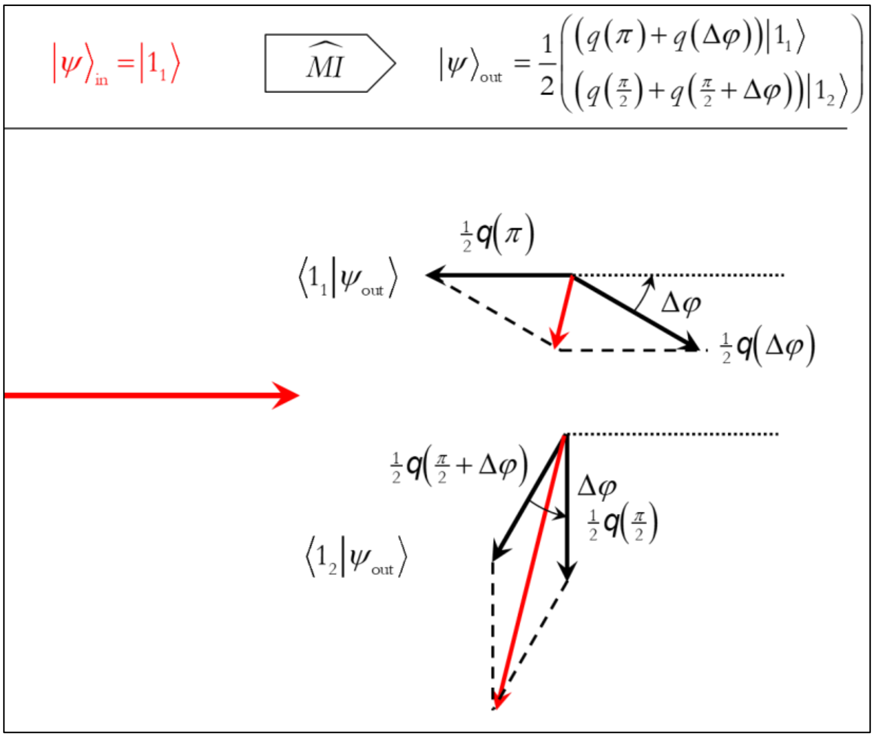


Figure A6. Pointer representation of single photon interference from a Michelson interferometer the transformation $|\psi_{in}\rangle \rightarrow |\psi_{out}\rangle$ is mediated by the matrix $\hat{M}i$.

References

1. Glauber, R. Coherent and incoherent states of the radiation field. *Phys. Rev.* **1963**, *131*, 2766–2788. [CrossRef]
2. Koopman, L.; Brouwer, N.; Heck, A.; Buma, W.J. Remedial mathematics for quantum chemistry. *J. Chem. Educ.* **2008**, *85*, 1233–1236. [CrossRef]
3. Müller, R.; Wiesner, H. Teaching quantum mechanics on an introductory level. *Am. J. Phys.* **2002**, *70*, 200–209. [CrossRef]
4. Dehlinger, D.; Mitchell, M.W. Entangled photon apparatus for the undergraduate laboratory. *Am. J. Phys.* **2002**, *70*, 898–901. [CrossRef]
5. Dehlinger, D.; Michtell, M.W. Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory. *Am. J. Phys.* **2002**, *70*, 903–910. [CrossRef]
6. Galvez, E.J.; Holbrow, C.H.; Pysher, M.J.; Martin, J.W.; Courtemanche, N.; Heilig, L.; Spencer, J. Interference with correlated photons: Five quantum mechanics experiments for undergraduates. *Am. J. Phys.* **2005**, *73*, 127–140. [CrossRef]
7. Thorn, J.J.; Neel, M.S.; Donato, V.W.; Bergreen, G.S.; Davies, R.E.; Beck, M. Observing the quantum behaviour of light in an undergraduate laboratory. *Am. J. Phys.* **2004**, *72*, 1210–1219. [CrossRef]
8. Scholz, R.; Wessnigk, S.; Weber, K.-A. A classical to quantum transition via key experiments. *Eur. J. Phys.* **2020**, *41*, 055304. [CrossRef]
9. Kuhlmann, M. Interpretationen der Quantenfeldtheorie. In *Philosophie der Physik.*; Elsfeld, M., Ed.; Suhrkamp: Berlin, Germany, 2012.
10. Bitzenbauer, P.; Meyn, J.P. A new teaching concept on quantum physics in secondary schools. *Phys. Educ.* **2020**, *55*, 055031. [CrossRef]
11. Krijtenburg-Lewerissa, K.; Pol, H.J.; Brinkman, A.; van Joolingen, W.R. Insights into teaching quantum mechanics in secondary and lower undergraduate education. *Phys. Rev. ST Phys. Educ. Res.* **2017**, *13*, 010109. [CrossRef]
12. Kidd, R.; Ardini, J.; Anton, A. Evolution of the modern photon. *Am. J. Phys.* **1989**, *57*, 27–35. [CrossRef]
13. Muthukrishan, A.; Scully, O.M.; Zubairy, M.S. The concept of the photon—Revisited. In *The Nature of Light*; Roychoudhuri, C., Kracklauer, A.F., Creath, K., Eds.; CRC Press: Boca Raton, FL, USA, 2008; Chapter 4. [CrossRef]
14. Hong, C.K.; Ou, Z.Y.; Mandel, L. Measurement of subpicosecond time intervals between two photons by interference. *Phys. Rev. Lett.* **1987**, *59*, 2044–2046. [CrossRef] [PubMed]
15. Quantum Flagship. The future is quantum. Available online: https://qt.eu (accessed on 1 September 2022).
16. Masshadi, A.; Woolnough, B. Insights into students' understanding of quantum physics: Visualizing quantum entities. *Eur. J. Phys.* **1999**, *20*, 511–516. [CrossRef]
17. Michelini, M.G.; Ragazzon, R.; Santi, L.; Stefanel, A. Proposal for quantum physics in secondary school. *Phys. Educ.* **2000**, *35*, 406–410. [CrossRef]

18. Escalada, L.T.; Rebello, N.S.; Zollman, D.A. Student explorations of quantum effects in LEDs and luminescent devices. *Phys. Teach.* **2004**, *42*, 173–179. [CrossRef]
19. Hoekzema, D.; van den Berg, E.; Schooten, G.; van Dijk, L. The particle/wave-in-a-boxmodel in Dutch secondary schools. *Phys. Educ.* **2007**, *42*, 391–398. [CrossRef]
20. Lautesse, P.; Vila Valls, A.; Ferlin, F.; Héraud, J.L.; Chabot, H. Teaching quantum physics in upper secondary school in France. *Sci. Educ.* **2015**, *24*, 937–955. [CrossRef]
21. Henriksen, E.K.; Bungum, B.; Angell, C.; Tellefsen, C.W.; Fragat, T.; Vetleseter Bøe, M. Relativity, quantum physics and philosophy in the upper secondary curriculum: Challenges, opportunities and proposed approaches. *Phys. Educ.* **2014**, *49*, 678–684. [CrossRef]
22. Karakostas, V.; Hadzidaki, P. Realism vs. constructivism in contemporary physics: The impact of the debate on the understanding of quantum theory and its instructional process. *Sci. Educ.* **2005**, *14*, 607–629. [CrossRef]
23. Kalkanis, G.; Hadzidaki, P.; Stavrou, D. An instructional model for a radical conceptual change towards quantum mechanics concepts. *Sci. Educ.* **2003**, *87*, 257. [CrossRef]
24. Chi, M.T.H. Two kinds and Four sub-types of misconceived knowledge, ways to change it, and the learning outcomes. In *International Handbook of Research on Conceptual Change*; Vosniadou, S., Ed.; Routledge: New York, NY, USA; London, UK, 2013; Chapter 3. Available online: https://www.routledgehandbooks.com/doi/10.4324/9780203154472.ch3 (accessed on 1 September 2022).
25. Mortimer, E.F. Conceptual change or conceptual profile change? *Sci. Educ.* **1995**, *4*, 267–285. [CrossRef]
26. Waitzmann, M. Wirkung Eines Quantenoptischen Arguments auf Quantenphysikalische Argumentation. Ph.D. Thesis, Leibniz University Hannover, Hanover, Germany, 2022.
27. Posner, G.J.; Strike, K.A.; Hewson, P.W.; Gertzog, W.A. Accommodation of a scientific conception: Toward a theory of conceptual change. *Sc. Edu.* **1982**, *66*, 211–227. [CrossRef]
28. diSessa, A.A. Toward an epistemology of physics. *Cogn. Instr.* **1993**, *10*, 165–255. [CrossRef]
29. diSessa, A.A. Conceptual change in a microcosm: Comparative learning analysis of a learning event. *Hum. Dev.* **2017**, *60*, 1–37. [CrossRef]
30. McNeill, K.L.; Krajcik, J. Inquiry and scientific explanations: Helping students use evidence and reasoning. In *Science as Inquiry in the Secondary Setting*; Luft, J., Bell, R., Gess-Newsome, J., Eds.; NSTA Press: Arlington, VA, USA, 2008; Chapter 11. Available online: https://dpi.wi.gov/sites/default/files/imce/science/Inquiry_and_Scientific_Explanations_McNeill-Krajcik.pdf (accessed on 1 September 2022).
31. Chiara, M.; Giuntini, R.; Greechie, R. *Reasoning in Quantum Theory. Sharp and Unsharp Quantum Logics*; Springer Science + Business Media: Dordrecht, The Netherlands, 2004. [CrossRef]
32. Küblbeck, J.; Müller, R. *Die Wesenszuege der Quantenphysik: Modelle, Bilder und Experimente*; Aulis Verlag Duebne GmbH & Co KG: Köln, 2003. Available online: https://docplayer.org/40006159-Die-wesenszuege-der-quantenphysik-modelle-bilder-und-experimente.html (accessed on 1 September 2022).
33. Friebe, C.; Kuhlmann, M.; Lyre, H.; Näger, P.; Passon, O.; Stöckler, M. *Philosophie der Quantenphysik*; Springer Spektrum: Beilin/Heidelberg, Germany, 2015. [CrossRef]
34. Bitzenbauer, P. Effect of an introductory quantum physics course using experiments with heralded photons on preuniversity students' conceptions about quantum physics. *Phys. Rev. ST Phys. Educ. Res.* **2021**, *17*, 020103. [CrossRef]
35. Aguayo, R.; Simms, G.; Siegel, P.B. Throwing nature's dice. *Am. J. Phys.* **1996**, *64*, 752–758. [CrossRef]
36. Bronner, P.; Strunz, A.; Silberhorn, C.; Meyn, J.P. Demonstrating quantum random with single photons. *Eur. J. Phys.* **2009**, *30*, 1189–1200. [CrossRef]
37. Ireson, G. The quantum understanding of pre-university physics students. *Phys. Educ.* **2000**, *35*, 15–21. [CrossRef]
38. Pospiech, G.; Schöne, M. Quantenphysik in Schule und Hochschule. *PhyDid B—Didaktik der Physik—Beiträge zur DPG-Frühjahrstagung*. 2012. *Mainz*, 20.01. Available online: https://ojs.dpg-physik.de/index.php/phydid-b/article/view/392 (accessed on 1 September 2022).
39. Zollman, D.A.; Rebello, N.S.; Hogg, K. Quantum mechanics for everyone: Hands-on activities integrated with technology. *Am. J. Phys.* **2002**, *70*, 252–259. [CrossRef]
40. Bronner, P. *Quantenoptische Experimente als Grundlage eines Curriculums zur Quantenphysik des Photons*; Logos Verlag: Berlin, Germany, 2010. Available online: https://d-nb.info/100496515X (accessed on 1 September 2022).
41. Häußler, P. *Physikalische Bildung: Eine curriculare Delphi-Studie. Teil 1., IPN-Arbeitsbericht 41*; IPN: Kiel, Germany, 1980.
42. Burkard, U.E. *Quantenphysik in der Schule: Bestandsaufnahme, Perspektiven und Weiterentwicklungsmöglichkeiten durch die Implementierung Eines Medienservers*; Logos-Verlag: Berlin, Germany, 2009. Available online: https://www.fachportal-paedagogik.de/literatur/vollanzeige.html?FId=1057488 (accessed on 1 September 2022).
43. Weber, K.-A.; Friege, G.; Scholz, R. Quantenphysik in der Schule—Was benötigen Lehrkräfte?—Ergebnisse einer Delphi-Studie. *Zeit. Didaktik Natur.—ZfDN* **2020**, *26*, 173–190. [CrossRef]
44. Bitzenbauer, P.; Meyn, J.P. Fostering students' conception about the quantum world—Results of an interview study. *Prog. Sc. Ed.* **2021**, *4*, 40–51.
45. Marshman, E.; Singh, C. Framework for understanding the patterns of student difficulties in quantum mechanics. *Phys. Rev. ST Phys. Educ. Res.* **2015**, *11*, 02011. [CrossRef]
46. Ubben, M.; Heusler, S. Gestalt and functionality as independent dimensions of mental models in science. *Res. Sc. Educ.* **2019**, *51*, 1349–1363. [CrossRef]

47. Kral, A.; Theis, C.; Schorn, B.; Heinke, H. Praxistauglicher Einstieg in die Quantenphysik mit Realexperimenten. In *Implementation fachdidaktischer Innovation im Spiegel von Forschung und Praxis. Gesellschaft für Didaktik der Chemie und Physik Jahrestagung in Zürich 2016*; Maurer, C., Ed. Universität Regensburg: Regensburg, Germany, 2017; pp. 400–404.
48. Niebert, K.; Gropengießer, H. Leitfadengestützte Interviews. In *Methoden in der Naturwissenschaftsdidaktischen Forschung*; Krüger, D., Parchmann, I., Schecker, H., Eds.; Springer Spektrum: Heidelberg/Berlin, Germany, 2014; pp. 121–132. [CrossRef]
49. Ayene, M.; Kriek, J.; Damtie, B. Wave-particle duality and uncertainty principle: Phenomenographic categories of description of tertiary physics students' depictions. *Phys. Rev. ST Phys. Educ. Res.* **2011**, *7*, 020113. [CrossRef]
50. Kuckartz, U. *Qualitative Inhaltsanalyse. Methoden, Praxis, Computerunterstützung*; Beltz Juventa: Weinheim, Germany, 2016.
51. Schreier, M. Varianten qualitativer Inhaltsanalyse: Ein Wegweiser im Dickicht der Begrifflichkeiten [59 Absätze]. *Forum Qual. Soc. Res.* **2014**, *15*, 18. [CrossRef]
52. Döring, N.; Bortz, J. *Forschungsmethoden und Evaluation in den Sozial- und Humanwissenschaften*; Springer: Berlin/Heidelberg, Germany, 2016. [CrossRef]
53. Mannheim, E.G.; Helmes, A.; Weis, J. Tanztherapie in der stationären Onkologischen Rehabilitation. *Forsch. Komplement.* **2013**, *20*, 33–41. [CrossRef]
54. Greenstein, G.; Zajonc, A.G. *The Quantum Challenge*; Jones and Barlett Publishers, Inc.: Sudbury, MA, USA, 1997. Available online: https://xn--webducation-dbb.com/the-quantum-challenge-pdf/ (accessed on 1 September 2022).
55. Messiah, A. *Quantum Mechanics*; Dover Publication: New York, NY, USA, 1999.
56. Filk, T. *Quantenmechanik (Nicht nur) für Lehramtsstudierende*; Springer Spektrum: Berlin/Heidelberg, Germany, 2019.
57. Scully, M.O.; Zubairy, M.S. *Quantum Optics*; Cambridge University Press: Cambridge, UK, 1997. [CrossRef]
58. Feynman, R.P.; Zee, A. *QED: The Strange Theory of Light and Matter*; Princeton University Press: Princeton, NJ, USA, 2006.
59. Born, M. Zur Quantenmechanik der Stoßvorgänge. *Z. Für Phys.* **1926**, *37*, 863–867. [CrossRef]
60. Loudon, R. *The Quantum Theory of Light*; Oxford University Press: Oxford, UK, 2000. Available online: http://rplab.ru/~as/2000%20-%20R.Loudon%20-%20The%20Quantum%20Theory%20of%20Light%20-%203rd%20ed%20Oxford%20Science%20Publications.pdf (accessed on 1 September 2022).